# Fiscal Policy as a Temptation Control Device

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**Abstract**

We formulate an overlapping generations model with temptation and self-control preferences and incomplete market for commitment devices to study the role of two fiscal programs: social security and saving subsidy. In our environment, the distortions created by such fiscal programs work as a corrective tool that mitigates the adverse effect of succumbing to temptation on inter-temporal allocation and releases severity of self-control problem. Our results indicate that both fiscal programs potentially lead to welfare gains; however, the driving mechanisms are different. Welfare gains associated with a social security program result mainly from releasing self-control costs while welfare gains associated with a saving subsidy program are mainly driven by mitigating inter-temporal allocation distortion. In addition, we also find that the direction and size of welfare effects vary substantially when allowing for different tax-financing instruments as well as when accounting for general equilibrium price adjustments.

**JEL Classification: D1, E2, H3**

**Keywords:** Temptation, Self-Control Problem, Taxation, Social Security, Welfare.

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# 1 Introduction

There is a long standing literature in psychology and economics that finds evidence suggesting that consumers suffer from self-control problems.[1] Gul and Pesendorfer (2001), Gul and Pesendorfer (2004) and Gul and Pesendorfer (2005) propose an axiomatic approach to modeling preference reversals and formalized the ideas of temptation and self-control. That axiomatization delivers a representation theorem with utility over consumption sets expressed in terms of two utility functions: commitment utility, which gives the ranking that consumers use to compare consumption bundles; and temptation utility, which plays a key role in determining how actual consumption choices depart from what commitment utility would dictate. This "temptation and self-control preferences" approach formulates the consumer's temptation and lack of self-control problem in terms of preferences over the choice sets. It does not necessitate splitting up the consumer into multiple selves as in the "time inconsistent preferences" literature, which dates back to Strotz (1956), Phelps and Pollak (1968) and Laibson (1997).

Intuitively, the Gul and Pesendofer approach builds on the idea that when an individual consumer facing temptation and lack of self-control problem chooses from his budget set, both the size and shape of that set matter as temptation creates an urge for any potentially tempting alternative which is costly to control. Yet, different from the standard preference case, the size and shape of the choice set directly influence the consumer's behavior and utility. Consequently, the consumer suffering from the pain of self-control has lower utility in an ex ante sense if tempting allocations are available in his choice set. In that environment, the consumer would be better off if he chooses out of a smaller set.

The departure from standard preferences has implications for the consumers' behaviors, for the role of markets as well as for the role of governments. Particularly, in such an environment where market mechanism for commitment is absent, the urge of temptation and severity of self-control give rise for fiscal intervention. The incentive distortions created by fiscal policies induce consumers to behave differently and change severity of self-control efforts, which subsequently has welfare implications.

Arguably, the presence of temptation and lack of self-control influence welfare via two transmission channels: distorting allocation of resource over lifetime, called *the inter-temporal allocation effect,* and increasing dis-utility costs of self-control, called *the self-control effect.* In such setting, there are induced preferences for commitment devices, that reflects a wish both to eliminate inter-temporal allocation distortion, and to reduce self-control cost. Yet, there are two underlying mechanisms through which fiscal policies influence welfare: $(i)$ mitigating the adverse effect of succumbing to temptation on inter-temporal choice, and $(ii)$ reducing the

[1] Frederick, Loewenstein and O'Donoughe (2002) provide a review of experimental evidences documenting that individuals indeed exhibit bias toward immediate gratification. Ameriks, Caplin, Leahy and Tyler (2007) conduct a survey to measure self-control problems and find that self-control problems are smaller in scale for older than for younger individuals. Moreover, in a recent paper Fang and Silverman (2009) empirically find the existence of time-inconsistency that stems from self-control problems. Huang, Liu and Zhu (2007) and Bucciol (forthcoming) study the empirical relevance of self-control preferences using household-level data from the Consumer Expenditure Survey and find evidence supporting the presence of temptation.

dis-utility cost of self-control.

