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The welfare effects of a partial tariff reduction under domestic distortion in Japan*

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Abstract

We examine a partial tariff reduction policy in a specific factor model with two import-competing sectors. When one of them enjoys a higher tariff and is also subject to a production subsidy, will a tariff reduction in the other import-competing sector be welfare improving? We argue that this second-best type question is highly relevant to Japan, and calibrate our model to its 2013 economy. We find that the policy is undesirable and that even a complete subsidy removal is insufficient to make it desirable. Making it welfare improving requires the other sector's tariff be more than halved.

Keywords: Tariff policy, domestic distortion, welfare, specific factor model, Japanese economy

JEL Classification: F11(Neoclassical Models of Trade), F13(Trade Policy; International Trade Organizations)

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1 Introduction

Japan is one of the world's largest agro-food importing countries and it is well known that significant support has been provided to its producers. According to OECD (2017), the producer support estimates (PSE) for Japanese farmers in 2013 was 52 per cent of gross farm receipts, which was well beyond the PSE in most other OECD countries (see Figure 1). Indeed, Table 1 indicates that high average tariff rates were imposed on Japanese agricultural imports such as Dairy products (116.9 per cent), Cereals and preparations (80.2 per cent), and Sugars and confectionery (50.2 per cent).

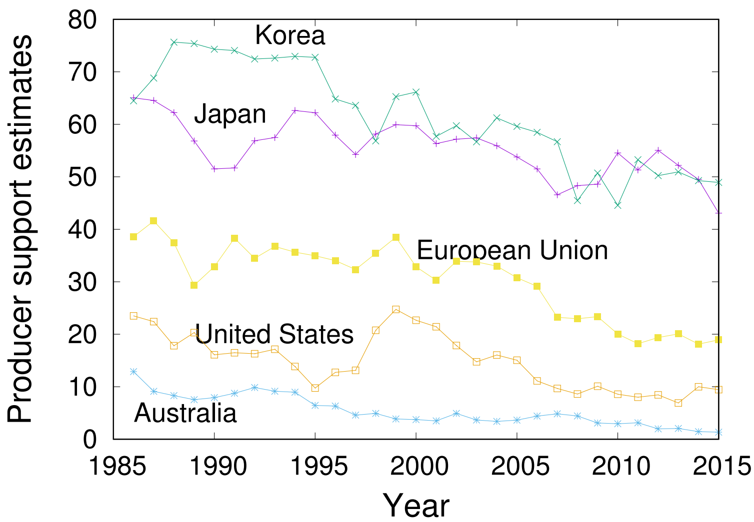

Figure 1: Producer support estimates (% of gross farm receipts, 2013). **Source:** OECD (2017)

Agricultural producer support in Japan is distinctive also in terms of the government subsidy. Table 2 suggests that the government subsidy allocated to the 'Agriculture, forestry and fishery' sector is the third largest after 'Finance, insurance and real estate' and 'Medical service, health, social security and nursing care' sectors. Amongst primary and manufacturing sectors, the subsidy allocated to the 'Agriculture, forestry and fishery' sector exceeds 80 per cent of the total subsidy. On the contrary, protection for other import-competing sectors — *e.g.* 'Textile products' and 'Wearing apparel and other textile products' — have been rather modest in terms of both import tariffs and subsidies.

Product group	Average tariff (%)
Dairy products	116.9
Cereals & preparations	80.2
Sugars and confectionery	50.2
Beverages & tobacco	16.8
Coffee, tea	14.4
Animal products	13.6
⋮	⋮
Clothing	9.2
Textiles	5.6
⋮	⋮

Source: WTO, ITC and UNCTAD (2013)

Table 1: Japan's tariff by product group in 2013

Sector	Subsidy	Ratio
Finance, insurance and real estate	1,011,578	26.09%
Medical service, health, social security and nursing care	906,039	23.37%
Agriculture, forestry and fishery	706,387	18.22%
Other civil construction	275,306	7.10%
Transport and postal service	253,531	6.54%
Water	252,001	6.50%
Other NPO service	125,863	3.25%
Food and beverages	114,399	2.95%
⋮	⋮	⋮
Total	3,877,586	100.00%

Source: Ministry of Economy, Trade and Industry, Japan (2013)

Table 2: Subsidy allocation in Japan by sector in 2013 (in million Japanese Yen)

In the process of the Trans-Pacific Partnership (TPP) negotiations, the Japanese government included the majority of the rice, wheat, meat, dairy products and sugar items in to the exception list. The Japanese public seems to have focussed on discussion as to whether these items should also be liberalised, but little seems to have been discussed as to whether removing tariffs for some items, whilst those for others — typically on which high tariffs are imposed — held intact, will benefit the economy. In this paper, with the above characteristics of the Japanese economy in mind, we wish study the welfare effect of such a partial tariff reduction policy.

Our question involves multiple distortions and hence it is a question under the theory of second best. In their seminal work, Lipsey and Lancaster (1956) have examined a similar question to ours as an example that illustrates the theory of second best, although they do not consider any domestic distortion. A strand of the literature has then followed to examine welfare improving piecemeal policy changes in a more general multi-sector framework.[1] Earlier papers include Foster and Sonnenschein (1970), Bruno (1972) and Hatta (1977a,b) amongst others, which have established the two types of tariff reforms that are welfare improving: (i) lower every distortion in an equal percentage (uniform reduction rule); and (ii) reduce the highest distortion to the next highest (concertina rule). The main results established in the earlier work have been extended by many succeeding papers — Fukushima (1979), Diewert et al. (1991), Turunen-Red and Woodland (1991), Abe (1992), Ju and Krishna (2000) and Anderson and Neary (2007), *etc.* — which have also focussed on sufficient conditions for the welfare improving policy rules. However, not much work has been done in terms of examining the welfare effect of the reform of our interest.

There seem to be two reasons for it. First, since the question of our interest involves multiple distortions, when one of them is reduced (or increased) the welfare effect is generally ambiguous. If one is interested in obtaining a concrete result for a welfare improving policy, it makes sense to limit the analysis to a particular set of policies.

[1]See Corden (1984), Dixit (1985) Vousden (1990) and Falvey and Kreickemeier (2013) for the survey on this topic as well as on trade and domestic distortion.

Second, once a distortion is added to any framework, the mathematical analysis in general becomes extremely complicated than otherwise, even in a standard two-sector model.[2] It is no surprise that classical work that has examined the relationship between trade and domestic distortion relies on graphical analysis (Bhagwati and Ramaswami, 1963; Bhagwati, 1967, for example).

To examine the tariff reform of our interest, we employ a specific factor model (Jones, 1971b; Mussa, 1974) that comprises two import-competing sectors (call them Sectors A and B) and an export sector (call it Sector C). Both import-competing sectors are protected by tariffs but only one of them (Sector A, namely Agriculture) is supported by a production subsidy. Our particular interest is the welfare effect of lowering the Sector B tariff when it is lower than the Sector A tariff. In this case, as has been established in the existing literature the welfare effect is ambiguous, and hence we resort to numerical simulations to understand how the welfare effect of the policy reform of our interest might be affected by consumer preferences and/or production parameters. We employ the specific factor model because (i) the parsimonious nature of the model helps interpret our numerical results clearly; and (ii) we are interested in examining the short-run static effect of a piecemeal tariff reform on resource allocation.

In our numerical simulations we show that there is a following case. A tariff reduction in Sector B lowers economies welfare, but if the Sector A subsidy is sufficiently reduced at the same time, the tariff reduction becomes welfare improving. Given the high tariffs and heavy subsidies in Sector A described previously, it is interesting to investigate if this situation applies to the current Japanese economy. To this end, we calibrate our model to the 2013 Japanese economy using various data. Our calibration result indicates that Sector B's tariff reduction lowers economy's welfare and the direction of the effect does not change even if the Sector A subsidy is reduced to zero. From the viewpoint of static

[2]For example, Johnson (1966) employs a numerical approach to sketch the production possibility frontier for the 2-by-2 model with a domestic distortion. Jones (1971a) provides mathematical explanation on the concavity of the production possibility frontier under the same situation, although complicated expressions make it difficult for us to interpret the results intuitively. Ohyama (1972) provides some mathematical analysis of tariff policies under domestic distortion, but it relies on an *ad hoc* way of modelling the distortion.

resource allocation, our result suggests that the partial tariff removal policy of our focus is undesirable and that it should be accompanied by a significant tariff reduction in Sector A.

Given the calibrated parameters, we also show that the main driving force of the (negative) welfare effect of this partial tariff removal policy is an efficiency loss in production. By lowering the protection in Sector B, more production resource is drawn in to Sector A, which already have had too much resource allocated in the first place. Further production inefficiency created by this policy explains roughly 70 per cent of the overall (negative) welfare effect, whilst the remaining effect can be attributed to an incremental efficiency loss in consumption.

In the following section, we start with the analysis of a two-sector specific factor model that incorporates domestic distortion, which helps understand the effect of a tariff policy under domestic distortion intuitively. In Section 3, we extend our model to three sectors by adding another import-competing sector. We first show that the concertina type tariff reform will increase welfare in our specific factor model setup. Then we conduct numerical simulations to examine how different consumer preferences or production parameters affect the welfare effect of the tariff reform of our interest. In Section 4, we calibrate our model to the Japanese economy and present the results. Section 5 concludes.

2 The two-sector model with a domestic distortion

2.1 The setup

We consider a two-sector specific factor model in a small open economy setting. We assume two goods, Good A and Good C. These goods are produced in two sectors, Sector A and Sector C, respectively. Outputs are denoted as Y_A and Y_C, respectively. International prices of the two goods are denoted as P_A and P_C.

