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Abstract

Extracting a trend component from nonstationary data is one of the first challenges in estimating a DSGE model. The misspecification of the component can distort structural parameter estimates and translate into a bias in policy-relevant statistic estimates. This paper investigates how important this bias is to estimated policy implications within a DSGE framework. The quantitative results suggest the bias in parameter estimates due to trend misspecification can result in significant inaccuracies in estimating statistics of interest. This then misleads policy conclusions. Particularly, a misspecified model is estimated using a deterministic-trend specification when the true process is a random-walk with drift.

1 Introduction

Macroeconomic data can typically be decomposed into a trend component and a cyclical component. Structural models such as Dynamic Stochastic General Equilibrium (DSGE) models are typically constructed around the stationary cyclical components of time series. Hence, the data are usually detrended in some way prior to estimation of the model. There has been a long debate on how to extract the cyclical component from the raw data in order to be used for the estimation of a structural model. Two common approaches have been proposed within the literature. Given a DSGE model, econometricians make an assumption regarding a trend specification in the data and extract the cyclical component either by (1) using filtering devices (e.g. linear detrending or first differencing) on the raw data before the estimation or (2) estimating the trend and cycle jointly with the DSGE model. Regardless of the approach, an incorrect assumption about the trend process can lead to mismeasurement of the cyclical component. This mismeasurement in turn causes a distortion in structural parameter estimates specified in the model. For policy makers, the distortion of structural parameter estimates is not the primary concern. Rather, they are more concerned about the accuracy of policy-relevant statistic estimates (e.g. impulse response functions and variance decompositions) and whether policy implications deduced from such estimates are misleading. These implied measures of policy implications are however dependent on potentially biased structural parameter estimates. This paper therefore investigates ways in which trend misspecifications can distort the accuracy of these policy-related statistics of interest given a DSGE framework.

There have been many studies estimating DSGE models to answer specific economic questions. The difference assumptions the econometricians make about trend processes in the data result in a different choice of detrending method used prior to estimation. Kim (2000), Dib (2003) and Smets and Wouters (2003), for example, specify a deterministic-trend in a model and linearly detrend the data prior to estimation. Ireland (2004), Del Negro et al. (2005) and Smets and Wouters (2007), on the other hand, assume a stochastic trend in their models and filter the data by first differencing prior to the estimation. Stock (1991), Rudebusch (1992) and Chang et al. (2007) show that there

is in fact an empirical difficulty in distinguishing between a stochastic and a deterministic-trend in macroeconomic time series. As a result, there is a potential problem of specifying an incorrect trend specification.

By misspecifying the trend, econometricians can induce a trend misspecification problem when they filter nonstationary data. Mismeasurement of the trend component can lead to problems in the estimation of a structural model and result in a bias in structural parameter estimates. To gain some understanding of this issue, simulation exercises become useful. Cogley (2001), Fukac and Pagan (2010) and Canova and Ferroni (2011) construct different data generating processes to investigate the use of statistical devices (e.g. linear trend, first difference and Hodrick-Prescott filter) and a model-based transformation on simulated data. Regardless of the filtering approach applied to the data, they find that structural parameter estimates are severely biased when an incorrect trend assumption is applied. The degree of bias varies depending on which approach is used and what was the underlying process. Further studies focus on the impact of this bias in parameter estimates on policy-relevant measurements. Canova and Ferroni (2011) and Filippo (2011), for instance, examine the implications of using inappropriate filtering devices for the impulse response function estimates while Clements and Hendry (2001) look into forecastability when trend misspecification exists. Following these studies, many researchers have proposed alternative methodologies to improve estimation (see e.g. Gorodnichenko and Ng, 2010; Filippo, 2011; Canova and Ferroni, 2011).

This paper builds upon our understanding and pursues further investigation of the consequences of trend misspecifications on policy implications. Unlike other studies, this paper does not focus only on determining the first-moment statistic of the potential bias in parameter and policy-relevant statistic estimates induced by trend misspecifications, relative to relevant true values. The paper studies properties of the distribution of the estimates as well as incorporates the sampling uncertainty into the results. To summarise this information, coverage rates are implemented.[1] The coverage rate for an estimator is defined as the fraction of times that some credible intervals (i.e. 95% Highest Posterior Density Interval) contain the relevant true values. We can therefore use the coverage rate

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[1]The idea of coverage rates has been implemented by, for example, Christiano et al. (2007) and Paustian (2007) to assess the accuracy of confidence intervals under a framework of a structural vector autoregression.

as a measure of accuracy of the estimator. In turn, we can also assess whether the size of the bias in structural parameter estimates is significant in affecting the accuracy of estimated statistics of interest. This is a useful information as we can answer a question of how likely we would obtain true values with some confidence level when a trend misspecification is allowed in a DSGE model. The main contribution of this paper is then to measure the accuracy of policy-relevant statistic estimates under a misspecified framework and demonstrate further compelling evidence of how important the trend misspecification is to policy analysis.

In this paper, I follow the econometric methodology proposed by Filippo (2011) to use nonstationary data and jointly estimate parameters governing trend processes with structural parameters specified in a DSGE model. The data generating process (DGP) is a DSGE model, as is the estimated model. In both the DGP and the estimated model, two trend specifications are considered; a deterministic-trend and a random-walk possibly with drift. The implications of trend misspecification are investigated by comparing all possible combinations of these two trend specifications in the DGP and the estimated model. Importantly, the trend specification in the estimated model may not be the same as one in the DGP. The bias in parameter estimates is then measured relative to the DGP's parameter values. As the trend process specified within the estimated model will in some cases differ from the trend process within the DGP, the structural parameter estimates become biased. This bias then changes the agents' equilibrium decisions and the dynamic of the model is altered to compensate for the mismeasurement of the cyclical component. For example, when the trend is misspecified, households become more inelastic in making a labour-consumption choice and exogenous processes are estimated to be more volatile and less persistent than they are in the DGP. These results are consistent with Filippo (2011).

The estimated structural parameters are then used to study consequences of trend misspecification on policy implications. The policy-relevant statistics considered in this paper are impulse response functions and variance decompositions. These measurements can be expressed as nonlinear functions of structural parameters. They can therefore be directly affected by any potential bias in structural parameter estimates. By using coverage rates, I find that the estimated parameters are not only distorted but the degree

of distortion is also large enough to reduce the accuracy of the policy statistics of interest to a practically important degree. This in turn misleads policy advice deduced from such statistics. By comparing two cases of misspecification, the mean squared errors and the coverage rates suggest that a misspecified model with a random-walk specification is able to provide smaller distortion of parameter estimates and has higher accuracy of policy-relevant measurements than a misspecified model with a deterministic-trend specification. Even when this is the case, however, policy makers still need to be cautious in interpreting the results from variance decompositions as the coverage rates are shown to be quite low for some variables.

This paper considers two different frameworks of DSGE models; a standard Real Business Cycle (RBC) and a New Keynesian (NK) model. This leads us to make another observation worth mentioning. Depending on which framework is implemented, the structural parameter estimates that become biased due to trend misspecification are different in interesting ways. In the case of an RBC model, the inverse elasticities associated with households are severely biased upwards, whereas the policy parameters specified in the Taylor rule are affected in the case of a NK model. As a NK-type model is a popular workhorse used by many central banks and the policy-rule parameters are particularly of interest, this result raises a concern for policy makers. This paper shows that these parameters governing the reaction of a central bank to a shock in an economy can be biased upwards if a trend component in nonstationary data is mistreated, and this may result in a substantial reduction in the accuracy of both impulse response functions and variance decompositions.

The rest of the paper is organised as follows. Section 2 provides an econometric methodology used to estimate a DSGE model and describes how simulation exercises are designed in this paper. Section 3 provides a description of the model used as a DGP and an estimated model and then discusses consequences of trend misspecification upon policy implications. Finally, Section 4 concludes and outlines a strategy for subsequent research.

2 Econometric Methodology

In this paper, I use the approach proposed by Filippo (2011) to estimate a DSGE model. Instead of filtering nonstationary data prior to the estimation, this approach computes the likelihood directly from a $n_y \times 1$ vector of observable variables, y_t. Filippo (2011) shows in detail that this approach is able to provide smaller parameter distortion compared to the filtering approach. The proposed methodology is called a one-step approach. The observable variables can be decomposed into a reduced form representation for the non-cyclical component (y_t^τ) and a structural representation for the cyclical component (y_t^c). The observable variables are then given by

$$y_t = y_t^\tau + y_t^c.$$

The cyclical behaviour of the data can be described by a stationary DSGE model whose linear state-space representation is given by

$$y_t^c = S\tilde{y}_t$$
$$\tilde{y}_t = \mathbf{g}(\Omega^m)\tilde{x}_t$$
$$\tilde{x}_t = \mathbf{h}(\Omega^m)\tilde{x}_{t-1} + \sigma_\epsilon \epsilon_t$$

where S is a selection matrix corresponding the cyclical component of the data y_t^c to the log-deviation variables from steady state values and trend components in the model $\tilde{y}_t$, $\tilde{x}_t$ is a $n_x \times 1$ vector of log-deviation unobservable state variables from steady state values, $\mathbf{g}(\Omega^m)$ and $\mathbf{h}(\Omega^m)$ are matrices of reduced-form parameters as a function of structural parameters of the model, $\epsilon_t \sim \mathcal{N}(\mathbf{0}_{n_\epsilon}, I_{n_\epsilon})$, $\mathbf{0}_{n_\epsilon}$ is a $n_\epsilon \times 1$ zero vector and $\sigma_\epsilon \sigma_\epsilon' = \Sigma_\epsilon$ is a variance-covariance matrix.

The trend component can be described by some filter functions, $y_t^\tau = \mathcal{F}_\tau(y_t)$, according to a trend assumption econometricians make on the observable variables. In this paper, given a set of simulated data, econometricians in the experiment consider two common trend specifications; a deterministic-trend (dt) and a random-walk (rw) process. The choice of filtering methodology depends on the econometricians' belief about the trend component in the data.

Suppose econometricians believe that the observable variables are stationary around a constant growth rate of θ_1 and integrated of order zero, I(0). Econometricians therefore consider a deterministic linear trend specification as a filter function. This model specification is denoted by $\mathcal{M}^{(dt)}$ and given by

$$y_t^\tau \quad = \quad \theta_0 + \theta_1 t + \sigma_v v_t$$

where θ_0 is a column vector representing means of the variables, $v_t \sim \mathcal{N}(\mathbf{0}_{n_y}, I_{n_y})$ and $\sigma_v \sigma_v' = \Sigma_v$ is a variance-covariance matrix.