Understanding the mechanics of these two transmission channels is essential to understand the potential role of fiscal policies as a corrective tool, and to shed some lights on which fiscal instruments should be used and the timing of intervention. In this paper we study relative importance of these two transmission channels and analyze welfare implications of two fiscal programs: saving subsidy and social security.

To that end we formulate a partial equilibrium, two-period overlapping generations model filled with individuals who have temptation and self-control preferences, and the absence of market mechanisms for commitment. Our model captures essential features of dynamic interactions between self-control problem and individuals' optimal inter-temporal allocation, while it is simple enough to allow us to obtain some analytical insights. Next, we conduct a quantitative analysis. Finally, we extend our analysis to include general equilibrium adjustment channels. Our key results are summarized as follows:

First, we analytically isolate two separate transmission channels through that temptation and lack of self-control distort an individual's inter-temporal allocation and trigger dis-utility costs of self-control. We decompose the welfare effect of temptation into these two transmission channels. We conduct a quantitative analysis and find that dis-utility cost of self-control is a main source of welfare losses.

Next, we analyze the role of two fiscal programs: saving subsidy and PAYG social security. We find that both social security and saving subsidy programs work as a temptation control device. That is, it corrects the temptation distortion by mitigating the adverse effect of succumbing to temptation on inter-temporal allocation, and by reducing dis-utility cost of self-control. Moreover, we decompose the welfare effects of such fiscal policies according to these two channels and find that underlying transmission channels are quite different. A saving subsidy program operates mainly through mitigating the distortions to individuals' inter-temporal allocation, while a PAYG social security program focuses on releasing self-control costs.

Our quantitative analysis indicates that both fiscal programs lead to welfare gains in a partial equilibrium model and alleviating severity of self-control costs is the driving channel. However, when accounting for general equilibrium effects we find that mitigating inter-temporal allocation distortions becomes a dominating channel. Indeed, general equilibrium price adjustments amplify the allocation distortions caused by the presence of temptation in preferences. As a result, we find that a PAYG social security program only dominates a saving subsidy program in terms of welfare in a partial equilibrium environment. However, when accounting for general equilibrium effects, this result reverses. We also find that tax-financing instruments play an important role.

Our study contributes to a branch of the macro/public finance literature employing Gul-Pesendofer type of preference in analyzing the role of fiscal policies. Krusell, Kuruscu and Smith (2009) and Krusell, Kuruscu and Smith (2010) characterize the role of tax policy in a model in which agents succumb to temptation. They show that optimal capital tax rate is negative, i.e. the optimal tax policy prescribes a subsidy to savings/investment, which is in contrast to the well-known Chamley-Judd result in the optimal taxation literature (see Chamley (1986) and Judd (1985)). Note that in their analysis Krusell and co-authors do not explicitly isolate transmission channels behind the welfare effect of a saving subsidy program, which we provide in this paper. We basically show the mechanics of how the distortions created by a savings subsidy program potentially correct the distortions caused by the presence of temptation in preferences. We propose a welfare decomposition exercise and highlight the key driving mechanism at work. That is, subsidizing saving mitigates inter-temporal allocation distortions. Moreover, we extend the previous literature to consider a wider range of tax financing instruments and show that tax-financing instruments also play a significant role in reducing the self-control costs caused by temptation and in neutralizing the additional self-control costs created by subsidizing saving. In addition, we compare differences in welfare outcomes between partial and general equilibrium analysis. We show that how a "small" distortion in preferences results in a "big" welfare consequence when accounting for general equilibrium channels. This explains why the effect of a saving subsidy program is more pronounced in a general equilibrium model.