There are two production factors. Labour is mobile across the two sectors, L_A and L_C. Capital in Sector A, K_A, and that in Sector C, K_C, are specific factors. We assume

Cobb-Douglas production technology for both sectors. More specifically,

$$Y_A = L_A^{\alpha} K_A^{1-\alpha}, \tag{1}$$

$$Y_C = L_C^{\gamma} K_C^{1-\gamma}, \tag{2}$$

where $0 < \alpha < 1$ and $0 < \gamma < 1$. The aggregate quantity of labour in the economy is given as $\overline{L}$, and hence:

$$\overline{L} = L_A + L_C. \tag{3}$$

We assume that Sector A is an import-competing sector, which is subject to an ad valorem tariff, $\tau_A \geq 0$, and also whose production is subsidised by the government. More specifically, the Sector A producer receives $\{P_A(1 + \tau_A) + s_A\}$ per unit of production, where s_A is the subsidy per unit of production. Sector C is an export sector.[3]

The profit functions of each sector are:

$$\pi_A = \{P_A(1 + \tau_A) + s_A\}Y_A - wL_A - r_A K_A,$$
$$\pi_C = P_C Y_C - wL_C - r_C K_C,$$

where w, r_A, and r_C are the wage and the rental prices of capital in each sector, respectively. Solving the profit maximisation problems, we obtain the following first-order

[3] $s_A < 0$ corresponds to the case when the Good A production is taxed. Although our interest is the case where $s_A > 0$, we do not impose any condition on its sign as it has an important implication on our discussion in the next section.

conditions.[4]

$$w = \{P_A(1+\tau_A) + s_A\}\alpha L_A^{\alpha-1} K_A^{1-\alpha}, \tag{4}$$

$$w = P_C \gamma L_C^{\gamma-1} K_C^{1-\gamma}, \tag{5}$$

$$r_A = \{P_A(1+\tau_A) + s_A\}(1-\alpha) L_A^{\alpha} K_A^{-\alpha}, \tag{6}$$

$$r_C = P_C(1-\gamma) L_C^{\gamma} K_C^{-\gamma}. \tag{7}$$

The consumer's utility stems from consumption of the two goods, denoted by Q_A and Q_C. We assume a standard Cobb-Douglas utility function as:

$$U(Q_A, Q_C) = Q_A^{\nu} Q_C^{1-\nu}, \tag{8}$$

where $0 < \nu < 1$. Then the consumer's utility maximisation requires:

$$\frac{Q_A}{Q_C} = \frac{\nu}{1-\nu} \frac{P_C}{P_A(1+\tau_A)}. \tag{9}$$

The consumer's budget constraint is given as:

$$P_A(1+\tau_A)Q_A + P_C Q_C = wL_A + wL_C + r_A K_A + r_C K_C - T, \tag{10}$$

where T is a lump-sum tax, which is the difference between the value of Sector A subsidy and the tariff revenue. That relationship is equivalent to the government budget constraint which is given as:

$$T = s_A Y_A - \tau_A P_A (Q_A - Y_A). \tag{11}$$

(1), (2), (4)-(7), (10), and (11) boil down to the balance-of-payment constraint, which is:

$$P_A M_A = P_C X_C, \tag{12}$$

[4]Throughout this paper we omit the second-order sufficient conditions for brevity as all our setups are standard and they are trivially satisfied.

where M_A and X_C are defined as:

$$M_A \equiv Q_A - Y_A, \tag{13}$$

$$X_C \equiv Y_C - Q_C. \tag{14}$$

The equilibrium of the model is characterised by the following equations: (1)-(9) and (12)-(14).

2.2 Welfare effects of a tariff policy

Let us examine the welfare effects of an increase in the import tariff, τ_A.[5] Partially differentiating $U(Q_A, Q_C)$ in (8) with respect to τ_A, we get:

$$\begin{aligned}\frac{\partial U}{\partial \tau_A} &= \nu \left(\frac{Q_A}{Q_C}\right)^{\nu-1} \frac{\partial Q_A}{\partial \tau_A} + (1-\nu)\left(\frac{Q_A}{Q_C}\right)^{\nu} \frac{\partial Q_C}{\partial \tau_A} \\ &= \nu \left\{\frac{\nu}{\nu-1}\frac{P_C}{P_A(1+\tau_A)}\right\}^{\nu-1} \frac{\partial Q_A}{\partial \tau_A} + (1-\nu)\left\{\frac{\nu}{1-\nu}\frac{P_C}{P_A(1+\tau_A)}\right\}^{\nu} \frac{\partial Q_C}{\partial \tau_A}. \end{aligned} \tag{15}$$

The second equality follows from (9). The close examination of various terms leads to the following proposition.

Proposition 1. *In the two-sector model, an increase in the import tariff reduces welfare for any (non-negative) level of government subsidy.*

Proof. We show that $\frac{\partial U}{\partial \tau_A} < 0$ for any $s_A \geq 0$ in Appendix B. □

There is a simple intuition behind this result. Initially, the economy is distorted by the two policies: the import tariff, τ_A, and the government subsidy, s_A. Both policies distort production by encouraging labour to be allocated in Sector A than otherwise, where the former also distorts consumption by affecting the consumer relative price. A further increase in the import tariff works to distort both production and consumption

[5] Although our focus is the welfare effect of a *reduction* in a tariff, in the most of the text we will discuss the opposite, *i.e.* the welfare effect of an *increase* in a tariff. Focussing on the latter is less confusing as it is what the (partial) derivative of the maximised utility with respect to a tariff in question means.

in the same direction, in which case, it should have a negative welfare effect. Of course, as in the argument for protection *à* la Hagen (1958), an increase in τ_A could be welfare improving if it alleviates an existing distortion. In our context, if s_A was (very) negative such that the initial production was biased towards Sector C, a marginal increase in τ_A would lead to an efficiency gain in production, and could increase economy's welfare so long as an efficiency loss in consumption it created was smaller.

3 A model with two import-competing sectors

3.1 The setup

We add another import-competing sector, Sector B, to the two-sector model we developed in the previous section. Good B is produced in this sector, which is denoted as Y_B. The government protects this sector by imposing an import tariff, $\tau_B \geq 0$, however, there is no production subsidy in this sector (recall Sector A is subject to both a tariff and a subsidy). For this sector, the specific factor and its price are denoted as K_B and r_B. Labour allocated to this sector is L_B.

The production function of Sector B is given as:

$$Y_B = L_B^{\beta} K_B^{1-\beta}, \tag{16}$$

where $0 < \beta < 1$.

The Sector B producer's profit maximisation requires the following first-order conditions be met.

$$w = P_B(1+\tau_B)\beta L_B^{\beta-1} K_B^{1-\beta}, \tag{17}$$

$$r_B = P_B(1+\tau_B)(1-\beta) L_B^{\beta} K_B^{-\beta}. \tag{18}$$

The labour market clearing condition is modified as:

$$L_A + L_B + L_C = \overline{L}. \tag{19}$$

We keep assuming Cobb-Douglas preferences for the consumer. Denoting the consumption of Good B as Q_B, the utility function is now given as:

$$U(Q_A, Q_B, Q_C) = Q_A^{\nu} Q_B^{\phi} Q_C^{1-\nu-\phi}, \tag{20}$$

where $0 < \nu < 1$, $0 < \phi < 1$, and $0 < \nu+\phi < 1$. Then the consumer's utility maximisation requires:

$$\frac{Q_A}{Q_B} = \frac{\nu}{\phi}\frac{P_B(1+\tau_B)}{P_A(1+\tau_A)}, \tag{21}$$

$$\frac{Q_B}{Q_C} = \frac{\phi}{1-\nu-\phi}\frac{P_C}{P_B(1+\tau_B)}. \tag{22}$$

The balance-of-payment constraint is now modified as:

$$P_A M_A + P_B M_B = P_C X_C, \tag{23}$$

where

$$M_B \equiv Q_B - Y_B. \tag{24}$$

The equilibrium of this economy is characterised by the following equations: (1), (2), (4)-(7), (13), (14), and (16)-(24).

3.2 Welfare effects of a tariff policy

Let us consider an increase in one of the tariffs and see how it affects economic welfare. As is well known, in the second best world, the welfare effect of magnifying (or alleviating) one distortion is generally ambiguous. However, in our model under a certain set of parameters, we can show that an increase in a tariff lowers economic welfare unambiguously. Unsurprisingly, when we set domestic distortion (subsidy) to zero, our analytical result conforms to the concertina rule.

Of course, the reduction in economic welfare may occur even when these sufficient conditions are not met, and indeed, so far as the tariff reform of our interest is concerned, unfortunately we cannot rely on these conditions. Therefore, to study the welfare effect of a tariff policy in general, we then conduct some numerical simulations. It allows us to uncover how various parameter values, including s_A, relate to the welfare effect of a tariff policy.

3.2.1 Analytical results

First, we consider an increase in τ_A. Taking the partial derivative of $U(Q_A, Q_B, Q_C)$ in (20) with respect to τ_A, we obtain:

$$\frac{\partial U}{\partial \tau_A} = Q_A^{\nu} Q_B^{\phi} Q_C^{1-\nu-\phi} \left\{ \nu \frac{1}{Q_A} \frac{\partial Q_A}{\partial \tau_A} + \phi \frac{1}{Q_B} \frac{\partial Q_B}{\partial \tau_A} + (1-\nu-\phi) \frac{1}{Q_C} \frac{\partial Q_C}{\partial \tau_A} \right\}. \tag{25}$$

It turns out that there are sufficient conditions for the sign of this partial derivative to be negative.

Proposition 2. *For any s_A,*

$$\frac{\partial U}{\partial \tau_A} < 0 \quad \textit{if} \quad \phi(\tau_A - \tau_B) + (1-\nu-\phi)\tau_A(1+\tau_B) \geq 0 \quad \textit{and} \quad \tau_A + \frac{s_A}{P_A} \geq \tau_B.$$

Proof. See Appendix C. □

Proposition 2 simply states that if a marginal increase in τ_A magnifies both consumption and production inefficiencies it surely harms economy. The left hand side of the first inequality, $\phi(\tau_A - \tau_B) + (1-\nu-\phi)\tau_A(1+\tau_B)$, represents an incremental consumption inefficiency caused by an increase in τ_A. It shows that, so long as $\tau_A, \tau_B > 0$, a marginal increase in τ_A always leads to a further over-consumption of Good C, and if $\tau_A > \tau_B$ Good B's over-consumption is also enlarged.

The second inequality, $\tau_A + \frac{s_A}{P_A} \geq \tau_B$, relates to a production inefficiency. The two sides of the inequality represent the protection levels for Sector A and Sector B, respectively. Even if $\tau_A > \tau_B$, a high enough production tax in Sector A, $s_A < 0$, may make Sector B's protection greater. Put it differently, even if $\tau_B > \tau_A$, a high enough production subsidy in Sector A, $s_A > 0$, can make Sector A a more protected sector. Hence, this part of proposition essentially says that, so long as Sector A is protected no less than Sector B, an increase in τ_A leads to a further inefficiency in production.

Our interest is the case where $s_A \geq 0$, which can be summarised in the following corollary.

Corollary 1. *For any* $s_A \geq 0$,

$$\frac{\partial U}{\partial \tau_A} < 0 \quad \textit{if} \quad \tau_A \geq \tau_B.$$

Since $s_A \geq 0$, so long as $\tau_A \geq \tau_B$, Sector A is no less protected than Sector B. Also, $\tau_A \geq \tau_B$ implies that the first inequality condition in Proposition 2 will be trivially met. It is because a marginal increase in τ_A magnifies an inefficiency in consumption as it leads to under consumption in both Good B and Good C so long as $\tau_A \geq \tau_B > 0$. Therefore it suffices to have $\tau_A \geq \tau_B$ for a marginal increase in τ_A to be harmful when $s_A \geq 0$.

A special case of Corollary 1 is when $s_A = 0$. It is the case under which the two tariffs are the only distortions. In this case, $\frac{\partial U}{\partial \tau_A} < 0$ if $\tau_A \geq \tau_B$, which means that a marginal increase in the highest tariff reduces the welfare of the economy. Hence, we have demonstrated the well-known concertina rule (*e.g.* Hatta, 1977a) holds in our specific factor model with two import-competing sectors.

An increase in τ_B can be examined in a similar fashion. Taking the partial derivative of $U(Q_A, Q_B, Q_C)$ in (20) with respect to τ_B,

$$\frac{\partial U}{\partial \tau_B} = Q_A^{\nu} Q_B^{\phi} Q_C^{1-\nu-\phi} \left\{ \nu \frac{1}{Q_A} \frac{\partial Q_A}{\partial \tau_B} + \phi \frac{1}{Q_B} \frac{\partial Q_B}{\partial \tau_B} + (1-\nu-\phi) \frac{1}{Q_C} \frac{\partial Q_C}{\partial \tau_B} \right\}. \tag{26}$$

The next proposition states the sufficient condition for an increase in τ_B to be harmful.