On the other hand, suppose the observable variables are assumed by econometricians to evolve according to a stochastic trend and be integrated of order one, I(1), then they consider a stochastic linear-trend process such as the so-called random walk specification as a filter function. This model specification is denoted by $\mathcal{M}^{(rw)}$ and given by

$$y_t^\tau \quad = \quad \theta_1 + \Gamma y_{t-1}^\tau + \sigma_v v_t$$

where θ_1 is a $n_y \times 1$ vector of the drift, Γ is a diagonal matrix which has zeros or ones on the main diagonal, $v_t \sim \mathcal{N}(\mathbf{0}_{n_y}, I_{n_y})$, $\sigma_v \sigma_v' = \Sigma_v$ is a variance-covariance matrix and y_0^τ is θ_0.

Therefore, the cyclical component y_t^c is explained as a function of structural parameters from the DSGE model Ω^m whereas the non-cyclical component y_t^τ is governed by filter parameters $\Omega^{\mathcal{F}} \equiv \{\theta_0, \theta_1, \Sigma_v\}$. With this approach, these two sets of parameters $\Omega \equiv \{\Omega^m, \Omega^{\mathcal{F}}\}$ are jointly estimated using a Bayesian method.

2.1 Policy Implications

Given an estimated DSGE model, I consider two important questions policy makers are interested in. These are: what is the impact of a shock in the economy, and what are the main driving forces? These questions can be answered by computing impulse response functions and variance decompositions from an estimated model. The impulse response functions help explain the movements of macroeconomic variables in response to different structural shocks and the variance decompositions measure the contribution of each type

of structural shock to the forecast error variance. Given an estimated DSGE model whose linear state-space representation is

$$\tilde{y}_t = \mathbf{g}(\hat{\Omega}^m)\tilde{x}_t$$

$$\tilde{x}_t = \mathbf{h}(\hat{\Omega}^m)\tilde{x}_{t-1} + \hat{\sigma}_\epsilon \epsilon_t,$$

the statistics of interest can be computed as follows.

Impulse response functions is defined as differences between the response of $\tilde{y}_{t+s}$ to a change in a structural shock ϵ_t by 1 standard deviation at period t and the one without the shock. Impulse response functions can then be expressed as a nonlinear function of structural parameter estimates given by

$$IRF(s; \hat{\Omega}^m) \equiv \left[vec\left(\frac{\partial \tilde{y}_{t+1}}{\partial \epsilon_t}\right)', \ldots, vec\left(\frac{\partial \tilde{y}_{t+s}}{\partial \epsilon_t}\right)' \right]'$$

$$= (\hat{\sigma}_\epsilon' \otimes \mathbf{g}(\hat{\Omega}^m) \otimes I_s) b(s; \hat{\Omega}^m)$$

where s is the number of periods ahead and $b(s; \hat{\Omega}^m) \equiv \left[vec(\mathbf{h}(\hat{\Omega}^m)^0, \ldots, vec(\mathbf{h}(\hat{\Omega}^m)^s) \right]$.

The variance decompositions can also be expressed as nonlinear functions of structural parameter estimates and s-period ahead, $VD(s; \hat{\Omega}^m)$. For each variable i, the contribution of shock j to variation of the underlying variable at s-period ahead can be expressed as

$$VD_{ij}(s; \hat{\Omega}^m) = \frac{\psi_{ij}(s; \hat{\Omega}^m)}{\sum_{j=1}^{n_\epsilon} \psi_{ij}(s; \hat{\Omega}^m)}$$

where, for any matrix $\mathbf{a}$, $\mathbf{a}_{i\cdot}$ is a ith row vector of matrix $\mathbf{a}$ and $\mathbf{a}_{\cdot j}$ is a jth column vector of matrix $\mathbf{a}$, $\psi_{ij}(s; \hat{\Omega}^m) = \hat{\sigma}_{\epsilon,j} \mathbf{g}_{i\cdot}(\hat{\Omega}^m) \mathbf{h}_{\cdot j}(\hat{\Omega}^m)^s$ which can also be seen as an impulse response of shock j to variable i, $\sum_{j=1}^{n_\epsilon} VD_{ij}(s; \hat{\Omega}^m) = 1$ and $\hat{\sigma}_{\epsilon,j}$ is an estimated standard deviation of shock j.

These statistics of interest are dependent on estimated structural parameters specified in the model. Any distortion in structural parameter estimates can therefore result in distortions in the estimates of these functions. The focus of this paper is on the distortion of parameter, impulse response and variance decomposition estimates induced by trend misspecification in an estimated DSGE model.

2.2 Simulation Exercises

I study the role of trend misspecification upon policy implications by considering two different trend specifications in the data generating process (DGP) and the DSGE model used in the estimation, which may not be the same. The trend specifications of interest are a deterministic-trend and a random-walk process. By alternating trend specifications of interest, I examine two cases of correct trend specification and another two cases of trend misspecification. The correct specification cases are a situation where the underlying trend process coincides with the one assumed by the econometricians (in our experiments) in their estimated models. The misspecification cases, on the other hand, are cases where the trend process constructed in the DGP and the one assumed in the model are different. The simulation exercises are summarised in Table 1. I consider two common specifications of DSGE models used in the literature; a Real Business Cycle (RBC) and a New Keynesian (NK) model.

In this paper, the distortion of parameter estimates is measured by mean squared errors (MSE) which can be decomposed into a variance and a bias of parameter estimates. The variance provides information on how far on average the posterior draws lie from a posterior mean, while the bias measures difference between a posterior mean and a true value specified in the DGP. The sign of the bias captures the directional bias of the estimates. MSE can then be expressed as

$$
\begin{aligned}
V(\Omega) &= \frac{1}{N} \sum_{n=1}^{N} \text{diag}[(\bar{\Omega} - \Omega^{(n)})(\bar{\Omega} - \Omega^{(n)})'] \\
\text{Bias}(\Omega, \Omega^0) &= \bar{\Omega} - \Omega^0 \\
MSE(\Omega, \Omega^0) &= V(\Omega) + \text{diag}[\text{Bias}(\Omega, \Omega^0)\text{Bias}(\Omega, \Omega^0)']
\end{aligned}
$$

where $\bar{\Omega}$ is a column vector of posterior means of parameter estimates, $\Omega^{(n)}$ is a column vector of parameter estimates for n^{th} draw, Ω^0 is a set of true parameter values and $N = 1,000$ is a total number of kept draws from a Markov chain.

The contribution of this paper is to demonstrate whether the parameter distortion measured previously is significant in affecting the accuracy of policy statistic estimates. I therefore measure the accuracy of these statistics by computing coverage rates of the 95%

Highest Posterior Density Intervals (HPDI) of such statistics. This coverage rate is defined as a fraction of times, across 150 simulations, that the 95% HPDI of statistic estimator contains the relevant true values. A high coverage rate (i.e. more than 90%) indicates that the distortion in parameter estimates due to trend misspecification is not significant in reducing the accuracy of policy-relevant measurements. Hence, the policy conclusions deduced from such statistics are considered reasonable.

3   Results and Discussion

This section begins with the specification of a standard Real Business Cycle (RBC) model, which does not have policy implications. I consider this model to illustrate the implementation of coverage rates as an accuracy measurement. Then, the consequences of trend misspecifications upon policy implications is investigated given a New Keynesian (NK) framework. At the end of each model section, results and implications from the coverage rates will be discussed.

3.1   Exercise 1: A Standard RBC Model

This model is a simple variation of Chang et al. (2007). I consider the specification where there is no adjustment cost of labour and exogenous processes evolve according to a stationary AR(1) process. Households in this economy optimise their expected discounted lifetime utility by choosing each period consumption (C_t), hours worked (H_t) and next-period capital holding (K_{t+1}) subject to their budget constraint and a capital accumulation equation. The endowment of time is normalised to be 1 which can be taken as leisure or hours worked. The sources of income for households are from supplying capital and labour services to firms. Income in this setting can be either consumed or invested. Firms optimise their profit subject to the labour-augmenting Cobb-Douglas production.

Let, for any variable a_t, a log-deviation from a steady state value (a_*) and a trend component (a_t^τ) be $\tilde{a}_t = ln\left(\frac{a_t}{a_* a_t^\tau}\right)$. The equilibrium conditions of this economy can be

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summarised in terms of log-deviation by the system of the following five equations.

$$\eta \tilde{C}_t = \mathbb{E}_t \left\{ \eta \tilde{C}_{t+1} - (1-\alpha)(1-\beta+\beta\delta)(\tilde{H}_{t+1} - \tilde{K}_{t+1} + \tilde{z}_{t+1}) \right\} \tag{1}$$

$$\eta \tilde{C}_t = \alpha \tilde{K}_t - (\alpha+\gamma)\tilde{H}_t + (1+\gamma)\tilde{B}_t + (1-\alpha)\tilde{z}_t \tag{2}$$

$$S_c \tilde{C}_t = \left(\alpha + \frac{(1-S_c)(1-\delta)}{\delta} \right) \tilde{K}_t - \left(\frac{1-S_c}{\delta} \right) \tilde{K}_{t+1} + (1-\alpha)\tilde{H}_t$$

$$+ (1-\alpha)\tilde{z}_t \tag{3}$$

$$\tilde{z}_{t+1} = \rho_z \tilde{z}_t + \epsilon_{z,t+1} \tag{4}$$

$$\tilde{B}_{t+1} = \rho_B \tilde{B}_t + \epsilon_{B,t+1} \tag{5}$$

where $S_c = C_*/Y_*$ is a share of consumption, $\beta \in (0,1)$ is a discount factor, $\alpha \in (0,1)$ is a capital share, $\eta > 0$ is an inverse intertemporal elasticity of substitution, $\gamma > 0$ is an inverse short-run (Frisch) labour supply elasticity, $\delta \in (0,1)$ is a depreciation rate of capital, $\rho_z \in (0,1)$ and $\rho_B \in (0,1)$ are measures of a persistence and $\epsilon_{z,t+1} \sim \mathcal{N}(0,\sigma_z^2)$ and $\epsilon_{B,t+1} \sim \mathcal{N}(0,\sigma_B^2)$ are Gaussian shocks in the exogenous technology and preference shock respectively.

These necessary conditions characterise the equilibrium decision rules for households and firms. Equation (1) is an Euler equation for consumption stating that the marginal rate of substitution between the consumption at period t and the consumption at period $t+1$ equals the marginal product of capital. Equation (2) is a labour supply equation stating that the marginal rate of substitution between consumption and leisure must equal the marginal product of labour. Equation (3) is a resource constraint which the equilibrium allocations from both households and firms need to satisfy. Equations (4) and (5) present the processes of technology and preference shocks respectively.