Proposition 3. *For any s_A,*

$$\frac{\partial U}{\partial \tau_B} < 0 \quad \textit{if} \quad \nu(\tau_B - \tau_A) + (1 - \nu - \phi)(1 + \tau_A)\tau_B \geq 0 \quad \textit{and} \quad \tau_B \geq \tau_A + \frac{s_A}{P_A}.$$

Proof. See Appendix D. □

Proposition 3 has the exactly same interpretation as Proposition 2. Namely, it is sufficient for the economic welfare to fall if the rise in τ_B does not create any efficiency gain in consumption ($\nu(\tau_B - \tau_A) + (1 - \nu - \phi)(1 + \tau_A)\tau_B \geq 0$) and if Sector B is at least as protected as Sector A even considering the subsidy in that sector ($\tau_B \geq \tau_A + \frac{s_A}{P_A}$). In fact, when $s_A \geq 0$, the latter condition implies $\tau_B \geq \tau_A$, which means that the condition on the consumption efficiency gain becomes redundant (*i.e.* trivially met). It follows that Proposition 3 collapses to the following corollary.

Corollary 2. *For any $s_A \geq 0$,*

$$\frac{\partial U}{\partial \tau_B} < 0 \quad \textit{if} \quad \tau_B \geq \tau_A + \frac{s_A}{P_A}.$$

Unfortunately we are unable to obtain as clear-cut a result as these ones for other situations. As explained in Introduction, the partial tariff removal policy of our interest corresponds to a change (reduction) in τ_B when $s_A > 0$ and $\tau_A > \tau_B$. Whilst Proposition 2 tells us that an increase in τ_A is harmful under this environment, it is silent as to the welfare effect of a change in τ_B. Hence we rely on numerical methods to uncover this question. In the following, we conduct a couple of numerical simulations that relate the welfare effect of an increase in τ_B to consumption/production parameters, and in particular, to the level of the production subsidy in Sector A, s_A.

3.2.2 Numerical simulations

We conduct two numerical simulations. In each simulation, we consider two economies which differ from one another in one respect. The following parameters take the same values for both the simulations: $\alpha = \beta = \gamma = 1/2$, $P_A = P_B = P_C = 1$, $\tau_A = 0.2$, $\tau_B = 0.1$. Table 3 summarises the other parameters that vary between the two economies in each of the simulations.

	Simulation 1		Simulation 2	
	Economy 1	Economy 2	Economy 2	Economy 3
K_A	2	1	1	1
K_B	4	4	4	4
K_C	8	9	9	9
ν	1/3	1/3	1/3	2/9
ϕ	1/3	1/3	1/3	1/3

Note: See text for the other parameter values.

Table 3: Parameter values for each simulation

- **Simulation 1**: In this simulation, the two economies differ only in the amounts of K_A and K_C. More specifically, the endowment of the specific factor in Sector A (C) in Economy 1 is greater (less, respectively) than that in Economy 2.

  Figure 2 plots the $\frac{\partial U}{\partial \tau_B}$ schedules for both economies, where the level of subsidy in Sector A is measured on the horizontal axis. From the figure, we can gather two bits of information. One is that the schedules are upward-sloping, meaning that the welfare effect of a marginal increase in τ_B is more likely to be positive as s_A increases. Intuitively, the more protected Sector A is, the more likely adding an extra distortion to Sector B alleviates the overall production inefficiency, and hence improves economic welfare.

  The other observation is that the $\frac{\partial U}{\partial \tau_B}$ schedule for Economy 1 lies above that for Economy 2. With the help of our intuition from the analytical section, we can interpret this result as follows. Since $s_A \geq 0$ as well as $\tau_A > \tau_B$, Sector A is more protected than Sector B. Hence, a marginal increase in τ_B will result in efficiency

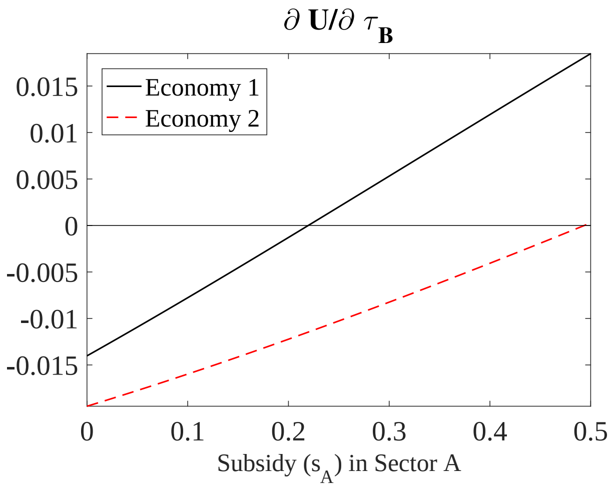

Figure 2: The effect of a marginal increase in τ_B on the welfare (Simulation 1)

gains in production by reducing the labour allocated to Sector A when Sector A and Sector B are concerned. However, a marginal increase in τ_B will also reduce the productive resource allocated to Sector C, which creates an efficiency loss in production. The overall efficiency change depends on the relative magnitude of these efficiency gains and losses. Figure 2 shows that the net efficiency *loss* is larger in Economy 2. It is because in Economy 2 compared to Economy 1 (i) a higher productivity of Sector C contributes to creating a greater efficiency loss as labour moves away; and (ii) the efficiency gain from the labour moving in to the less productive Sector A is smaller.

- **Simulation 2**: In this simulation, we altered Economy 2's preferences to create Economy 3. That is, they are different only because of the value of ν, which is the expenditure share of Good A. The simulation results are presented in Figure 3. As in the results from the previous simulation, we again observe an upward-sloping $\frac{\partial U}{\partial \tau_B}$ schedules with respect to s_A. It also turns out that the $\frac{\partial U}{\partial \tau_B}$ schedule for Economy 2 lies above that for Economy 3.

  This tax policy affects the two economies differently because of their different preferences. A marginal increase in τ_B discourages consumption in Good B and encourages

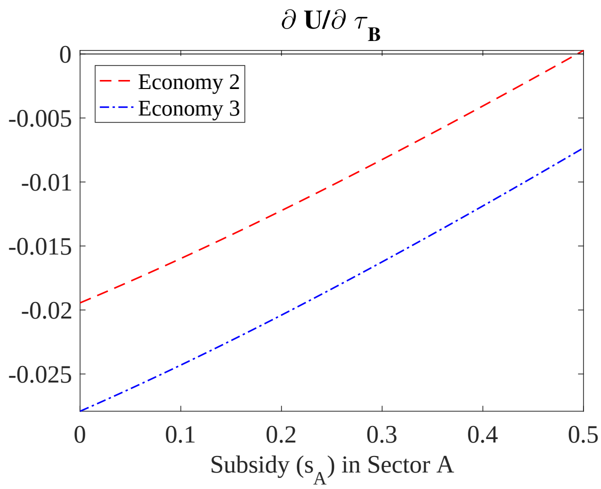

Figure 3: The effect of a marginal increase in τ_B on the welfare (Simulation 2)

consumption of the other two goods. Therefore, whilst a further distortion in the Good C consumption creates an efficiency loss, some efficiency gain is realised by alleviating Good A's under consumption. In Economy 3, the expenditure share of Good A (C) is lower (higher, respectively) than in Economy 2. The net efficiency *loss*, then, must be greater in Economy 3, and it is evident by the relative positions of the two economies' $\frac{\partial U}{\partial \tau_B}$ schedules in Figure 3.

The upward sloping $\frac{\partial U}{\partial \tau_B}$ schedules in these simulations suggest that the welfare effect of a policy to reduce τ_B depends on the level of production subsidy in Sector A, s_A. Focus on Economy 1 for example. A marginal *reduction* in τ_B is a welfare improving policy so long as the subsidy is sufficiently low, but is a harmful policy otherwise. If τ_B is to be lowered, should the subsidy be lowered together (if changing τ_A is not an option)? If so, how much reduction in the subsidy is necessary to make the tariff policy beneficial? They are quantitative questions that depend on the current level of the subsidy, which can be only addressed using the real data.

Indeed, given complicated efficiency trade-offs entailed in the real world, the welfare effect of any tariff reform ought to be assessed using the real data. All in all, the lesson we can learn from our numerical simulations per se is a modest one. A marginal increase in a tariff induces reallocations of production and consumption activities, and whether the policy is beneficial for the economy depends on whether a net efficiency gain is realised after these reallocations. In the next section, we calibrate our model to the Japanese economy to assess the welfare effect of a policy to reduce τ_B.

4 Calibrating our model to the Japanese economy

4.1 Adding a non-traded goods sector

So far our model only concerns traded-goods sectors. In calibrating our model to the Japanese economy, this feature of our model may be problematic given the magnitude of its service industry where a variety of non-traded goods are mainly produced. According to the Updated Input-Output Table 2013 (Ministry of Economy, Trade and Industry, Japan, 2013, IO Table hereafter), the service industry accounts for around 85 per cent of the Japanese economy both in terms of the final demand and the number of employees. In this paper we regard the service industry as the non-traded goods sector and incorporate it into our three-sector model. Our four-sector specific factor model hence resembles the specific factor model in Corden and Neary (1982), which has a non-traded goods sector and two traded-goods sectors, except that ours has an extra import-competing sector (as well as domestic distortion).

We denote the non-traded goods sector as N, and its production function is given as:

$$Y_N = L_N^{\delta} K_N^{1-\delta}, \tag{27}$$

where, L_N is the labour allocated to Sector N, K_N is the specific factor, and $0 < \delta < 1$.

Solving the producer's profit maximisation problem, we obtain the following first-order conditions.

$$w = P_N \delta L_N^{\delta-1} K_N^{1-\delta}, \tag{28}$$

$$r_N = P_N (1-\delta) L_N^{\delta} K_N^{-\delta}. \tag{29}$$

The labour market clearing condition is rewritten as:

$$L_A + L_B + L_C + L_N = \overline{L}. \tag{30}$$

Denoting the consumption of Good N as Q_N, the utility function is modified as:

$$U\left(Q_A, Q_B, Q_C, Q_N\right) = Q_A^{\nu} Q_B^{\phi} Q_C^{\omega} Q_N^{1-\nu-\phi-\omega}, \tag{31}$$

where $0 < \nu < 1$, $0 < \phi < 1$, $0 < \omega < 1$, and $0 < \nu + \phi + \omega < 1$. The consumer's utility maximisation requires:

$$\frac{Q_A}{Q_B} = \frac{\nu}{\phi} \frac{P_B(1+\tau_B)}{P_A(1+\tau_A)}, \tag{32}$$
$$\frac{Q_B}{Q_C} = \frac{\phi}{\omega} \frac{P_C}{P_B(1+\tau_B)}, \tag{33}$$
$$\frac{Q_C}{Q_N} = \frac{\omega}{1-\nu-\phi-\omega} \frac{P_N}{P_C}. \tag{34}$$

The market-clearing condition for the non-traded goods sector is given as:

$$Y_N = Q_N. \tag{35}$$

We also introduce a production subsidy s_B in Sector B into our extended model as we are interested in a counterfactual policy to marginally change s_B. It is a useful counterfactual experiment as we can isolate the welfare effect of a tariff change in Sector B that is caused by a change in the producer relative prices.

In any event, the producer's profit maximisation conditions involving Sector B has now become the following.

$$w = \{P_B(1+\tau_B) + s_B\}\beta L_B^{\beta-1} K_B^{1-\beta}, \tag{36}$$

$$r_B = \{P_B(1+\tau_B) + s_B\}(1-\beta) L_B^{\beta} K_B^{-\beta}. \tag{37}$$

We set s_B equal to zero in our calibration as the data suggest that the Sector B subsidy is insignificant, and hence these equations are essentially identical to (17) and (18), respectively. However, for the sake of accurately presenting our calibration exercise later on, we use (36) and (37) as part of the equations that characterise the equilibrium of this economy. Together with these two equations, (1), (2), (4)-(7), (13), (14), (16), (23), (24) and (27)-(35) describe the equilibrium of this economy. Our calibration's primary focus is the partial derivative of $U(Q_A, Q_B, Q_C, Q_N)$ in (31) with respect to τ_B:

$$\frac{\partial U}{\partial \tau_B} = Q_A^{\nu} Q_B^{\phi} Q_C^{\omega} Q_N^{1-\nu-\phi-\omega} \left\{ \nu \frac{1}{Q_A} \frac{\partial Q_A}{\partial \tau_B} + \phi \frac{1}{Q_B} \frac{\partial Q_B}{\partial \tau_B} + \omega \frac{1}{Q_C} \frac{\partial Q_C}{\partial \tau_B} + (1-\nu-\phi-\omega) \frac{1}{Q_N} \frac{\partial Q_N}{\partial \tau_B} \right\}. \quad (38)$$

4.2 Calibration strategy

Our calibration strategy follows two main procedures. First, we determine the values of the parameters which do not rely on the solution of the model, namely tariff rates (τ_A and τ_B), the aggregate quantity of labour in the economy ($\overline{L}$), the share of labour in each sector (α, β, γ, and δ), and the levels of capital in each sector (K_A, K_B, K_C, and K_N). Next, we calibrate remaining parameters, *i.e.* prices of the goods (P_A, P_B, P_C, and P_N), the production subsidy in Sector A (s_A), and the preference parameters (ν, ϕ, and ω), by matching the equilibrium conditions of the model.

We use the Japanese data for 2013. Table 6 in Appendix A shows how we have converted the 54 sectors in the IO Table in to our four sectors.[6] As shown in Table 4 in Appendix A, τ_A and τ_B are calculated as the weighted averages of average tariff rates of product groups provided in World Tariff Profiles 2013 (WTO, ITC and UNCTAD, 2013).

To determine α, β, γ, and δ, we are guided by the following definition of labour share in Sugou and Nishizaki (2002):

$$\text{Labour share} = \frac{\text{Personnel expenses}}{\text{Personnel expenses} + \text{Operating profit} + \text{Depreciation}}.$$

For personnel expenses, operating profit, and depreciation, we use the corresponding IO

[6] We have omitted Sector 30 (Other manufacturing) and Sector 54 (Activities not elsewhere classified). They account for less than one per cent of economic activities in Japan.

Table entries of compensation for employees, operating surplus, and provision for the consumption of fixed capital, respectively. It follows that $\alpha = 0.40$, $\beta = 0.57$, $\gamma = 0.68$, and $\delta = 0.59$.

We normalise $\overline{L}$ to unity without loss of generality. K_A, K_B, K_C, and K_N are pinned down using the labour equipment ratio, *i.e.* the ratio of tangible fixed assets to the number of employees, obtained from Financial Statements Statistics of Corporations by Industry (Ministry of Finance, Policy Research Institute, 2014).[7] In our model, this ratio corresponds to K/L for each sector. After normalising the ratios by the total sum of employees obtained from the employment table attached to the IO Table, we obtain $K_A/L_A = 0.14$, $K_B/L_B = 0.26$, $K_C/L_C = 0.18$, and $K_N/L_N = 0.20$. Given these ratios as well as the number of paid officials and employees in each sector, it follows that $K_A = 0.005$, $K_B = 0.005$, $K_C = 0.01$, and $K_N = 0.17$.

Given the pre-determined parameters described above, P_A, P_B, P_C, P_N, and s_A are calibrated to match the first-order conditions of the producer's profit maximisation. First, P_N is computed using (28) given w, δ, and K_N/L_N. We use $w = 0.08$, which results from normalising the compensation of employed by the number of employees, both of which are available in the IO Table. Similarly we can calculate P_C using (5) given w, γ, and K_C/L_C as can we obtain P_B with the help of (36) given w, τ_B, β, and K_B/L_B. The calibrated values for these prices are: $P_B = 0.24$, $P_C = 0.20$, $P_N = 0.26$. Calibrating P_A and s_A is a little tricky as using (4), we can only compute $P_A(1 + \tau_A) + s_A$ given w, α, and K_A/L_A. However, the total of gross value added and subsidies for Sector A, which correspond to $\{P_A(1 + \tau_A) + s_A\} Y_A$ and $s_A Y_A$, respectively, are available in the IO Table. Combining these bits of information with the value of $P_A(1 + \tau_A) + s_A$ we already have, we can separately obtain $P_A = 0.50$ and $s_A = 0.02$.

Finally, we calibrate ν, ϕ, and ω to match the consumer's utility maximisation. Using (32), (33), and (34), the ratios of consumption expenditure between two goods are

[7] See Table 5 in Appendix A for how their sectors correspond to our four sectors.

described as:

$$\frac{P_A(1+\tau_A)Q_A}{P_B(1+\tau_B)Q_B} = \frac{\nu}{\phi}, \tag{39}$$
$$\frac{P_B(1+\tau_B)Q_B}{P_C Q_C} = \frac{\phi}{\omega}, \tag{40}$$
$$\frac{P_C Q_C}{P_N Q_N} = \frac{\omega}{1-\nu-\phi-\omega}. \tag{41}$$

We obtain the values of private consumption expenditure of each sector from the IO Table and substitute them into the left hand side of each of these equations. Then (39), (40), and (41) together become a system of three equations with three unknowns, namely ν, ϕ, and ω. Solving this system, we obtain $\nu = 0.10$, $\phi = 0.04$, and $\omega = 0.05$.

4.3 Results

Figure 4 illustrates the $\frac{\partial U}{\partial \tau_B}$ schedule for the Japanese economy.

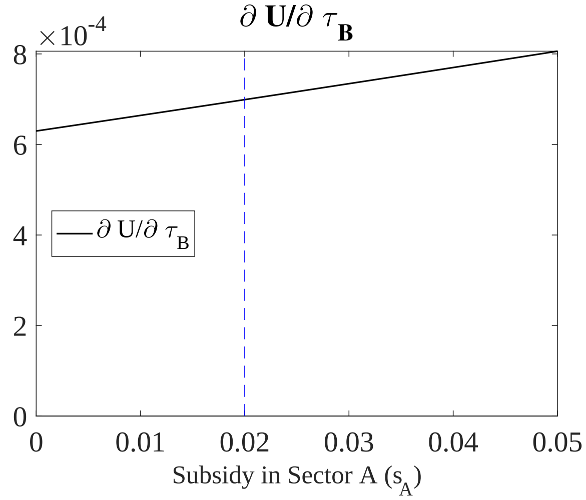

Figure 4: The welfare effect of an increase in τ_B on the Japanese economy

It shows, given the calibrated value of the production subsidy in Sector A, $s_A = 0.02$, that a marginal *reduction* in τ_B is harmful for the Japanese economy. In fact, even if the production subsidy in Sector A is completely removed, a marginal reduction in τ_B is still harmful for the Japanese economy. It indicates that τ_A is so high that Sector A is much more protected than Sector B is, even after s_A is fully removed.

To get a better sense of Sector A's high protection, we ask the following question. How much reduction in τ_A is necessary to render the marginal reduction in τ_B welfare improving? It turns out that, to have $\frac{\partial U}{\partial \tau_B} = 0$ when $s_A = 0.02$, τ_A needs to be more than halved ($\tau_A = 0.10$). Given $\tau_A = 0.10$, the counterfactual $\frac{\partial U}{\partial \tau_B}$ schedule is illustrated in Figure 5. It indicates that, if $\tau_A = 0.10$, then the policy to marginally reduce τ_B accompanied by a reduction in s_A would be beneficial for the Japanese economy.

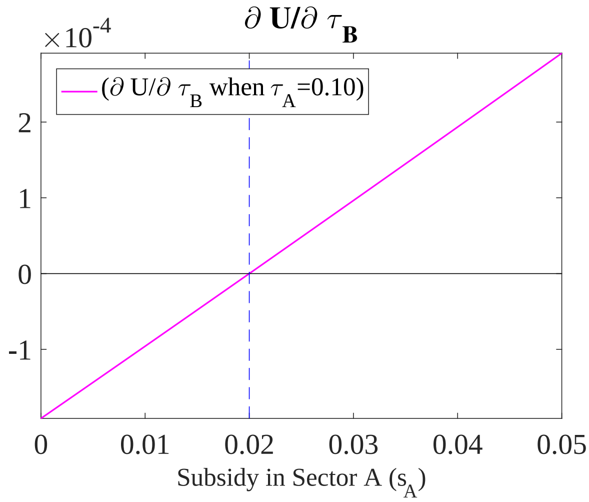

Figure 5: Counterfactual welfare effect of an increase in τ_B on the Japanese economy when $\tau_A = 0.10$

A marginal reduction in τ_B lowers Sector B's protection, but lowering s_B can do the same job. However, these two policies are different in that whilst the former affects the relative prices that both consumers and producers face, the latter only affects the relative prices faced by producers. Hence we can decompose the welfare effect of a marginal reduction in τ_B into two components: (i) the effect which is caused by s_B; and (ii) the remaining effect. Since the first effect is brought about by the resource reallocation in production only, the remaining effect can be interpreted as that caused by the change in consumption choice.

In Figure 6, the $\frac{\partial U}{\partial s_B}$ schedule is depicted together with the $\frac{\partial U}{\partial \tau_B}$ schedule. On appearance the $\frac{\partial U}{\partial s_B}$ schedule lies well above the $\frac{\partial U}{\partial \tau_B}$ schedule, but we need to be careful about

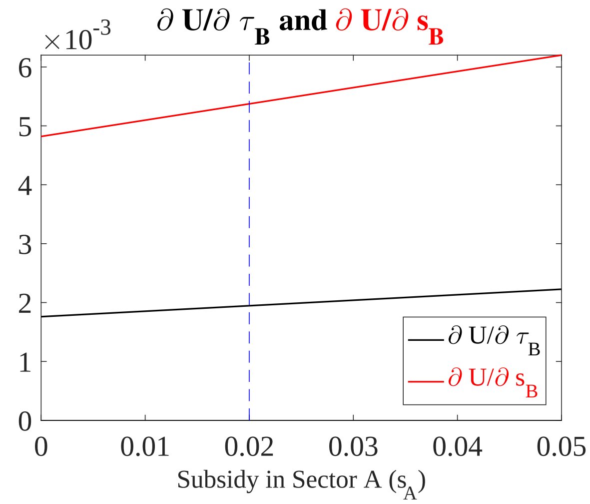

Figure 6: The welfare effect of an increase in s_B on the Japanese economy

comparing these welfare effects. Note that whilst a marginal increase in s_B affects the producer price of Good B by ds_B, a marginal increase in τ_B increases the producer price of Good by $P_B d\tau_B$. Therefore, if we want to examine the comparable welfare effect of $d\tau_B$ only on the production side, we need to focus on the welfare effect of ds_B multiplied by P_B.