Data Generating Process

The simulated data of consumption and hours worked, $\{C_t, H_t\}_{t=0}^{T}$, used in this exercise are constructed following the specification presented in Section 2,

$$\begin{pmatrix} C_t \\ H_t \end{pmatrix} = \begin{pmatrix} C_t^\tau \\ H_t^\tau \end{pmatrix} + \begin{pmatrix} C_t^c \\ H_t^c \end{pmatrix}$$

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where the cyclical component $y_t^c \equiv (C_t^c, H_t^c)'$ is obtained from the log-linearised solution of the RBC model with $\tilde{y}_t \equiv (\tilde{C}_t, \tilde{H}_t)'$ and $\tilde{x}_t \equiv (\tilde{K}_t, \tilde{z}_t, \tilde{B}_t)'$.

For the non-cyclical component $y_t^\tau \equiv (C_t^\tau, H_t^\tau)'$, I consider two cases of data generating processes. In the first case, a deterministic-trend specification is assumed in the non-cyclical component. Hereinafter, I refer this case as $DGP^{(dt)}$ which is given by

$$\begin{pmatrix} C_t^\tau \\ H_t^\tau \end{pmatrix} = \begin{pmatrix} \theta_0 \\ 0 \end{pmatrix} + \begin{pmatrix} \theta_1 \\ 0 \end{pmatrix} t + \begin{pmatrix} \sigma_C & 0 \\ 0 & 0 \end{pmatrix} v_t.$$

For the second case, a random-walk specification is assumed which can be presented as

$$\begin{pmatrix} C_t^\tau \\ H_t^\tau \end{pmatrix} = \begin{pmatrix} \theta_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} C_{t-1}^\tau \\ H_{t-1}^\tau \end{pmatrix} + \begin{pmatrix} \sigma_C & 0 \\ 0 & 0 \end{pmatrix} v_t.$$

This case will be denoted as $DGP^{(rw)}$. Given these specifications, only consumption contains the trend component. The trend in both cases has an initial value of θ_0 and an average growth rate of θ_1. Therefore, the only difference between these two specifications is the trend of consumption. The filter parameters are set as follows; $\theta_0 = 2$, $\theta_1 = 0.0063$, $\sigma_C = 0.007$ and $v_t \sim \mathcal{N}(\mathbf{0}_2, I_2)$. The values of the structural parameters used in the data generating process are summarised in Table 5 in the Appendix. For each case, $T = 200$ observations are generated, which is equivalent to 50 years of quarterly data.

Parameter Estimation and Qualitative Analysis

Given the framework of the RBC model, the structural parameters are $\Omega^m \equiv \{\beta, \alpha, \delta, \gamma, \eta, \rho_z, \rho_B, \sigma_z, \sigma_B\}$ and the filter parameters are $\Omega^{\mathcal{F}} \equiv \{\theta_0, \theta_1, \sigma_C\}$. For each realisation of a DGP, econometricians in our study implement the methodology proposed by Komunjer and Ng (2011) to check necessary and sufficient conditions for structural parameters Ω^m to be identified.[2] Consequently, I assume they fix the discount rate $\beta = 0.99$ corresponding to a 4% annual interest rate and the capital share $\alpha = 0.33$

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[2]The procedure is briefly described in the Appendix. The identifiability of structural parameters is important as the lack of identification can contaminate the likelihood and affect the parameter estimation. There are many studies such as Canova and Sala (2009), Schorfheide (2011) and Guerron-Quintana et al. (2013) investigating the issue and the consequences for parameter estimation as well as inference of these estimates. By ensuring identifiability, the analysis in this paper is isolated from any bias induced by lack of identification.

to ensure the identifiability of structural parameters. The prior distributions of other parameters are specified to be centred around the true values. The prior distribution of σ_C is however bounded away from zero to guarantee that an estimated model with a random-walk process is always misspecified when the underlying trend process is a deterministic trend. The reason is that, once σ_C is allowed to be zero, there is a nonzero probability such that a random-walk specification can collapse to a deterministic-trend specification. The parameter distortions for the correct specification cases and the misspecification cases are summarised in Table 2. Figures 1 to 4 depict the coverage rates of the impulse response functions and the variance decompositions in each period under both correct specification and misspecification cases.

Under the correct specification cases, the nonzero mean squared errors (MSE) of parameter estimates in the first two blocks of Table 2 suggest that there are still some distortions in the estimation that are unaccounted for. These distortions therefore affect the dynamic of the model. Note that a bias in parameter estimates is presented as a percentage deviation from a parameter's DGP value. Given this set of trend specifications, the inverse short-run labour elasticity (γ) and the inverse elasticity of intertemporal substitution (η) are estimated about 20% and 3% higher than the true values respectively. These estimates suggest that households become less elastic and therefore they are less responsive on their consumption-labour equilibrium choice to any shock in the economy, especially a shock to their preference (B_t). Estimated impulse responses thus understate the true dynamic of macroeconomic variables. Moreover, the perception of the expected size of exogenous shocks in the system is also distorted. In particular, these shocks are estimated to be more volatile but less persistent than they are in their corresponding DGPs. The magnitude of the bias varies depending on which trend specification is considered. For instance, the standard deviation of the technology shock is estimated to be approximately 13% higher than it really is in the case of a deterministic-trend process while it is only 2% higher in the case of a random-walk process. The estimated impulse responses thus overstate the initial true impact as the size of the initial shock is normalised to 1 standard deviation, and converges back to zero relatively faster than in the DGP. However, the 95% HPDI of the impulse response functions is able to capture the associated true values more than 95% of the time as

shown in Figures 1.[3] Therefore, once the model is correctly specified, the coverage rates suggest that the level of distortion in parameter estimates due to some unaccounted factors is small and does not affect the accuracy of policy-relevant statistics.

The last two blocks of Table 2 report the parameter distortions under the misspecification cases. Due to the trend misspecification, it is not surprising that econometricians are unable to extract the trend component accurately and obtain nonzero biases in filter parameter estimates. The structural parameter estimates in these cases then become distorted to compensate for the misspecification. The block on the left shows the parameter distortion in the case where the true trend component is deterministic and estimated as if it were stochastic. The notation used for this case is $DGP^{(dt)} - \mathcal{M}^{(rw)}$. According to the MSEs, these parameter estimates are as much distorted as the ones in the correct specification cases, except for the persistence of technology shock (ρ_z). Due to less persistence of technology shock, the impulse responses to the shock converge back to zero relatively faster than the ones in the correct specification cases. The coverage rates of the corresponding impulse responses, displayed by dashed lines in Figure 3, therefore show a slight drop in later periods. The rates pick up once the impulse responses converge to zero. However, the overall coverage rates of impulse response functions in this case are still more than 90% throughout the entire period. This result indicates that the accuracy of impulse response functions can still be safely maintained. It is also worth mentioning that even though the MSEs are similar to the correct specification cases, most of the parameter estimates in this case have smaller bias but larger variance than in the cases of correct specification.

The block on the right shows the case where an econometrician estimates the model with a deterministic-trend while the underlying process is in fact a stochastic trend. This case is denoted by $DGP^{(rw)} - \mathcal{M}^{(dt)}$. Note that the numbers in parentheses are a ratio of the distortions in the case of $DGP^{(rw)} - \mathcal{M}^{(dt)}$ relative to the ones in the case of $DGP^{(dt)} - \mathcal{M}^{(rw)}$. A number greater than 1 thus implies that an estimated model with a deterministic-trend process provides a larger distortion than an estimated model with

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[3]I also consider another commonly used 95% credible interval to compute coverage rates in this paper. In contrast to 95% HPDI, this interval for some parameter Ω_i is defined as an interval $[a, b]$ such that $p(\Omega_i < a|y) = p(\Omega_i > b|y) = 0.025$. Even though the figures for the coverage rates are different, I arrive at similar conclusions as using 95% HPDI. Therefore, I do not present the results in this paper but they are available upon request.

a random-walk process. As we can see, this misspecification induces a larger distortion of structural parameter estimates compared to the competing misspecified case, except for the persistence of technology shock (ρ_z). The upward biased inverse elasticity of intertemporal substitution (η) and inverse short-run labour elasticity (γ) further disrupt the equilibrium behaviour of agents in the system, and the perception of the expected size of the exogenous shocks is further distorted. Particularly, the technology shock (z_t) is estimated to be even more volatile whereas the preference shock (B_t) is estimated to be even less persistent and more volatile than in the case of $DGP^{(dt)} - \mathcal{M}^{(rw)}$. The impact of these parameter distortions upon the accuracy of the impulse response functions can be reflected by the coverage rates. From the solid lines in Figure 3, the upward biased standard deviation causes the coverage rates of the impulse responses of the technology shock to start off low before picking up higher rates in later periods. Due to a well-estimated measure of persistence of the technology shock, reflected by an almost zero bias, the coverage rates of the corresponding responses in later periods can be even higher than the ones in the case of $DGP^{(dt)} - \mathcal{M}^{(rw)}$. The changes in the estimated equilibrium behaviour regarding labour choice and the dynamic of the preference shock, on the other hand, keep the coverage rates of the responses to the preference shock very low. Once again, rates increase when the responses converge to zero. These low coverage rates imply that the parameter distortions in this misspecified case are severe enough to greatly reduce the accuracy of the impulse response functions. One general observation worth mentioning here is that, despite having smaller variances compared to the case of $DGP^{(dt)} - \mathcal{M}^{(rw)}$, the large biases in parameter estimates cause the 95% HPDI to concentrate and situate in a wrong portion of the parameter space. This in turn leads to low coverage rates of impulse response functions.

Another policy-related statistic that I consider is variance decompositions. Figure 4 shows coverage rates of variance decompositions using 95% HPDI. As variance decompositions are closely related to impulse response functions, a poor performance in estimating impulse response functions leads to a reduction in the accuracy of the the function as can be seen in the case of $DGP^{(rw)} - \mathcal{M}^{(dt)}$. In particular, the poor performance in estimating impulse responses of the preference shock contributes greatly to inaccuracy of variance decompositions. However, it is not always the case that having

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high accuracy of impulse response functions will lead to high accuracy of variance decompositions. In the case of $DGP^{(dt)} - \mathcal{M}^{(rw)}$, for instance, the variance decompositions of consumption have very low coverage rates throughout the entire period of interest and the coverage rates in the case of output decline significantly when errors are forecast further ahead. This is due to nonlinearity of the variance decomposition as a function of the impulse response function. Any small distortion in estimated impulse response functions can be amplified and thus contribute to the distortion incurred in estimated variance decompositions. As a result, policy makers need to be careful in interpreting results of the variance decompositions when the trend misspecification might be present. By comparing coverage rates between misspecification cases, the case of $DGP^{(dt)} - \mathcal{M}^{(rw)}$ still performs better in estimating variance decompositions than in the case of $DGP^{(rw)} - \mathcal{M}^{(dt)}$.