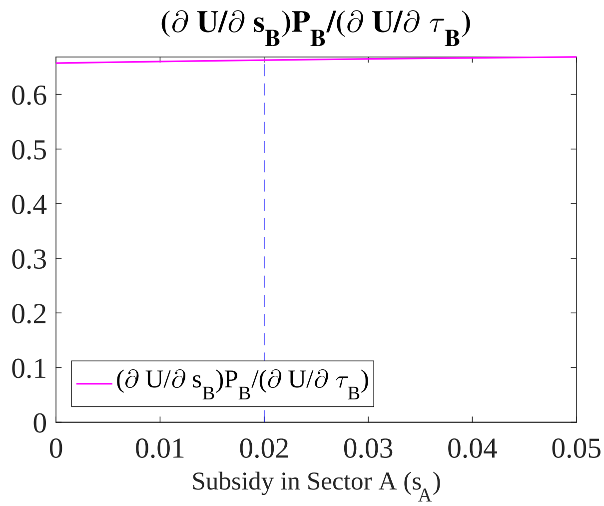

Figure 7: Comparison of two policies to protect Sector B

To see the relative importance of the welfare effect of $P_B ds_B$, we illustrate the ratio of

$\frac{\partial U}{\partial s_B}P_B$ and $\frac{\partial U}{\partial \tau_B}$ in Figure 7. This figure and Figure 4 indicate that: (i) a marginal reduction in τ_B, for the calibrated value of the production subsidy in Sector A ($s_A = 0.02$), harms the Japanese economy by creating further distortions both on production and consumption sides; and (ii) roughly 70 per cent of the (negative) welfare effect is attributable to further inefficient resource allocations on the production side.

5 Conclusion

Agricultural producer support in Japan is distinctive in terms of both tariffs imposed on agricultural products and the government subsidy, whilst both forms of protection in other import-competing sectors are rather modest. To incorporate these characteristics of the Japanese economy, we have constructed a specific factor model that has two import-competing sectors, both of which are protected by tariffs but only one of them enjoys a production subsidy. Perhaps, often politically, since the protection in the most heavily protected sector is hugely costly to be reduced, a general movement towards freetrade tends to end up in a tariff reduction only in modestly protected sectors, which we view as the outcome of the recent TPP negotiations. Using our model, we have examined the welfare effect of a policy to reduce a tariff in the less protected sector, by calibrating it to the 2013 Japanese economy.

Our calibration result suggests that the partial tariff reduction policy is harmful by making both production and consumption activities inefficient where the former explains roughly 70 per cent of the overall inefficiency losses. A complete removal of the production subsidy in the agricultural sector hardly changes the negative welfare effect of this tariff policy, which is indicative of the enormity of the tariff protection in the agricultural sector. Indeed, for the partial tariff reduction policy to be welfare improving, ceteris paribus, the tariff in the agricultural sector must be more than halved.

Our specific factor model is tractable, which helps follow and interpret our calibration results clearly. However, the use of a simple economic model imposes some limitations. For example, a specific factor model assumes a set of fixed international prices, *i.e.* a country in question is considered a small open economy, but whether it is applied to Japan is debatable. A similar analysis may be conducted by constructing a two-country model, where the prices of the traded-goods are also endogenously determined, but whatever result that comes out of it could be difficult to interpret. A set of fixed international prices also implies that no tariff change has occurred elsewhere. Of course, in reality, trade policy negotiations occur with other economies, and our setup is restricted to examining a unilateral tariff reform. These agenda are beyond the scope of the current paper and are left for future research.

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Appendix A Sector classification

Product group	Ave. tariff (%)	Import share (%)	Sector in our model
Animal products	13.6	1.6	
Dairy products	116.9	0.2	
Fruit, vegetables, plants	9.9	1.1	
Coffee, tea	14.4	0.4	
Cereals & preparations	80.2	1.6	
Oilseeds, fats & oils	9.8	0.8	
Sugars and confectionery	50.2	0.2	
Beverages & tobacco	16.8	1.2	
Cotton	0	0	
Other agricultural products	5.4	0.7	
Fish & fish products	4.9	2.1	
	($\tau_A = 24.4$)		**A**
Minerals & Metals	1	24.5	
Petroleum	8.2	20.2	
Wood, paper, *etc.*	1	2.9	
Textiles	5.6	1.9	
Clothing	9.2	3.7	
Chemicals	2.3	9.3	
	($\tau_B = 4.1$)		**B**
Non-electrical machinery	0	7.7	
Electrical machinery	0.2	10.3	
Transport equipment	0	2.6	**C**

Table 4: Product groups in World Tariff Profiles 2013 (WTO, ITC and UNCTAD, 2013) and traded-goods sectors in our model

Sector name	Sector in our model
Agriculture, forestry, and fisheries	
Foodstuffs and beverages	**A**
Textile mill products	
Pulp and paper products	
Petroleum and coal products	
Mining	
Chemical products	**B**
General-purpose machinery	
Production machinery	
Business oriented machinery	
Electrical machinery, equipment and supplies	
Information and communication electronics equipment	
Motor vehicles, parts and accessories	
Miscellaneous transportation equipment	**C**
Construction	
Electricity	
Gas, heat supply and water	
Information and communications	
Wholesale and retail trade	
Real estate	
Goods rental and leasing	
Services	**N**

Table 5: Sectors in Financial Statements Statistics of Corporations by Industry (Ministry of Finance, Policy Research Institute, 2014) and in our model

Sector No.	Sector name	Sector in our model
1	Agriculture, forestry and fishery	
4	Beverages and Foods	**A**
2	Mining	
3	Coal mining, crude petroleum and natural gas	
5	Textile products	
6	Wearing apparel and other textile products	
7	Timber, wooden products and furniture	
8	Pulp, paper, building paper	
9	Chemical basic products	
10	Final chemical products	
11	Petroleum and coal products	**B**
12	Plastic and rubber products	
13	Ceramic, stone and clay products	
14	Iron and steel	
15,16	Non-ferrous metals and metal products	
17-19	Machinery	
20-26	Electric and electronic equipments	
27-29	Motor vehicles, parts and accessories	
30	Other transportation equipment	**C**
32-34	Construction and public work	
35	Other civil engineering and construction	
36-38	Electricity, gas, heat and water supply	
39	Waste management service	
40	Commerce	
41	Finance, insurance and real estate	
42	Transport and postal activities	
43	Communication and broadcasting	
44	Information services	
45	Other information and communications	
46	Public administration	
47	Education and research	
48	Medical service, health, social security and nursing care	
49	Other non-profit organization services	
50	Goods rental and leasing services	
51	Advertising services	
52	Other business services	
53	Personal services	**N**

Table 6: Sectors in the Input-Output Table 2013 (Ministry of Economy, Trade and Industry, Japan, 2013) and in our model

Appendix B Proof of Proposition 1

Proof. Given that $M_A = Q_A - Y_A$ and $X_C = Y_C - Q_C$, (12) implies that Q_A is written as:

$$Q_A = \left(P_A + P_C \frac{Q_C}{Q_A}\right)^{-1} (P_A Y_A + P_C Y_C). \tag{42}$$

It follows from (9) that Q_C is obtained as a function of Q_A as:

$$Q_C = \frac{1-\nu}{\nu} \frac{P_A(1+\tau_A)}{P_C} Q_A. \tag{43}$$

Now we take the partial derivatives of Q_A and Q_C in (42) and (43) with respect to τ_A. Utilising the facts that $Y_A = L_A^{\alpha} K_A^{1-\alpha}$, $Y_C = L_C^{\gamma} K_C^{1-\gamma}$, and $\frac{\partial L_C}{\partial \tau_A} = -\frac{\partial L_A}{\partial \tau_A}$ implied by (3), we obtain:

$$\begin{aligned}\frac{\partial Q_A}{\partial \tau_A} &= -\frac{1}{P_A}\left\{\frac{\nu}{\nu+(1-\nu)(1+\tau_A)}\right\}^2 \left(\frac{1-\nu}{\nu}\right)\left(P_A L_A^{\alpha} K_A^{1-\alpha} + P_C L_C^{\gamma} K_C^{1-\gamma}\right)\\ &+\frac{1}{P_A}\left\{\frac{\nu}{\nu+(1-\nu)(1+\tau_A)}\right\}\left\{\frac{\partial L_A}{\partial \tau_A}\left(P_A \alpha L_A^{\alpha-1} K_A^{1-\alpha} - P_C \gamma L_C^{\gamma-1} K_C^{1-\gamma}\right)\right\},\end{aligned} \tag{44}$$

$$\begin{aligned}\frac{\partial Q_C}{\partial \tau_A} &= \frac{(1-\nu)\nu}{P_C}\left\{\frac{1}{\nu+(1-\nu)(1+\tau_A)}\right\}^2 \left(P_A L_A^{\alpha} K_A^{1-\alpha} + P_C L_C^{\gamma} K_C^{1-\gamma}\right)\\ &+\frac{1-\nu}{P_C}\left\{\frac{1+\tau_A}{\nu+(1-\nu)(1+\tau_A)}\right\}\left\{\frac{\partial L_A}{\partial \tau_A}\left(P_A \alpha L_A^{\alpha-1} K_A^{1-\alpha} - P_C \gamma L_C^{\gamma-1} K_C^{1-\gamma}\right)\right\}.\end{aligned} \tag{45}$$

Substituting (44) and (45) into (15), it follows that:

$$\begin{aligned}\frac{\partial U}{\partial \tau_A} &= -\frac{\nu^2(1-\nu)\tau_A}{P_A(1+\tau_A)\left\{\nu+(1-\nu)(1+\tau_A)\right\}^2}\left\{\frac{\nu}{1-\nu}\frac{P_C}{P_A(1+\tau_A)}\right\}^{\nu-1}\left(P_A L_A^{\alpha} K_A^{1-\alpha} + P_C L_C^{\gamma} K_C^{1-\gamma}\right)\\ &+\frac{\nu}{P_A\left\{\nu+(1-\nu)(1+\tau_A)\right\}}\left\{\frac{\nu}{1-\nu}\frac{P_C}{P_A(1+\tau_A)}\right\}^{\nu-1}\frac{\partial L_A}{\partial \tau_A}\left(P_A \alpha L_A^{\alpha-1} K_A^{1-\alpha} - P_C \gamma L_C^{\gamma-1} K_C^{1-\gamma}\right),\end{aligned} \tag{46}$$

where $-\frac{\nu^2(1-\nu)\tau_A}{P_A(1+\tau_A)\{\nu+(1-\nu)(1+\tau_A)\}^2}\left\{\frac{\nu}{1-\nu}\frac{P_C}{P_A(1+\tau_A)}\right\}^{\nu-1} \leq 0$ and $\frac{\nu}{P_A\{\nu+(1-\nu)(1+\tau_A)\}}\left\{\frac{\nu}{1-\nu}\frac{P_C}{P_A(1+\tau_A)}\right\}^{\nu-1} > 0$ since $0 < \nu < 1$. It is straightforward to see that $P_A L_A^{\alpha} K_A^{1-\alpha} + P_C L_C^{\gamma} K_C^{1-\gamma} > 0$. Therefore, to analyse the sign of (46), we are now left with analysing the sign of $\frac{\partial L_A}{\partial \tau_A}$ and $P_A \alpha L_A^{\alpha-1} K_A^{1-\alpha} - P_C \gamma L_C^{\gamma-1} K_C^{1-\gamma}$.