To sum up the results of the RBC model, the trend misspecification does impact the estimation of structural parameters and policy-relevant statistics to a significant degree. The structural parameters which are severely affected by the misspecification are the two inverse elasticities associated with households, $\{\gamma, \eta\}$, and the parameters governed the exogenous processes, $\{\rho_z, \rho_B, \sigma_z, \sigma_B\}$. The coverage rates show that the degree of distortion among these parameter estimates is significant in affecting the accuracy of policy-related statistics, especially variance decompositions.

3.2 Exercise 2: A New Keynesian Model

In this section, the RBC model in the previous exercise is extended by incorporating a price stickiness and a central bank conducting a monetary policy rule. This is a slightly larger model in terms of the number of parameters to be considered in the experiment. As this type of DSGE model is popularly used by many policy makers, the experiment under this framework will provide useful implications concerning the policy analysis.

The economy under this framework can be briefly described as follows. In the beginning of period t, households have initial assets and allocate their income to purchase a portfolio of Arrow securities in addition to the consumption of a basket of differentiated goods and capital goods. A continuum of firms on the compact interval $[0, 1]$ stays in a monopolistically competitive environment and follows the calvo-price

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setting. That is, as firms produce differentiated goods, they are able to set up their own prices. However, the firms receive signals with only constant probability in each period to adjust the price levels. The central bank adjusts the monetary instrument, the nominal interest rate in this case, according to a Taylor (1993) rule.

The equilibrium conditions of this economy can be expressed in terms of log-deviation as follows. Define C_t as consumption, Y_t as output, I_t as investment, K_t as capital goods, i_t as nominal interest rate, π_t as inflation and mc_t as real marginal cost whereas z_t, $e_{B,t}$ and $e_{\pi,t}$ are exogenous processes of technology, demand and marginal cost shock respectively. For any variable a_t, let a log-deviation from a steady state value and a trend component be $\tilde{a}_t = ln\left(\frac{a_t}{a_* a^\tau}\right)$.

$$\tilde{Y}_t = \mathbb{E}_t \tilde{Y}_{t+1} - (1 - S_c)(\mathbb{E}_t \tilde{I}_{t+1} - \tilde{I}_t)$$
$$- \frac{1}{\eta}(\tilde{i}_t - \mathbb{E}_t \tilde{\pi}_{t+1}) + e_{B,t} \tag{6}$$

$$\tilde{i}_t - \mathbb{E}_t \tilde{\pi}_{t+1} = (1 - \beta + \beta\delta)\mathbb{E}_t(\tilde{mc}_{t+1} + \tilde{Y}_{t+1} - \tilde{K}_{t+1}) \tag{7}$$

$$\tilde{\pi}_t = \beta\mathbb{E}_t \tilde{\pi}_{t+1} + \tilde{mc}_t + e_{\pi,t} \tag{8}$$

$$\tilde{mc}_t = \left(\frac{\gamma + \alpha}{1 - \alpha}\right)\tilde{Y}_t - \left(\frac{1 + \gamma}{1 - \alpha}\right)\tilde{K}_t + \eta\tilde{C}_t - (1 + \gamma)\tilde{z}_t \tag{9}$$

$$\delta\tilde{I}_t = \tilde{K}_{t+1} - (1 - \delta)\tilde{K}_t \tag{10}$$

$$\tilde{Y}_t = S_c\tilde{C}_t + (1 - Sc)\tilde{I}_t \tag{11}$$

$$\tilde{i}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{Y}_t \tag{12}$$

$$\tilde{z}_{t+1} = \rho_z \tilde{z}_t + \epsilon_{z,t+1} \tag{13}$$

$$e_{B,t+1} = \rho_B e_{B,t} + \epsilon_{B,t+1} \tag{14}$$

$$e_{\pi,t+1} = \rho_\pi e_{\pi,t} + \epsilon_{\pi,t+1} \tag{15}$$

where $S_c = C_*/Y_*$ is a share of consumption, $\beta \in (0,1)$ is a discount factor, $\alpha \in (0,1)$ is a capital share, $\eta > 0$ is an inverse intertemporal elasticity of substitution, $\gamma > 0$ is an inverse short-run (Frisch) labour supply elasticity, $\delta \in (0,1)$ is a depreciation rate of capital, $\theta \in (0,1)$ is a fraction of sticky price firms, ϕ_π is a policy response to inflation, ϕ_y is a policy response to output, $\rho_z \in (0,1)$, $\rho_B \in (0,1)$ and $\rho_\pi \in (0,1)$ are measures of persistence and $\epsilon_{z,t+1} \sim \mathcal{N}(0, \sigma_z^2)$, $\epsilon_{B,t+1} \sim \mathcal{N}(0, \sigma_B^2)$ and $\epsilon_{\pi,t+1} \sim \mathcal{N}(0, \sigma_\pi^2)$ are Gaussian

shocks in the exogenous technology, demand and marginal cost shock respectively.

Equations (6) to (15) summarise the equilibrium conditions of the agents in this economy. Equation (6) is IS equation representing the relation of output, expected inflation and policy instrument. As there is capital in the economy, Equation (7) explains the connection between the real interest rate and marginal product of capital. The NK Phillips curve presented in Equation (8) shows the dependency of current inflation on expected future inflation and the real marginal cost. Equation (9) is the marginal cost equation. Equations (10) and (11) are the capital accumulation and resource constraint equation respectively. The reaction function of policy makers is given by Equation (12) stating that the nominal interest rate responses according to the Taylor principle where the degrees of monetary policy response to inflation and output are $\phi_\pi > 1$ and $\phi_y > 0$ respectively. The last three equations show that exogenous shocks in this setting are governed by a stationary AR(1) process.

Data Generating Process

Similar to the data generating process in the RBC model, the simulated data of output, nominal interest rate and inflation, $\{Y_t, i_t, \pi_t\}_{t=0}^{T}$, used in the exercise are constructed as

$$
\begin{pmatrix} Y_t \\ i_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} Y_t^\tau \\ i_t^\tau \\ \pi_t^\tau \end{pmatrix} + \begin{pmatrix} Y_t^c \\ i_t^c \\ \pi_t^c \end{pmatrix}
$$

where the cyclical component $y_t^c \equiv (Y_t^c, i_t^c, \pi_t^c)'$ is obtained from the log-linearised solution of the NK model with $\tilde{y}_t \equiv (\tilde{C}_t, \tilde{I}_t, \tilde{Y}_t, \tilde{i}_t, \tilde{\pi}_t, \tilde{mc}_t)'$ and $\tilde{x}_t \equiv (\tilde{K}_t, \tilde{z}_t, e_{B,t}, e_{\pi,t})'$.

Two cases of DGPs for the non-cyclical component $y_t^\tau \equiv (Y_t^\tau, i_t^\tau, \pi_t^\tau)'$ are considered; a deterministic-trend and a random-walk process. The former case is denoted by $DGP^{(dt)}$ and can be expressed as

$$
\begin{pmatrix} Y_t^\tau \\ i_t^\tau \\ \pi_t^\tau \end{pmatrix} = \begin{pmatrix} \theta_0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \theta_1 \\ 0 \\ 0 \end{pmatrix} t + \begin{pmatrix} \sigma_Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} v_t.
$$

The latter case is presented as

$$
\begin{pmatrix} Y_t^\tau \\ i_t^\tau \\ \pi_t^\tau \end{pmatrix} = \begin{pmatrix} \theta_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Y_{t-1}^\tau \\ i_{t-1}^\tau \\ \pi_{t-1}^\tau \end{pmatrix} + \begin{pmatrix} \sigma_Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \upsilon_t.
$$

This case is denoted as $DGP^{(rw)}$. Here, only output contains a trend component. The filter parameters in both cases are set as $\theta_0 = 2$, $\theta_1 = 0.0063$, $\sigma_Y = 0.007$ and $\upsilon_t \sim \mathcal{N}(\mathbf{0}_3, I_3)$. The values of structural parameters used in the data generating process are summarised in Table 5 in the Appendix. For each case, I generate 200 observations, which is equivalent to 50 years of quarterly data.

Parameter Distortion and Qualitative Analysis

Given the framework of a NK model, I have $\Omega^m \equiv \{\beta, \alpha, \delta, \gamma, \eta, \phi_\pi, \phi_y, \rho_z, \rho_B, \rho_\pi, \sigma_z, \sigma_B, \sigma_\pi\}$ as the structural parameters and $\Omega^{\mathcal{F}} \equiv \{\theta_0, \theta_1, \sigma_Y\}$ as the filter parameters. The setting of the experiment is similar to the experiment under a RBC model. That is, the priors are specified to be centred around the true values. To ensure the identifiability of structural parameters under this framework, I therefore assume that econometricians fix the share of consumption $S_c = 0.6$ and the fraction of sticky firms $\theta = 0.75$ in addition to the discount rate and the capital share.[4] The parameter distortions for both correct specification cases and misspecification cases are summarised in Table 3. Figures 5 to 7 and 9 plot the coverage rates of the impulse response functions and the variance decompositions in each period under the correct specification and misspecification cases.

The results under the correct specification cases are similar to the ones estimated using a RBC model. The nonzero MSEs reflect some distortion of parameter estimates incurred during the estimation. However, the coverage rates arrive at the same conclusion, suggesting that this level of distortion is not significant in affecting the accuracy of either impulse response functions or variance decompositions. This can be seen as, more than 90% of the time, the relevant true values of both impulse response functions and variance decompositions are maintained within 95% HPDI.

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[4]See Appendix A for the procedure to check for the identifiability of structural parameters.

As one would expect, once the trend process is misspecified, the filter parameters are biased as suggested by nonzero biases. Similar to the RBC case, estimating the model with an assumption of a random-walk process while the underlying process is a deterministic-trend process, denoted by $DGP^{(dt)} - \mathcal{M}^{(rw)}$, provides a similar distortion of structural parameter estimates as in the correct specification cases. The high coverage rates of impulse response functions presented by the dashed lines in Figure 7 suggest that the 95% HPDI of estimated impulse response functions does very well in capturing the corresponding true values. However, in the case of $DGP^{(rw)} - \mathcal{M}^{(dt)}$, the policy parameters responding to the deviation of output and inflation (ϕ_y and ϕ_π respectively) in the Taylor rule are distorted to compensate for this misspecification. The degree of distortion, on average, is substantially larger (i.e. more than 5 and 10 times) relative to the alternative misspecified case. The estimation errors of these policy parameters in our studies can also be seen clearly in Figure 8. This figure plots the estimated policy parameters from each simulation of both misspecification cases. While the estimates in the case of random walk process are mass around the true values presented by the intersection of the two dashed lines, the estimates in the case of deterministic trend disperse widely away from the true values. The bias in the policy-rule parameter estimates implies drastically different behaviour of the central bank reacting to a shock in the economy, especially a shock to the marginal cost. In particular, the central bank is estimated to be much more aggressive in conducting monetary policy to stabilise the economy. This causes other agents in the economy to alter their equilibrium decisions accordingly. The degree of bias in these policy parameter estimates is severe enough to distort the accuracy of impulse response functions. As can be seen by the straight lines in Figure 7, the coverage rates of impulse responses to the marginal cost shock are very low (i.e. less than 50% of the time) throughout the entire period. As other parameters are equally distorted, the coverage rates of impulse responses of both technology and demand shocks in these misspecification cases are similar and high. The result here at least gives positive evidence that we can still implement this misspecified model to study these shocks' propagation.

Once again, estimated DSGE models with an incorrect trend assumption cannot maintain the relevant true values of variance decompositions within 95% HPDI as well as when they are implemented to compute impulse response functions. Figure 9 shows the

coverage rates of variance decompositions using 95% HPDI. For the case of $DGP^{(dt)} - \mathcal{M}^{(rw)}$ represented by dashed lines, only the variance decompositions of the preference and marginal cost shock have high coverage rates. The coverage rates drop for the variance decompositions of the technology shock where the rates are only a bit higher than 50% of the time. The coverage rates become even worse in the case of $DGP^{(rw)} - \mathcal{M}^{(dt)}$. The rates for all variables and shocks are not even above 50% of the time, suggesting how inaccurate the estimated variance decompositions are. By comparing the coverage rates between the misspecification cases, the case of $DGP^{(dt)} - \mathcal{M}^{(rw)}$ is still able to provide a better estimation of variance decompositions than the case of $DGP^{(rw)} - \mathcal{M}^{(dt)}$.

In conclusion, the structural parameter estimates that are severely distorted by the trend misspecification in this framework are the policy-rule parameters responding to the deviation of output and inflation, $\{\phi_y, \phi_\pi\}$. This result is crucial to policy makers. Many studies have an interest in estimating interest rate rules to study the role of monetary policy in response to any shock in the economy over time and the determinacy of a rational expectation equilibrium (see e.g. Judd and Rudebusch, 1998; Clarida et al., 2000). However, the results above demonstrate a potential problem that should be given careful consideration when conducting empirical work with this DSGE model. The misspecification of a trend component in nonstationary macroeconomic data can result in potential bias in policy-rule parameter estimates and thus affect implications of monetary policy rules deduced from an estimated model.

Furthermore, similar to the RBC case considered previously, a misspecified NK model with a random-walk process can provide smaller parameter distortion and higher accuracy in estimating policy-relevant statistics compared to a misspecified NK model with a deterministic-trend process. A possible explanation why a model with a deterministic-trend process performs worse when the underlying process is in fact a stochastic trend is discussed as follows. Given a random-walk specification in a DGP, the trend specification can be expressed as a function of time period t and a sum of residuals,

$$y_t^\tau \quad = \quad \theta_1 t + \sigma_\upsilon \sum_{i=1}^{t} \upsilon_i.$$

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This expression is similar to the deterministic-trend process except for the second term which is due to a unit root. Our attention is then on the variance of trend innovation σ_v^2. When σ_v is large to some extent, the sum of residuals does matter to trend innovation. Incorrectly imposing a deterministic-trend specification as a filter function thus results in larger estimation error in filter parameters as the specification is not flexible enough to capture this permanent innovation. This in turn causes a bias in structural parameter estimates. Even so, it is therefore possible that, if we consider a small true value of σ_v in a DGP with a random-walk process, a misspecified model with a deterministic trend might be able to perform as well as an alternative misspecification case. However, I do not explore the sensitivity of the results to these parameter values in this paper.

3.3   Sensitivity Analysis

In this section, I consider small sample bias and illustrate the rate at which bias reduces as the sample size grows. This gives a feel for the usefulness of asymptotic behaviour of estimators. However, only sample sizes that are relevant for macroeconometrics are considered. I redo the experiments for both the RBC and NK modesl and, this time, increase the sample size from 200 to 500 observations. I find that increasing sample size only improves the estimation of parameters that are not affected by trend misspecification as can be seen by smaller MSEs, biases and variances. Otherwise, affected parameter estimates such as estimates of inverse elasticities associated with households $\{\eta, \gamma\}$, the policy-rule parameters $\{\phi_y, \phi_\pi\}$ and parameters governed exogenous processes $\{\rho_z, \rho_B, \rho_\pi, \sigma_z, \sigma_B, \sigma_\pi\}$, remain at least as distorted as in the case of having a smaller sample. Therefore, even in a larger sample size, the parameter distortion induced by trend misspecification cannot be alleviated and inaccuracies of policy-relevant statistics remain.[5]

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[5]The coverage rates derived from these experiments suggest the same conclusions as in the case of using a small sample. As a result, I do not present the results in this paper but they are available upon request.

4 Conclusion

This paper investigates the role of trend misspecifications upon policy implications given a framework of a DSGE model. Trend misspecifications can be induced when econometricians do not know the underlying trend process in nonstationary data. To demonstrate the importance of trend misspecification, simulation exercises are conducted where simulated data exhibits a deterministic-trend and econometricians estimate a model with a stochastic trend, and vice versa. To incorporate sampling uncertainty, the implications of trend misspecifications drawn in this paper are based on 150 simulations. The findings can be summed up as follows. First, the misspecification of a trend component can lead to distortion of structural parameter estimates. The degree of distortion is significant in reducing the accuracy of policy-relevant statistics, especially for estimated variance decompositions as suggested by coverage rates. Second, between two competing misspecified models, an estimated model with a random-walk process can provide a smaller parameter distortion and a higher accuracy of statistics of interest compared to an estimated model with a deterministic-trend process. Third, the structural parameter estimates that become biased due to trend misspecification are different depending on a DSGE framework. Finally, the parameter distortion induced by trend misspecification still remains even in a large sample size.

Even though an estimated model with a random-walk specification has better performance than the other competing model, the accuracy of the estimated variance decompositions under the presence of trend misspecification is still low for some variables. Therefore, strategies to reduce or accommodate trend misspecification should be implemented. Effort invested in determining the appropriate form of the trend is warranted. Alternatively, one might employ Bayesian Model Averaging to extract information on trend specifications and obtain a better estimation in terms of a bias in estimates. The aim would be to obtain more accurate estimates of policy-related statistics.

Table 1: All Possible Combinations of Trend Specifications in DGP and Estimated Model

DGP	Model ($\mathcal{M}$)	
	Deterministic Trend (dt)	Random Walk (rw)
Deterministic Trend (dt)	Correct Specification 1 $DGP^{(dt)} - \mathcal{M}^{(dt)}$	Misspecification 1 $DGP^{(dt)} - \mathcal{M}^{(rw)}$
Random Walk (rw)	Misspecification 2 $DGP^{(rw)} - \mathcal{M}^{(dt)}$	Correct Specification 2 $DGP^{(rw)} - \mathcal{M}^{(rw)}$

Figure 1: Coverage Rates of Impulse Responses under Correct Specification Cases given a RBC framework
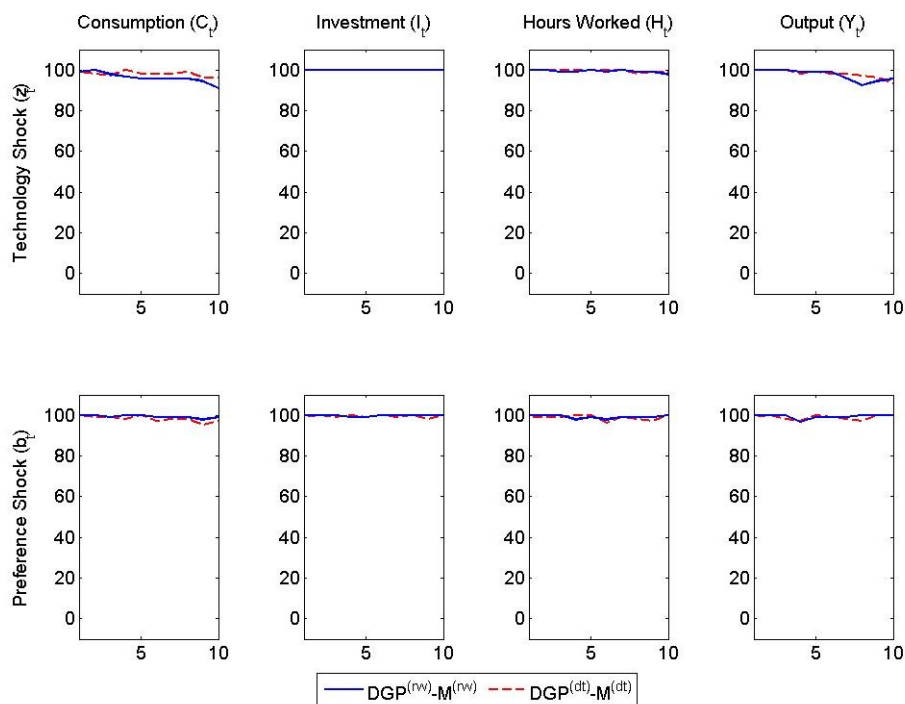
Table 2: Parameter Distortions given a Standard RBC Model