The sign of $\frac{\partial L_A}{\partial \tau_A}$ follows from an implicit function of L_A. Due to the wage equalisation, (3), (4), and (5) yield:

$$\{P_A(1+\tau_A)+s_A\}\,\alpha L_A^{\alpha-1}K_A^{1-\alpha}-P_C\gamma(\overline{L}-L_A)^{\gamma-1}K_C^{1-\gamma}=0, \tag{47}$$

where the left hand side is an implicit function of L_A. Setting the left hand side as $f(L_A,\tau_A)$, and using the implicit function theorem,

$$\frac{\partial L_A}{\partial \tau_A}=-\frac{f_{\tau_A}}{f_{L_A}}, \tag{48}$$

where

$$f_{\tau_A}=P_A\alpha L_A^{\alpha-1}K_A^{1-\alpha}>0,$$

and

$$f_{L_A}=\{P_A(1+\tau_A)+s_A\}\,\alpha(\alpha-1)L_A^{\alpha-2}K_A^{1-\alpha}+P_C\gamma(\gamma-1)(\overline{L}-L_A)^{\gamma-2}K_C^{1-\gamma}<0,$$

where the inequality follows from $0<\alpha<1$ and $0<\gamma<1$. Therefore, $\frac{\partial L_A}{\partial \tau_A}>0$.

Next, due to the wage equalisation, (4) and (5) straightforwardly yield:

$$P_A\alpha L_A^{\alpha-1}K_A^{1-\alpha}-P_C\gamma L_C^{\gamma-1}K_C^{1-\gamma}=-P_A\left(\tau_A+\frac{s_A}{P_A}\right)\alpha L_A^{\alpha-1}K_A^{1-\alpha}\leq 0, \tag{49}$$

when $\tau_A+\frac{s_A}{P_A}\geq 0$.

Collectively, it follows that:

$$\frac{\partial U}{\partial \tau_A}\leq 0, \tag{50}$$

when $\tau_A+\frac{s_A}{P_A}\geq 0$. For $\frac{\partial U}{\partial \tau_A}=0$ to occur, (46) implies that both τ_A and $P_A\alpha L_A^{\alpha-1}K_A^{1-\alpha}-P_C\gamma L_C^{\gamma-1}K_C^{1-\gamma}$ must be zero. This requires $\tau_A=s_A=0$. Hence, except for the trivial case where there is no distortion ($\tau_A=s_A=0$), we have shown that:

$$\frac{\partial U}{\partial \tau_A} < 0,$$

for any $s_A \geq 0$. $\square$

Appendix C Proof of Proposition 2

Proof. Taking the partial derivatives of Q_A, Q_B, and Q_C with respect to τ_A, we obtain the following.

$$\frac{\partial Q_A}{\partial \tau_A} = -\left(P_A + P_B\frac{Q_B}{Q_A} + P_C\frac{Q_C}{Q_A}\right)^{-2}\left\{P_B\frac{\partial\left(\frac{Q_B}{Q_A}\right)}{\partial \tau_A} + P_C\frac{\partial\left(\frac{Q_C}{Q_A}\right)}{\partial \tau_A}\right\}(P_AY_A + P_BY_B + P_CY_C)$$
$$+\left(P_A + P_B\frac{Q_B}{Q_A} + P_C\frac{Q_C}{Q_A}\right)^{-1}\left(P_A\frac{\partial Y_A}{\partial \tau_A} + P_B\frac{\partial Y_B}{\partial \tau_A} + P_C\frac{\partial Y_C}{\partial \tau_A}\right), \tag{51}$$

$$\frac{\partial Q_B}{\partial \tau_A} = -\left(P_A\frac{Q_A}{Q_B} + P_B + P_C\frac{Q_C}{Q_B}\right)^{-2}\left\{P_A\frac{\partial\left(\frac{Q_A}{Q_B}\right)}{\partial \tau_A}\right\}(P_AY_A + P_BY_B + P_CY_C)$$
$$+\left(P_A\frac{Q_A}{Q_B} + P_B + P_C\frac{Q_C}{Q_B}\right)^{-1}\left(P_A\frac{\partial Y_A}{\partial \tau_A} + P_B\frac{\partial Y_B}{\partial \tau_A} + P_C\frac{\partial Y_C}{\partial \tau_A}\right), \tag{52}$$

$$\frac{\partial Q_C}{\partial \tau_A} = -\left(P_A\frac{Q_A}{Q_C} + P_B\frac{Q_B}{Q_C} + P_C\right)^{-2}\left\{P_A\frac{\partial\left(\frac{Q_A}{Q_C}\right)}{\partial \tau_A}\right\}(P_AY_A + P_BY_B + P_CY_C)$$
$$+\left(P_A\frac{Q_A}{Q_C} + P_B\frac{Q_B}{Q_C} + P_C\right)^{-1}\left(P_A\frac{\partial Y_A}{\partial \tau_A} + P_B\frac{\partial Y_B}{\partial \tau_A} + P_C\frac{\partial Y_C}{\partial \tau_A}\right). \tag{53}$$

Using (51), (52), and (53), we can rewrite (25) as follows:

$$\frac{\partial U}{\partial \tau_A} = Q_A^{\nu}Q_B^{\phi}Q_C^{1-\nu-\phi}\left\{\Omega_1(P_AY_A + P_BY_B + P_CY_C) + \Omega_2\left(P_A\frac{\partial Y_A}{\partial \tau_A} + P_B\frac{\partial Y_B}{\partial \tau_A} + P_C\frac{\partial Y_C}{\partial \tau_A}\right)\right\}, \tag{54}$$

where

$$\Omega_1 \equiv -\frac{\nu}{Q_A}\left(P_A + P_B\frac{Q_B}{Q_A} + P_C\frac{Q_C}{Q_A}\right)^{-2}\left\{P_B\frac{\partial\left(\frac{Q_B}{Q_A}\right)}{\partial\tau_A} + P_C\frac{\partial\left(\frac{Q_C}{Q_A}\right)}{\partial\tau_A}\right\}$$
$$-\frac{\phi}{Q_B}\left(P_A\frac{Q_A}{Q_B} + P_B + P_C\frac{Q_C}{Q_B}\right)^{-2}\left\{P_A\frac{\partial\left(\frac{Q_A}{Q_B}\right)}{\partial\tau_A}\right\}$$
$$-\frac{1-\nu-\phi}{Q_C}\left(P_A\frac{Q_A}{Q_C} + P_B\frac{Q_B}{Q_C} + P_C\right)^{-2}\left\{P_A\frac{\partial\left(\frac{Q_A}{Q_C}\right)}{\partial\tau_A}\right\}, \tag{55}$$

and

$$\Omega_2 \equiv \frac{\nu}{Q_A}\left(P_A + P_B\frac{Q_B}{Q_A} + P_C\frac{Q_C}{Q_A}\right)^{-1} + \frac{\phi}{Q_B}\left(P_A\frac{Q_A}{Q_B} + P_B + P_C\frac{Q_C}{Q_B}\right)^{-1}$$
$$\frac{1-\nu-\phi}{Q_C}\left(P_A\frac{Q_A}{Q_C} + P_B\frac{Q_B}{Q_C} + P_C\right)^{-1}. \tag{56}$$

To analyse the sign of $\frac{\partial U}{\partial\tau_A}$, we need the signs of Ω_1, Ω_2, $P_AY_A + P_BY_B + P_CY_C$, and $P_AY_A + P_BY_B + P_CY_C$ and $P_A\frac{\partial Y_A}{\partial\tau_A} + P_B\frac{\partial Y_B}{\partial\tau_A} + P_C\frac{\partial Y_C}{\partial\tau_A}$ in (54), respectively. Since it straightforwardly follows that $P_AY_A + P_BY_B + P_CY_C > 0$ and $\Omega_2 > 0$ in (56), we consider the rest of the terms, namely $P_A\frac{\partial Y_A}{\partial\tau_A} + P_B\frac{\partial Y_B}{\partial\tau_A} + P_C\frac{\partial Y_C}{\partial\tau_A}$ and Ω_1.

We first derive the sign of $P_A\frac{\partial Y_A}{\partial\tau_A} + P_B\frac{\partial Y_B}{\partial\tau_A} + P_C\frac{\partial Y_C}{\partial\tau_A}$ by rewriting the expression as:

$$P_A\frac{\partial Y_A}{\partial\tau_A} + P_B\frac{\partial Y_B}{\partial\tau_A} + P_C\frac{\partial Y_C}{\partial\tau_A} = P_A\frac{\partial Y_A}{\partial L_A}\frac{\partial L_A}{\partial\tau_A} + P_B\frac{\partial Y_B}{\partial L_B}\frac{\partial L_B}{\partial\tau_A} + P_C\frac{\partial Y_C}{\partial L_C}\frac{\partial L_C}{\partial\tau_A}$$
$$= P_A\frac{\partial Y_A}{\partial L_A}\left(-\frac{\partial L_B}{\partial\tau_A} - \frac{\partial L_C}{\partial\tau_A}\right) + P_B\frac{\partial Y_B}{\partial L_B}\frac{\partial L_B}{\partial\tau_A} + P_C\frac{\partial Y_C}{\partial L_C}\frac{\partial L_C}{\partial\tau_A}$$
$$= \frac{\partial L_B}{\partial\tau_A}\left(P_B\frac{\partial Y_B}{\partial L_B} - P_A\frac{\partial Y_A}{\partial L_A}\right) + \frac{\partial L_C}{\partial\tau_A}\left(P_C\frac{\partial Y_C}{\partial L_C} - P_A\frac{\partial Y_A}{\partial L_A}\right). \tag{57}$$

To obtain the sign of $P_A\frac{\partial Y_A}{\partial\tau_A} + P_B\frac{\partial Y_B}{\partial\tau_A} + P_C\frac{\partial Y_C}{\partial\tau_A}$ in (57), we first derive the sign of $\frac{\partial L_C}{\partial\tau_A}$. Given the result, we consider $\frac{\partial L_B}{\partial\tau_A}$. The sign of $P_C\frac{\partial Y_C}{\partial L_C} - P_A\frac{\partial Y_A}{\partial L_A}$ follows. Finally, we analyse the sign of $P_B\frac{\partial Y_B}{\partial L_B} - P_A\frac{\partial Y_A}{\partial L_A}$.