| Parameter	DGP	Correct Specification Case $DGP^{(dt)}-\mathcal{M}^{(dt)}$ MSE	Bias	Variance	$DGP^{(rw)}-\mathcal{M}^{(rw)}$ MSE	Bias	Variance	Misspecification Case $DGP^{(dt)}-\mathcal{M}^{(rw)}$ MSE	Bias	Variance	$DGP^{(rw)}-\mathcal{M}^{(dt)}$ MSE	Bias	Variance
						Structural Parameters							
δ	0.025	0.0000	1.62%	0.0000	0.0000	2.09%	0.0000	0.0000	1.37%	0.0000	0.0000 (1.048)	2.12% (1.551)	0.0000 (1.005)
γ	1.89	0.6584	24.18%	0.4333	0.6548	22.88%	0.4583	0.6153	19.26%	0.4776	0.9585 (1.558)	38.07% (1.976)	0.4069 (0.852)
η	1	0.0039	3.31%	0.0027	0.0039	3.17%	0.0028	0.0038	2.60%	0.0030	0.0055 (1.475)	5.43% (2.087)	0.0023 (0.770)
ρ_z	0.95	0.0025	-3.21%	0.0014	0.0045	-4.80%	0.0023	0.0055	-5.66%	0.0026	0.0007 (0.121)	0.34% (-0.060)	0.0005 (0.194)
ρ_B	0.8	0.0016	-0.52%	0.0010	0.0018	-1.07%	0.0011	0.0018	-0.72%	0.0011	0.0020 (1.112)	-2.19% (3.033)	0.0010 (0.888)
σ_z	0.007	0.0000	13.42%	0.0000	0.0000	2.59%	0.0000	0.0000	-13.91%	0.0000	0.0000 (4.860)	77.90% (-5.602)	0.0000 (0.808)
σ_B	0.007	0.0000	2.10%	0.0000	0.0000	1.48%	0.0000	0.0000	1.22%	0.0000	0.0000 (1.031)	4.43% (3.640)	0.0000 (0.712)
						Filter Parameters							
θ_0	2	0.0001	0.001%	0.0001	-	-	-	-	-	-	0.0001	0.00%	0.0001
θ_1	0.0063	0.0000	-0.14%	0.0000	0.0000	0.84%	0.0000	0.0000	0.52%	0.0000	0.0000 (0.830)	-1.36% (-2.618)	0.0000 (0.361)
σ_C	0.007	0.0000	1.58%	0.0000	0.0000	1.56%	0.0000	0.0000	46.54%	0.0000	0.0000 (1.297)	-54.33% (-1.168)	0.0000 (0.556)

Note: Numbers in parentheses are a ratio of the distortion in the case of $DGP^{(rw)}-\mathcal{M}^{(dt)}$ relative to the ones in the case of $DGP^{(dt)}-\mathcal{M}^{(rw)}$.

Table 3: Parameter Distortions given a New Keynesian Model

| | | Correct Specification Case | | | | | | Misspecification Case | | | | | |
| | | $DGP^{(dt)} - \mathcal{M}^{(dt)}$ | | | $DGP^{(rw)} - \mathcal{M}^{(rw)}$ | | | $DGP^{(dt)} - \mathcal{M}^{(rw)}$ | | | $DGP^{(rw)} - \mathcal{M}^{(dt)}$ | | |
Parameter	DGP	MSE	Bias	Variance	MSE	Bias	Variance	MSE	Bias	Variance	MSE	Bias	Variance
							Structural Parameters						
δ	0.025	0.0000	2.64%	0.0000	0.0000	3.28%	0.0000	0.0000	2.55%	0.0000	0.0000 (0.990)	2.40% (0.942)	0.0000 (1.001)
γ	1.89	0.3385	11.87%	0.2315	0.3476	12.16%	0.2352	0.3474	12.74%	0.2361	0.3906 (1.124)	14.75% (1.158)	0.2309 (0.978)
η	1	0.0499	4.10%	0.0371	0.0514	5.15%	0.0366	0.0480	3.72%	0.0373	0.0517 (1.078)	5.39% (1.451)	0.0371 (0.994)
ϕ_π	1.107	0.0000	0.001%	0.0000	0.0000	0.01%	0.0000	0.0000	0.03%	0.0000	0.0003 (11.394)	0.164% (5.080)	0.0000 (1.505)
ϕ_y	0.12	0.0000	0.007%	0.0000	0.0000	0.23%	0.0000	0.0000	0.47%	0.0000	0.0005 (13.680)	4.70% (9.961)	0.0000 (1.708)
ρ_z	0.95	0.0050	-5.23%	0.0025	0.0049	-5.22%	0.0025	0.0049	-5.22%	0.0025	0.0050 (1.005)	-5.23% (1.002)	0.0025 (1.006)
ρ_B	0.8	0.0022	-0.92%	0.0016	0.0021	-0.08%	0.0015	0.0022	-0.67%	0.0016	0.0021 (0.983)	-1.03% (1.526)	0.0016 (1.001)
ρ_π	0.95	0.0011	-2.04%	0.0004	0.0010	-1.82%	0.0004	0.0011	-1.94%	0.0004	0.0011 (1.024)	-1.89% (0.972)	0.0004 (0.989)
σ_z	0.007	0.0000	16.02%	0.0000	0.0000	15.43%	0.0000	0.0000	14.21%	0.0000	0.0000 (0.982)	13.04% (0.918)	0.0000 (0.979)
σ_B	0.007	0.0000	1.69%	0.0000	0.0000	-0.22%	0.0000	0.0000	2.05%	0.0000	0.0000 (0.990)	-0.27% (-0.129)	0.0000 (0.903)
σ_π	0.007	0.0000	5.92%	0.0000	0.0000	5.88%	0.0000	0.0000	5.95%	0.0000	0.0000 (1.219)	5.57% (0.935)	0.0000 (0.999)
							Filter Parameters						
θ_0	2	0.0001	0.000%	0.0001	-	-	-	-	-	-	0.0001	0.001%	0.0001
θ_1	0.0063	0.0000	0.001%	0.0000	0.0000	2.22%	0.0000	0.0000	0.30%	0.0000	0.0000 (0.712)	-1.12% (-3.762)	0.0000 (0.003)
σ_Y	0.007	0.0000	-0.57%	0.0000	0.0000	0.62%	0.0000	0.0000	43.24%	0.0000	0.0002 (23.085)	198.37% (4.588)	0.0000 (4.614)

Note: Numbers in parentheses are a ratio of the distortion in the case of $DGP^{(rw)} - \mathcal{M}^{(dt)}$ relative to the ones in the case of $DGP^{(dt)} - \mathcal{M}^{(rw)}$.

Figure 2: Coverage Rates of Variance Decomposition under Correct Specification Cases given a RBC framework
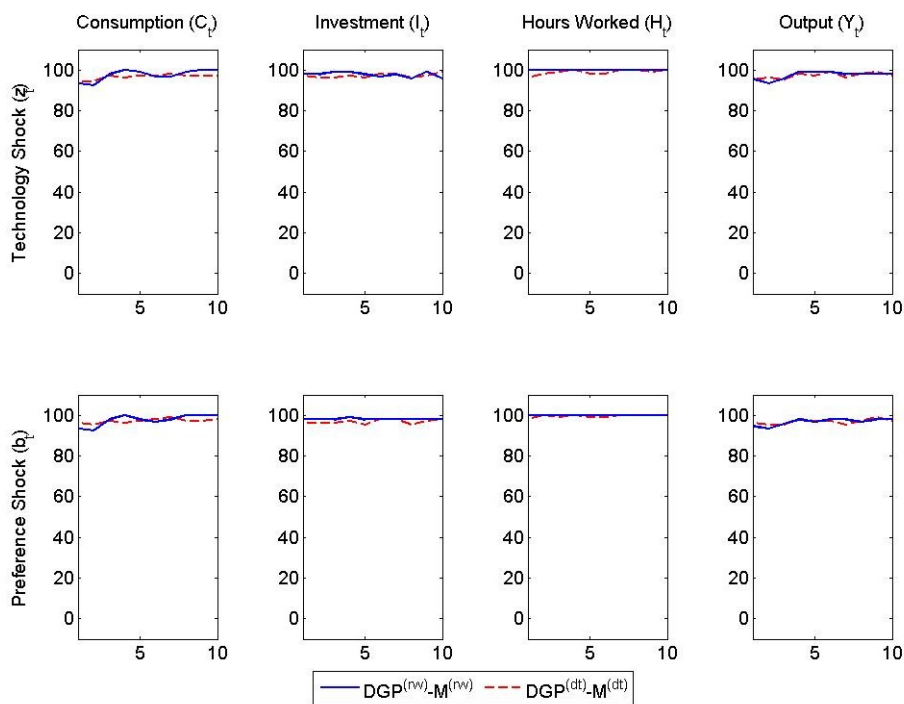
Notes: The x-axis indicates the s-steps ahead error.

Figure 3: Coverage Rates of Impulse Responses under Misspecification Cases given a RBC framework
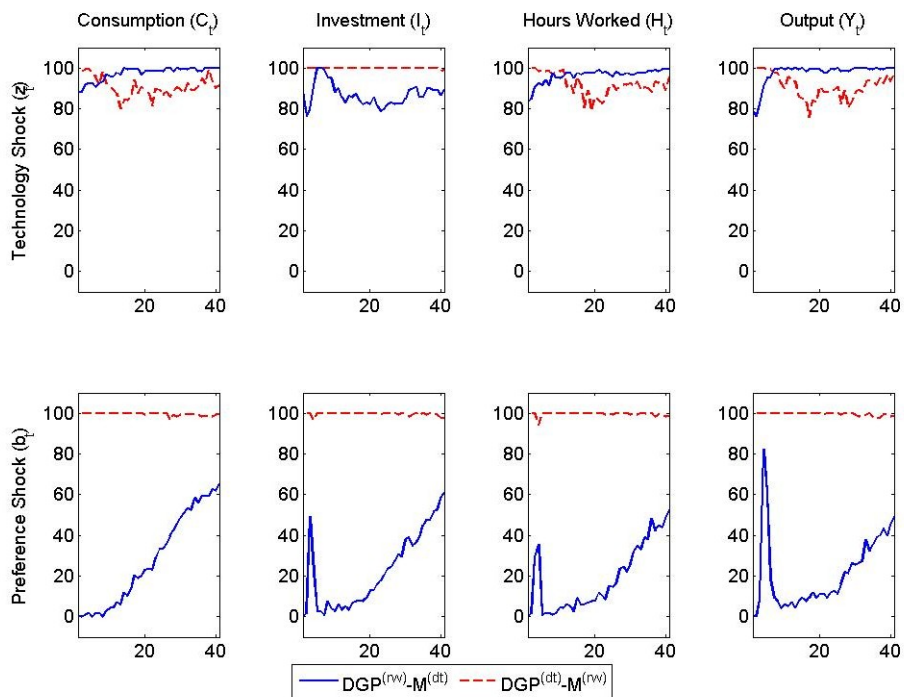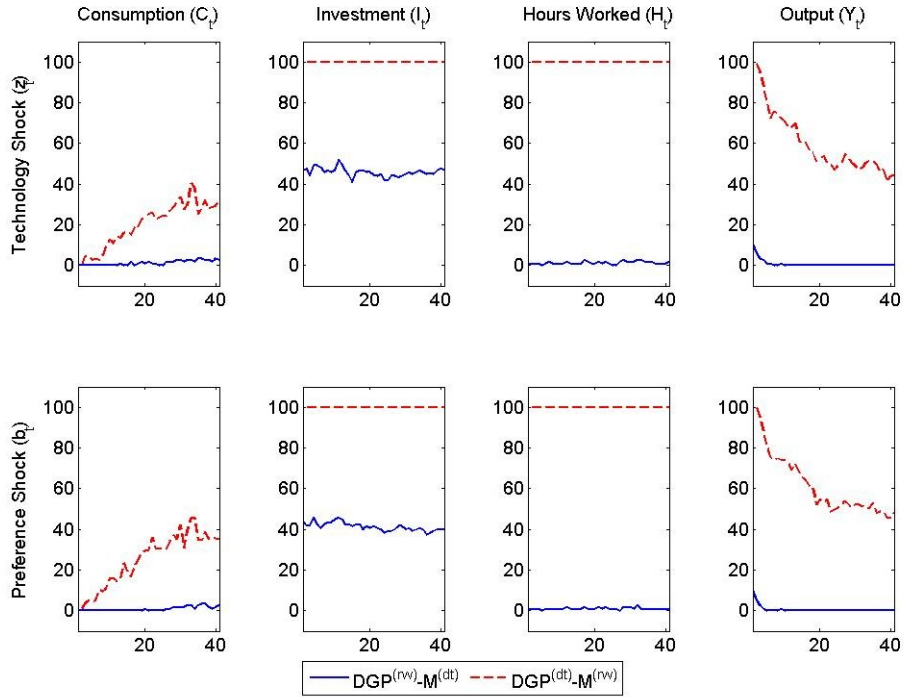

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Figure 4: Coverage Rates of Variance Decompositions under Misspecification Cases given a RBC framework

Notes: The x-axis indicates the s-steps ahead error.

Figure 5: Coverage Rates of Impulse Responses under Correct Specification Cases given a NK framework
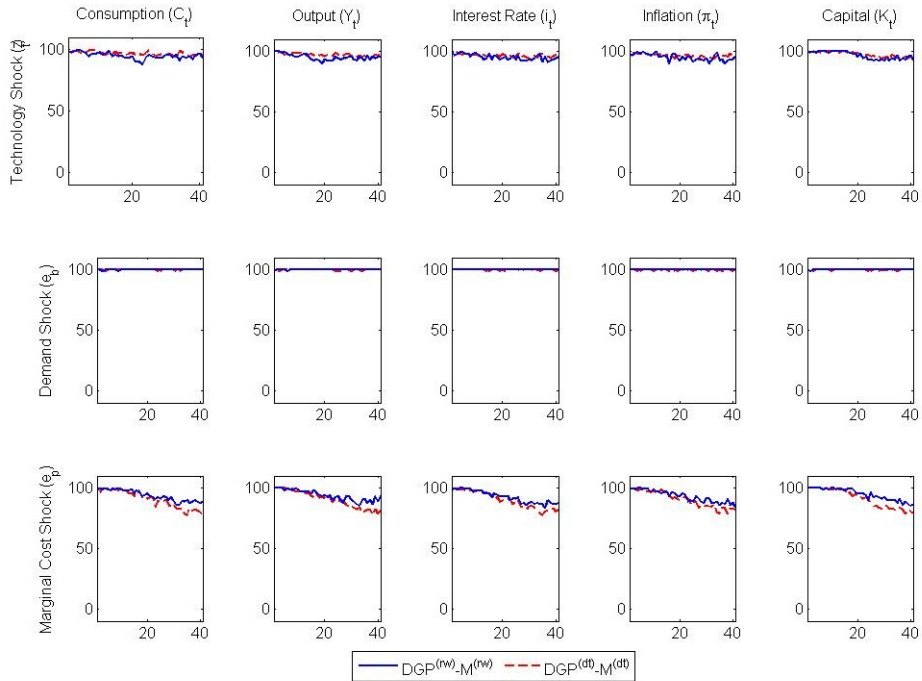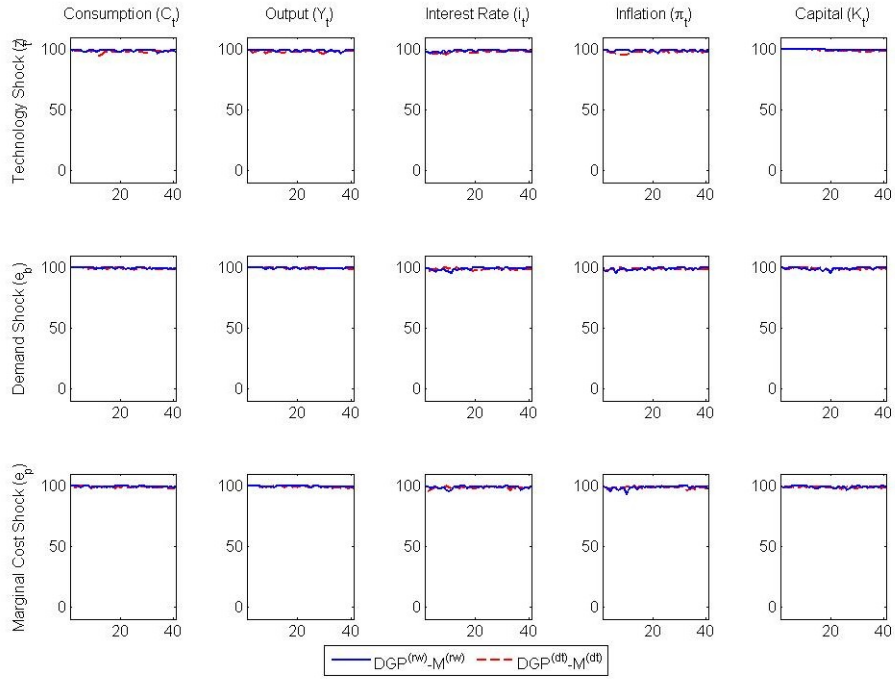
Figure 6: Coverage Rates of Variance Decomposition under Correct Specification Cases given a NK framework

Notes: *The x-axis indicates the s-steps ahead error.*

Figure 7: Coverage Rates of Impulse Responses under Misspecification Cases given a NK framework
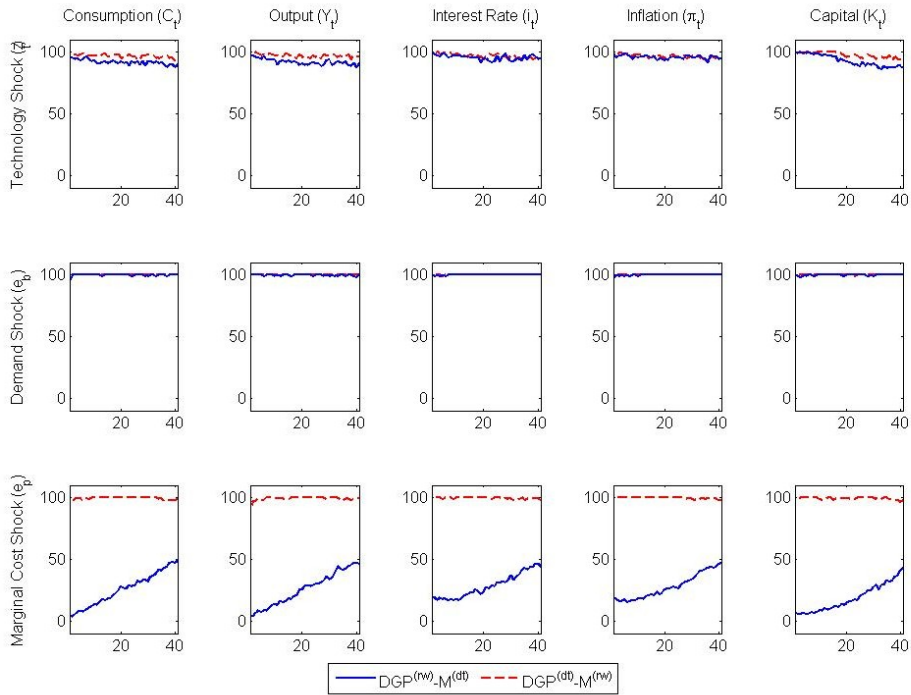
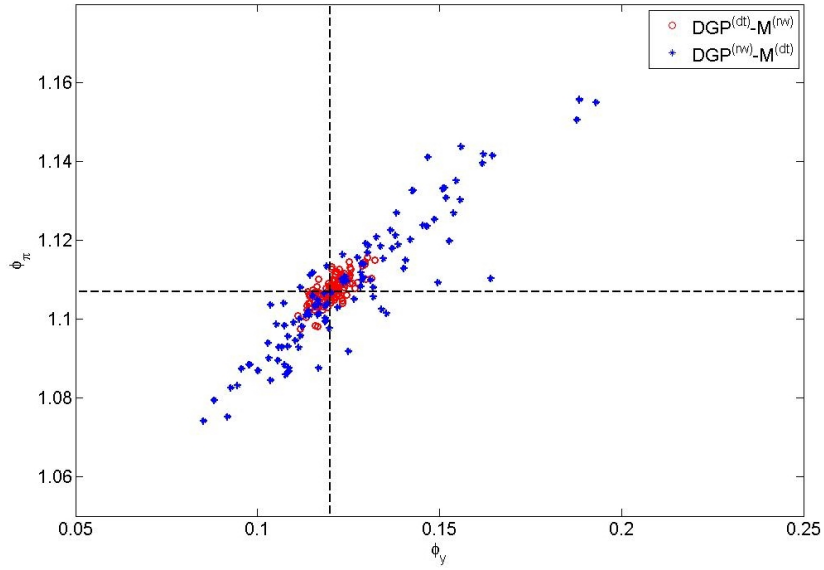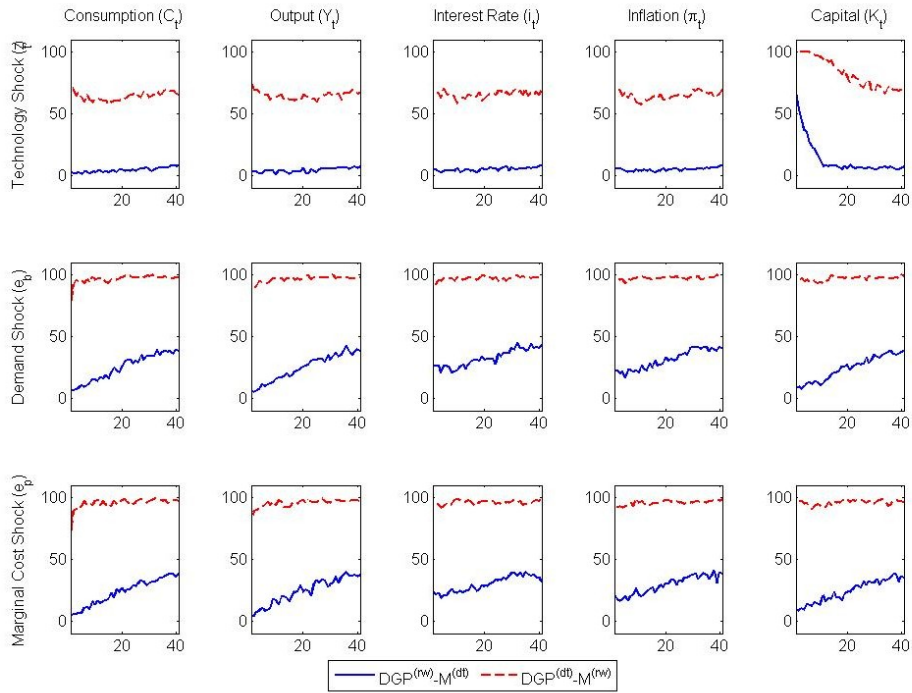Figure 8: Estimated Policy Parameters in the Taylor Interest Rate Rule

Figure 9: Coverage Rates of Variance Decompositions under Misspecification Cases given a NK framework

Notes: The x-axis indicates the s-steps ahead error.