- **The sign of $\frac{\partial L_C}{\partial\tau_A}$**

We first need to solve for $\frac{\partial L_A}{\partial \tau_A}$, which entails implicit differentiation as follows. Equating (4) and (17), we obtain:

$$L_B = \left[\frac{\{P_A(1+\tau_A)+s_A\}\,\alpha K_A^{1-\alpha} L_A^{\alpha-1}}{P_B(1+\tau_B)\beta K_B^{1-\beta}} \right]^{\frac{1}{\beta-1}}. \tag{58}$$

Equating (5) and (17) and utilising (19), we obtain the following equation.

$$P_B(1+\tau_B)\beta L_B^{\beta-1} K_B^{1-\beta} - P_C \gamma (\overline{L} - L_A - L_B)^{\gamma-1} K_C^{1-\gamma} = 0. \tag{59}$$

Substituting (58) into (59), we obtain an implicit function of L_A. Defining the left hand side of (59) as $f(L_A, \tau_A)$ and using the implicit function theorem, we obtain the partial derivative of L_A with respect to τ_A as:

$$\frac{\partial L_A}{\partial \tau_A} = -\frac{f_{\tau_A}}{f_{L_A}}, \tag{60}$$

where

$$\begin{aligned} f_{\tau_A} &= \frac{P_A P_B(1+\tau_B)}{P_A(1+\tau_A)+s_A} \beta L_B^{\beta-1} K_B^{1-\beta} + \frac{P_A P_C}{P_A(1+\tau_A)+s_A} \frac{\gamma(\gamma-1)}{\beta-1} L_B L_C^{\gamma-2} K_C^{1-\gamma} \\ &= \frac{P_A P_B(1+\tau_B)}{P_A(1+\tau_A)+s_A} \frac{\partial Y_B}{\partial L_B} + \frac{P_A P_C}{P_A(1+\tau_A)+s_A} \frac{\gamma-1}{\beta-1} \frac{L_B}{L_C} \frac{\partial Y_C}{\partial L_C} \\ &= P_A \left(1 + \frac{\gamma-1}{\beta-1} \frac{L_B}{L_C} \right) \frac{\partial Y_A}{\partial L_A}. \end{aligned} \tag{61}$$

The second equality in (61) follows from $\frac{\partial Y_B}{\partial L_B} = \beta L_B^{\beta-1} K_B^{1-\beta}$ and $\frac{\partial Y_C}{\partial L_C} = \gamma L_C^{\gamma-1} K_C^{1-\gamma}$. The third equality in (61) follows from the wage equalisation conditions, which follow from (4), (5), (17), and the fact that $\frac{\partial Y_A}{\partial L_A} = \alpha L_A^{\alpha-1} K_A^{1-\alpha}$. Next, f_{L_A} is solved as:

$$f_{L_A} = \{P_A(1+\tau_A)+s_A\} \frac{(\alpha-1)(\beta-1)L_C + (\beta-1)(\gamma-1)L_A + (\alpha-1)(\gamma-1)L_B}{(\beta-1)L_A L_C} \frac{\partial Y_A}{\partial L_A} \tag{62}$$

Substituting (61) and (62) into (60), we obtain the partial derivative of L_A with respect to τ_A as:

$$\begin{aligned}\frac{\partial L_A}{\partial \tau_A} &= \frac{P_A\{(1-\beta)L_A L_C + (1-\gamma)L_A L_B\}}{\{P_A(1+\tau_A)+s_A\}\{(\beta-1)(\gamma-1)L_A + (\alpha-1)(\gamma-1)L_B + (\alpha-1)(\beta-1)L_C\}} \\ &> 0. \end{aligned} \tag{63}$$

The inequality in (63) follows from the facts that $0 < \alpha < 1$, $0 < \beta < 1$, and $0 < \gamma < 1$.

Given (58) and (63), we obtain:

$$\frac{\partial L_B}{\partial \tau_A} = \frac{L_B^{2-\beta}}{P_B(1+\tau_B)\beta(\beta-1)K_B^{1-\beta}}\frac{\partial Y_A}{\partial L_A}\left[P_A + \{P_A(1+\tau_A)+s_A\}(\alpha-1)\frac{1}{L_A}\frac{\partial L_A}{\partial \tau_A}\right]. \tag{64}$$

Substituting (63) and (64), it also follows that:

$$\begin{aligned}&\frac{\partial L_C}{\partial \tau_A} = -\frac{\partial L_A}{\partial \tau_A} - \frac{\partial L_B}{\partial \tau_A} \\ &= \frac{P_A(\beta-1)L_A L_C}{\{P_A(1+\tau_A)+s_A\}\{(\beta-1)(\gamma-1)L_A + (\alpha-1)(\gamma-1)L_B + (\alpha-1)(\beta-1)L_C\}} \\ &< 0, \end{aligned} \tag{65}$$

where the inequality follows from the facts that $0 < \alpha < 1$, $0 < \beta < 1$, and $0 < \gamma < 1$.

- **The sign of $\frac{\partial L_B}{\partial \tau_A}$**

(5) and (17) result in the following equalisation condition.

$$P_B(1+\tau_B)\beta L_B^{\beta-1}K_B^{1-\beta} = P_C\gamma L_C^{\gamma-1}K_C^{1-\gamma}. \tag{66}$$

Taking the partial derivative of both sides of this equation with respect to τ_A, we obtain:

$$P_B(1+\tau_B)\beta(\beta-1)K_B^{1-\beta}L_B^{\beta-2}\frac{\partial L_B}{\partial \tau_A} = P_C\gamma(\gamma-1)K_C^{1-\gamma}L_C^{\gamma-2}\frac{\partial L_C}{\partial \tau_A}. \tag{67}$$

Since $\frac{\partial L_C}{\partial \tau_A} < 0$ as obtained in (65) and $0 < \gamma < 1$, the right hand side of (67) is positive. Since $0 < \beta < 1$, from the left hand side of (67), we obtain:

$$\frac{\partial L_B}{\partial \tau_A} < 0. \tag{68}$$

- **The sign of** $P_C \frac{\partial Y_C}{\partial L_C} - P_A \frac{\partial Y_A}{\partial L_A}$

  From the wage equalisation implied by (4) and (5) it follows that:

$$\begin{aligned} P_C \frac{\partial Y_C}{\partial L_C} - P_A \frac{\partial Y_A}{\partial L_A} &= P_C \gamma L_C^{\gamma-1} K_C^{1-\gamma} - P_A \alpha L_A^{\alpha-1} K_A^{1-\alpha} \\ &= P_A \left(\tau_A + \frac{s_A}{P_A} \right) \alpha L_A^{\alpha-1} K_A^{1-\alpha} \geq 0, \end{aligned} \tag{69}$$

  when $\tau_A + \frac{s_A}{P_A} \geq 0$.

- **The sign of** $P_B \frac{\partial Y_B}{\partial L_B} - P_A \frac{\partial Y_A}{\partial L_A}$

  (4) and (17) yield

$$\begin{aligned} P_B \frac{\partial Y_B}{\partial L_B} - P_A \frac{\partial Y_A}{\partial L_A} &= P_B \beta L_B^{\beta-1} K_B^{1-\beta} - P_A \alpha L_A^{\alpha-1} K_A^{1-\alpha} \\ &= \left\{ \frac{P_A(1+\tau_A) + s_A - P_A(1+\tau_B)}{1+\tau_B} \right\} \alpha L_A^{\alpha-1} K_A^{1-\alpha} \geq 0, \end{aligned}$$

  when $\tau_A + \frac{s_A}{P_A} \geq \tau_B$.

Therefore, (57), (65), (68), (69), and (70) together imply that:

$$P_A \frac{\partial Y_A}{\partial \tau_A} + P_B \frac{\partial Y_B}{\partial \tau_A} + P_C \frac{\partial Y_C}{\partial \tau_A} \leq 0 \text{ when } \tau_A + \frac{s_A}{P_A} \geq \tau_B. \tag{70}$$

We are left with the analysis of the sign of Ω_1 in (54). Following from (55) and using

(21), (22), and their partial derivatives with respect to τ_A, Ω_1 is further written as

$$\begin{aligned}\Omega_1 &= -\frac{1}{(P_AQ_A+P_BQ_B+P_CQ_C)^2} \\ &\quad \left[Q_A\left\{\frac{\phi P_A}{1+\tau_B}+(1-\nu-\phi)P_A\right\}-Q_B\frac{\nu P_B(1+\tau_B)}{(1+\tau_A)^2}-Q_C\frac{\nu P_C}{(1+\tau_A)^2}\right] \\ &= -\frac{1}{(P_AQ_A+P_BQ_B+P_CQ_C)^2} \\ &\quad \left[\frac{P_AQ_A}{(1+\tau_A)(1+\tau_B)}\left\{\phi(\tau_A-\tau_B)+(1-\nu-\phi)\tau_A(1+\tau_B)\right\}\right].\end{aligned} \tag{71}$$

Therefore, it follows that:

$$\Omega_1 \leq 0 \ \text{ if } \ \phi(\tau_A-\tau_B)+(1-\nu-\phi)\tau_A(1+\tau_B) \geq 0. \tag{72}$$

Finally, using (70) and (72), we obtain from (54) that:

$$\frac{\partial U}{\partial \tau_A} \leq 0 \ \text{ if } \ \phi(\tau_A-\tau_B)+(1-\nu-\phi)\tau_A(1+\tau_B) \geq 0 \ \text{ and } \ \tau_A+\frac{s_A}{P_A} \geq \tau_B. \tag{73}$$

Now, for $\frac{\partial U}{\partial \tau_A}=0$ to occur, (54) suggests that both Ω_1 and $P_A\frac{\partial Y_A}{\partial \tau_A}+P_B\frac{\partial Y_B}{\partial \tau_A}+P_C\frac{\partial Y_C}{\partial \tau_A}$ must be zero. It follows from (70) and (72) that it requires $\tau_A=\tau_B=s_A=0$. Hence, except for the trivial case where there is no distortion ($\tau_A=\tau_B=s_A=0$), we have shown that for any s_A,

$$\frac{\partial U}{\partial \tau_A} < 0 \ \text{ if } \ \phi(\tau_A-\tau_B)+(1-\nu-\phi)\tau_A(1+\tau_B) \geq 0 \ \text{ and } \ \tau_A+\frac{s_A}{P_A} \geq \tau_B. \tag{74}$$

□

Appendix D Proof of Proposition 3

Proof. Taking the partial derivatives of Q_A, Q_B, and Q_C with respect to τ_B, we obtain the following.