Appendix

A Identification of Structural Parameters in DSGE Models

The methodology proposed by Komunjer and Ng (2011) to check the identifiability of structural parameters can be described briefly as follows. Consider a DSGE model with structural parameters Ω^m in a set $\Theta \subseteq \mathbb{R}^{n_{\Omega^m}}$. The linear state-space representation of a DSGE model presented in Section 2 can be rewritten as

$$
\begin{aligned}
\tilde{x}_{t+1} &= \mathbf{A}(\Omega^m)\tilde{x}_t + \mathbf{B}(\Omega^m)\epsilon_{t+1} \\
\tilde{y}_{t+1} &= \mathbf{C}(\Omega^m)\tilde{x}_t + \mathbf{D}(\Omega^m)\epsilon_{t+1}
\end{aligned}
$$

where $\tilde{x}_t$ is a $n_x \times 1$ vector of unobservable state variables, $\tilde{y}_t$ is a $n_y \times 1$ vector of log-deviation variables, $\epsilon_t \sim \mathcal{N}(\mathbf{0}_{n_\epsilon}, I_{n_\epsilon})$ and $\mathbb{E}(\sigma_\epsilon \sigma_\epsilon') = \Sigma_\epsilon$ is a variance-covariance matrix.

Given the system, assumptions required to obtain the rank and order conditions for identification are as follows. First, the innovations ϵ_t is white noise. Second, the system is stable, left-invertible and minimal. The system is minimal when it contains only the smallest vector of state variables x_t such that it is able to fully characterise the properties of the model. Last, the mapping $\Lambda(\Omega^m) : \Omega^m \mapsto \Lambda(\Omega^m)$ is continuously differentiable on Θ where $\Lambda(\Omega^m)$ is the hyperparameter in the state-space solution given by

$$
\Lambda(\Omega^m) \equiv ((\text{vec}\mathbf{A}(\Omega^m))', (\text{vec}\mathbf{B}(\Omega^m))', (\text{vec}\mathbf{C}(\Omega^m))', (\text{vec}\mathbf{D}(\Omega^m))', (\text{vech}\Sigma_\epsilon)')'.
$$

If all assumptions hold then a necessary and sufficient rank condition for Ω^m to be locally identified at a point Ω_0^m is

$$
\text{rank}\Delta(\Omega_0^m) = \text{rank}(\Delta_\Lambda(\Omega_0^m) \quad \Delta_T(\Omega_0^m) \quad \Delta_U(\Omega_0^m)) = n_{\Omega^m} + n_x^2 + n_\epsilon^2
$$

where

$$
\Delta(\Omega_0^m) = \begin{pmatrix} \frac{\partial \Phi(\Omega^m)}{\partial \Omega^m} & \mathbf{0}_{n_R^2 \times n_x^2} & \mathbf{0}_{n_R^2 \times n_\epsilon^2} \\[2mm] \frac{\partial \mathrm{vec} \mathbf{A}(\Omega^m)}{\partial \Omega^m} & \mathbf{A}(\Omega^m)' I_{n_x} - I_{n_x} \otimes \mathbf{A}(\Omega^m) & \mathbf{0}_{n_x^2 \times n_\epsilon^2} \\[2mm] \frac{\partial \mathrm{vec} \mathbf{B}(\Omega^m)}{\partial \Omega^m} & \mathbf{B}(\Omega^m) \otimes I_{n_x} & I_{n_\epsilon} \otimes \mathbf{B}(\Omega^m) \\[2mm] \frac{\partial \mathrm{vec} \mathbf{C}(\Omega^m)}{\partial \Omega^m} & -I_{n_\epsilon} \otimes \mathbf{C}(\Omega^m) & \mathbf{0}_{n_y n_x \times n_\epsilon^2} \\[2mm] \frac{\partial \mathrm{vec} \mathbf{D}(\Omega^m)}{\partial \Omega^m} & \mathbf{0}_{n_y n_x \times n_\epsilon^2} & I_{n_\epsilon} \otimes \mathbf{D}(\Omega^m) \\[2mm] \frac{\partial \mathrm{vech} \Sigma_\epsilon}{\partial \Omega^m} & \mathbf{0}_{n_\epsilon(n_\epsilon+1)/2 \times n_x^2} & -2\varepsilon_{n_\epsilon}[\Sigma_\epsilon \otimes I_{n_\epsilon}] \end{pmatrix}_{\Omega^m = \Omega_0^m}
$$

$$
\equiv \begin{pmatrix} \frac{\partial \Phi(\Omega^m)}{\partial \Omega^m} & \mathbf{0}_{n_R^2 \times n_x^2} & \mathbf{0}_{n_R^2 \times n_\epsilon^2} \\[2mm] \Delta_\Lambda(\Omega_0^m) & \Delta_T(\Omega_0^m) & \Delta_U(\Omega_0^m)) \end{pmatrix}_{\Omega^m = \Omega_0^m},
$$

$\Phi(\Omega^m)$ is a set of n_R priori restrictions satisfying $\Phi(\Omega_0^m) = 0$, $\varepsilon_n = (\mathcal{G}_n' \mathcal{G}_n)^{-1} \mathcal{G}_n'$ and $\mathcal{G}_n$ is an $n^2 \times n(n+1)/2$ duplication matrix of 0s and 1s with a single 1 in each row.

Further, a necessary order condition is $n_{\Omega^m} + n_x^2 + n_\epsilon^2 \leq n_\Lambda$, where $n_\Lambda = (n_x + n_y)(n_x + n_\epsilon) + (n_\epsilon + 1)/2$. This condition requires the system to have the number of equations at least as large as the number of unknowns in those equations.

I check all assumptions for both RBC and NK frameworks evaluated Ω^m at both estimated values and DGP values and then proceed with the rank and order conditions. The restrictions are set such that some parameters are fixed at DGP values and a steady state condition is satisfied. As the rank of $\Delta(\Omega_0^m)$ varies according to a tolerance value used in Matlab, I consider the values ranging from $1e-5$ to $1e-11$. Given these values of tolerance, both rank and order conditions are all satisfied ensuring the identifiability of structural parameters specified in both frameworks. Table 4 presents the results of identifiability evaluated at DGP values.

Table 4: Rank and Order Conditions given a RBC and NK Frameworks

		RBC							NK				
	Δ_Λ	Δ_T	Δ_U	Δ	Rank	Order	Δ_Λ	Δ_T	Δ_U	Δ	Rank	Order
Required	9	9	4	22			14	16	9	39		
1e-5	9	9	4	22	Pass	Pass	14	16	9	39	Pass	Pass
1e-6	9	9	4	22	Pass	Pass	14	16	9	39	Pass	Pass
1e-7	9	9	4	22	Pass	Pass	14	16	9	39	Pass	Pass
1e-8	9	9	4	22	Pass	Pass	14	16	9	39	Pass	Pass
1e-9	9	9	4	22	Pass	Pass	14	16	9	39	Pass	Pass
1e-10	9	9	4	22	Pass	Pass	14	16	9	39	Pass	Pass
1e-11	9	9	4	22	Pass	Pass	14	16	9	39	Pass	Pass

B Parameter Values and Priors

Table 5 summarises the parameter values used in the data generating processes (DGP) in both RBC and NK exercises and the priors used in a Bayesian estimation. These parameter values are in a reasonable range and common within the literature. For priors, they are quite standard and centred around the true values.

Table 5: True structural parameter values used in DGP and Priors used in the estimation

	Parameter	True Value	Prior			Model
			Density	Mean	S.D.	
	Structural parameters					
β	Discounted factor	0.99	-	-	-	RBC, NK
α	Capital share	0.33	-	-	-	RBC, NK
δ	Depreciation rate of capital	0.025	Beta	0.025	0.005	RBC, NK
γ	Inverse short-run labor elasticity	1.89	Normal	1.9	0.75	RBC, NK
η	Inverse intertemporal elasticity of substitution	1	Normal	1	0.1	RBC, NK
θ	A fraction of sticky price firms	0.75	-	-	-	NK
ϕ_π	Response of monetary policy to inflation	1.107	Gamma	1	0.1	NK
ϕ_y	Response of monetary policy to output	0.12	Gamma	0.1	0.1	NK
ρ_z	Persistence in technology shock	0.95	Beta	0.9	0.05	RBC, NK
ρ_B	Persistence in preference shock	0.8	Beta	0.8	0.05	RBC, NK
ρ_p	Persistence in cost-push shock	0.95	Beta	0.9	0.05	NK
σ_z	Standard deviation of technology shock	0.007	IGamma	0.01	Inf	RBC, NK
σ_B	Standard deviation of preference shock	0.007	IGamma	0.01	Inf	RBC, NK
σ_p	Standard deviation of cost-push shock	0.007	IGamma	0.01	Inf	NK
	Filtered parameters					
θ_0	Average mean	2	Normal	2	0.01	RBC, NK
θ_1	Average growth rate	0.0063	Normal	0.006	0.01	RBC, NK
σ_C		0.007	IGamma	0.01	0.01	RBC
σ_Y		0.007	IGamma	0.01	0.01	NK

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