$$\frac{\partial Q_A}{\partial \tau_B} = -\left(P_A + P_B\frac{Q_B}{Q_A} + P_C\frac{Q_C}{Q_A}\right)^{-2}\left(P_B\frac{\partial\left(\frac{Q_B}{Q_A}\right)}{\partial \tau_B}\right)(P_AY_A + P_BY_B + P_CY_C)$$
$$+\left(P_A + P_B\frac{Q_B}{Q_A} + P_C\frac{Q_C}{Q_A}\right)^{-1}\left(P_A\frac{\partial Y_A}{\partial \tau_B} + P_B\frac{\partial Y_B}{\partial \tau_B} + P_C\frac{\partial Y_C}{\partial \tau_B}\right), \tag{75}$$

$$\frac{\partial Q_B}{\partial \tau_B} = -\left(P_A\frac{Q_A}{Q_B} + P_B + P_C\frac{Q_C}{Q_B}\right)^{-2}\left(P_A\frac{\partial\left(\frac{Q_A}{Q_B}\right)}{\partial \tau_B} + P_C\frac{\partial\left(\frac{Q_C}{Q_B}\right)}{\partial \tau_B}\right)(P_AY_A + P_BY_B + P_CY_C)$$
$$+\left(P_A\frac{Q_A}{Q_B} + P_B + P_C\frac{Q_C}{Q_B}\right)^{-1}\left(P_A\frac{\partial Y_A}{\partial \tau_B} + P_B\frac{\partial Y_B}{\partial \tau_B} + P_C\frac{\partial Y_C}{\partial \tau_B}\right), \tag{76}$$

$$\frac{\partial Q_C}{\partial \tau_B} = -\left(P_A\frac{Q_A}{Q_C} + P_B\frac{Q_B}{Q_C} + P_C\right)^{-2}\left(P_B\frac{\partial\left(\frac{Q_B}{Q_C}\right)}{\partial \tau_B}\right)(P_AY_A + P_BY_B + P_CY_C)$$
$$+\left(P_A\frac{Q_A}{Q_C} + P_B\frac{Q_B}{Q_C} + P_C\right)^{-1}\left(P_A\frac{\partial Y_A}{\partial \tau_B} + P_B\frac{\partial Y_B}{\partial \tau_B} + P_C\frac{\partial Y_C}{\partial \tau_B}\right). \tag{77}$$

Using (75), (76), and (77), we can rewrite (26) as follows.

$$\frac{\partial U}{\partial \tau_B} = Q_A^{\nu}Q_B^{\phi}Q_C^{1-\nu-\phi}\left\{\Omega_3\left(P_AY_A + P_BY_B + P_CY_C\right) + \Omega_2\left(P_A\frac{\partial Y_A}{\partial \tau_B} + P_B\frac{\partial Y_B}{\partial \tau_B} + P_C\frac{\partial Y_C}{\partial \tau_B}\right)\right\}, \tag{78}$$

where

$$\Omega_3 \equiv -\frac{\nu}{Q_A}\left(P_A + P_B\frac{Q_B}{Q_A} + P_C\frac{Q_C}{Q_A}\right)^{-2}\left(P_B\frac{\partial\left(\frac{Q_B}{Q_A}\right)}{\partial\tau_B}\right)$$
$$-\frac{\phi}{Q_B}\left(P_A\frac{Q_A}{Q_B} + P_B + P_C\frac{Q_C}{Q_B}\right)^{-2}\left(P_A\frac{\partial\left(\frac{Q_A}{Q_B}\right)}{\partial\tau_B} + P_C\frac{\partial\left(\frac{Q_C}{Q_B}\right)}{\partial\tau_B}\right)$$
$$-\frac{1-\nu-\phi}{Q_C}\left(P_A\frac{Q_A}{Q_C} + P_B\frac{Q_B}{Q_C} + P_C\right)^{-2}\left(P_B\frac{\partial\left(\frac{Q_B}{Q_C}\right)}{\partial\tau_B}\right). \tag{79}$$

It is straightforward to see that $P_AY_A + P_BY_B + P_CY_C > 0$ and $\Omega_2 > 0$, which is defined in (56). To analyse the sign of $\frac{\partial U}{\partial\tau_B}$, we further need the signs of Ω_3 and $P_A\frac{\partial Y_A}{\partial\tau_B} + P_B\frac{\partial Y_B}{\partial\tau_B} + P_C\frac{\partial Y_C}{\partial\tau_B}$ in (78).

We first derive the sign of $P_A\frac{\partial Y_A}{\partial\tau_B} + P_B\frac{\partial Y_B}{\partial\tau_B} + P_C\frac{\partial Y_C}{\partial\tau_B}$ by rewriting the expression as:

$$P_A\frac{\partial Y_A}{\partial\tau_B} + P_B\frac{\partial Y_B}{\partial\tau_B} + P_C\frac{\partial Y_C}{\partial\tau_B} = P_A\frac{\partial Y_A}{\partial L_A}\frac{\partial L_A}{\partial\tau_B} + P_B\frac{\partial Y_B}{\partial L_B}\frac{\partial L_B}{\partial\tau_B} + P_C\frac{\partial Y_C}{\partial L_C}\frac{\partial L_C}{\partial\tau_B}$$
$$= P_A\frac{\partial Y_A}{\partial L_A}\left(-\frac{\partial L_B}{\partial\tau_B} - \frac{\partial L_C}{\partial\tau_B}\right) + P_B\frac{\partial Y_B}{\partial L_B}\frac{\partial L_B}{\partial\tau_B} + P_C\frac{\partial Y_C}{\partial L_C}\frac{\partial L_C}{\partial\tau_B}$$
$$= \frac{\partial L_B}{\partial\tau_B}\left(P_B\frac{\partial Y_B}{\partial L_B} - P_A\frac{\partial Y_A}{\partial L_A}\right) + \frac{\partial L_C}{\partial\tau_B}\left(P_C\frac{\partial Y_C}{\partial L_C} - P_A\frac{\partial Y_A}{\partial L_A}\right) \tag{80}$$

To obtain the sign of (80), we first derive the sign of $\frac{\partial L_C}{\partial\tau_B}$. Given the result, we consider $\frac{\partial L_B}{\partial\tau_B}$. The signs of $P_C\frac{\partial Y_C}{\partial L_C} - P_A\frac{\partial Y_A}{\partial L_A}$ and $P_B\frac{\partial Y_B}{\partial L_B} - P_A\frac{\partial Y_A}{\partial L_A}$ are given in (69) and (70) , respectively.

- **The sign of $\frac{\partial L_C}{\partial\tau_B}$**

  We first need to solve for $\frac{\partial L_A}{\partial\tau_B}$. From (59) and using the implicit function theorem, we obtain:

$$\frac{\partial L_A}{\partial\tau_B} = -\frac{f_{\tau_B}}{f_{L_A}}, \tag{81}$$

$$f_{\tau_B} = P_C \gamma(\gamma - 1)(\overline{L} - L_A - L_B)^{\gamma-2} \frac{K_C^{1-\gamma} L_B}{(1-\beta)(1+\tau_B)}, \tag{82}$$

where L_B is given as a function of L_A as written in (58), and f_{L_A} is given in (62). Substituting these into (81), it follows that:

$$\frac{\partial L_A}{\partial \tau_B} = \frac{(\gamma - 1) L_A L_B}{(1+\tau_B)\{(\beta-1)(\gamma-1)L_A + (\alpha-1)(\gamma-1)L_B + (\alpha-1)(\beta-1)L_C\}} < 0, \tag{83}$$

where the inequality follows from $0 < \alpha < 1$, $0 < \beta < 1$, and $0 < \gamma < 1$. (4) and (5) imply that:

$$\{P_A(1+\tau_A) + s_A\} \alpha K_A^{1-\alpha} L_A^{\alpha-1} = P_C \gamma K_C^{1-\gamma} L_C^{\gamma-1}. \tag{84}$$

Taking the partial derivative of both sides of this equation with respect to τ_B,

$$\{P_A(1+\tau_A) + s_A\} \alpha(\alpha-1) K_A^{1-\alpha} L_A^{\alpha-2} \frac{\partial L_A}{\partial \tau_B} = P_C \gamma(\gamma-1) K_C^{1-\gamma} L_C^{\gamma-2} \frac{\partial L_C}{\partial \tau_B}. \tag{85}$$

Since $\frac{\partial L_A}{\partial \tau_B} < 0$ as obtained in (83) and $0 < \alpha < 1$, the left hand side of (85) is positive. Since $0 < \gamma < 1$, from the right hand side of (85), we obtain:

$$\frac{\partial L_C}{\partial \tau_B} < 0. \tag{86}$$

- **The sign of $\frac{\partial L_B}{\partial \tau_B}$**

  From (19), it follows that:

$$\frac{\partial L_B}{\partial \tau_B} = -\frac{\partial L_A}{\partial \tau_B} - \frac{\partial L_C}{\partial \tau_B} > 0, \tag{87}$$

  where the inequality follows from (83) and (86).

Therefore, (69), (70), (80), (86), and (87) together imply that:

$$P_A \frac{\partial Y_A}{\partial \tau_B} + P_B \frac{\partial Y_B}{\partial \tau_B} + P_C \frac{\partial Y_C}{\partial \tau_B} \leq 0 \text{ when } \tau_B \geq \tau_A + \frac{s_A}{P_A}. \tag{88}$$

We are left with the analysis of the sign of Ω_3 in (78). Following from (79), Ω_3 is further rewritten as:

$$\begin{aligned}
&\Omega_3 = \frac{1}{(P_A Q_A + P_B Q_B + P_C Q_C)^2} \\
&\left\{ Q_A \frac{\phi P_A (1+\tau_A)}{(1+\tau_B)^2} - Q_B \frac{\nu P_B + (1-\nu-\phi) P_B (1+\tau_A)}{1+\tau_A} + Q_C \frac{\phi P_C}{(1+\tau_B)^2} \right\} \\
&= -\frac{1}{(P_A Q_A + P_B Q_B + P_C Q_C)^2} \\
&\left[\frac{\phi P_A Q_A}{\nu (1+\tau_B)^2} \left\{ \nu(\tau_B - \tau_A) + (1-\nu-\phi)(1+\tau_A)\tau_B \right\} \right]. \qquad (89)
\end{aligned}$$

Therefore, it follows that,

$$\Omega_3 \leq 0 \quad \text{if} \quad \nu(\tau_B - \tau_A) + (1-\nu-\phi)(1+\tau_A)\tau_B \geq 0. \qquad (90)$$

Finally, using (88) and (90) in (78), we obtain:

$$\frac{\partial U}{\partial \tau_B} \leq 0 \quad \text{if} \quad \nu(\tau_B - \tau_A) + (1-\nu-\phi)(1+\tau_A)\tau_B \geq 0 \quad \text{and} \quad \tau_B \geq \tau_A + \frac{s_A}{P_A}. \qquad (91)$$

Similarly to the proof of Proposition 2, $\frac{\partial U}{\partial \tau_B} = 0$ occurs only in the trivial case where there is no distortion. Hence, except for the trivial case ($\tau_A = \tau_B = s_A = 0$), we have proven that for any s_A,

$$\frac{\partial U}{\partial \tau_B} < 0 \quad \text{if} \quad \nu(\tau_B - \tau_A) + (1-\nu-\phi)(1+\tau_A)\tau_B \geq 0 \quad \text{and} \quad \tau_B \geq \tau_A + \frac{s_A}{P_A}. \qquad (92)$$

□