Kumru and Thanopoulos (2008) and Bucciol (2011) study the role of social security and show that temptation and self-control problems give rise for a social security program as a device to reduce self-control cost. Kumru and Tran (2012) analyze the role of social security when self-control problem and altruistic concern are both present. Kumru and Thanopoulos (2011) study the effects of privatizing social security systems with a model in which agents have self-control preferences. Notice that these authors conduct analysis in dynamic general equilibrium overlapping generations models with heterogenous agents generated by earning shocks and uncertain lifetime. In their frameworks, PAYG social security plays two roles: a form of social insurance against interruption or loss of earnings (redistributive) and a temptation control device (corrective). That modelling approach is more realistic for policy analysis. However, in that complex framework it is not possible to separate out the two channels of welfare effects as well as to highlight the corrective function of a PAYG social security program. In our simplified framework, we are able to demonstrate, qualitatively and quantitatively, PAYG social security as a temptation control device. We also conduct a cross-program comparison and show how a social security program loses its welfare dominance to a saving subsidy program when accounting for general equilibrium effects.

In sum, our contribution enhances the understanding of key driving forces behind the welfare effects of fiscal policies in an environment where individuals suffering from temptation and self-control problems, and quantifies which one is more important. We also provide a comparison of a saving subsidy program to a social security program, and highlight the mechanisms at work in both a partial equilibrium and general equilibrium models. The novelty of this paper is that it highlights the distinction between two main drivers of welfare variation.

The paper is organized as follows. Section 2 presents a basic model in our analysis. In section 3, we analyze the effects of temptation on inter-temporal allocation and welfare. In section 4, we study the role of fiscal policies and welfare implications. Section 5 concludes. Appendix presents the additional algebra.

## 2 Basic model

We consider a simple partial equilibrium overlapping generations model.

**Endowment and constraint**. Individuals live for two periods, period 1: young/current and period 2: old/future. Each period, individuals are endowed with 1 unit of time, but work only during the first. They supply one unit of time inelastically to labor markets at market wage rate ($w$) in the first period, and are retired in the second period. Individuals values consumption today ($c_1$) and tomorrow ($c_2$). There is a saving technology available so that individuals can save ($s$) to smooth their consumption over the lifetime; however, they are not allowed to borrow, $s \geq 0$. The net and gross real rates of return are $r$ and $R = 1 + r$, respectively. The corresponding period-by-period budget constraints are given by

$$c_1 + s = w \text{ and } c_2 = (1 + r)\, s.$$

**Temptation and self-control preference**. In our model, individuals have temptation and self-control preferences as described in Gul and Pesendofer (2001, 2004). Basically, Gul and Pesendofer define preferences over consumption sets rather than over consumption sequences and propose an axiomatic approach to modeling preference reversals as follows. For a set of consumption $B$ in a two-period setting and under a specific assumption on choice sets (set betweenness) combined with other standard axioms a typical individual with temptation and lack of self-control has the utility function as $U(B) = \max_{c \in B} \{u(c) + v(c)\} - \max_{\widetilde{c} \in B} \{v(\widetilde{c})\}$, where $U(B)$ is the utility that the individual associates with set $B$. In this representation, there are two utility components: commitment utility and temptation utility. The function $u(.)$ represents the individual's ranking over alternatives when he is committed to a single choice; while when he is not committed to a single choice, his welfare is affected by the temptation utility represented by $v(.)$. Note that when $B$ is a singleton, the terms involving $v(.)$ will vanish leaving only the $u(.)$ terms to represent preferences. However, if it is e.g. $B = \{c, \widetilde{c}\}$ with $u(\widetilde{c}) > u(c)$ the individual will succumb to the temptation only if the latter provides a sufficiently high temptation utility $v(.)$ and offsets the fact that $u(\widetilde{c}) > u(c)$, i.e., when $u(c) + v(c) > u(\widetilde{c}) + v(\widetilde{c})$. In this case the individual wishes he had only $c$ as the available alternative, since under the presence of $\widetilde{c}$, he cannot resist the temptation of choosing the latter. When the above inequality is reversed, however, the individual will pick $c$, albeit at a cost of $v(c) - v(\widetilde{c})$, which is referred as "cost of self-control."

**Optimization problem.** A typical individual suffering from temptation and self-control problem makes a decision on consumption and saving to maximize his lifetime utility as follows. The individual make decisions on consumption and savings to maximize commitment utility while resisting the temptation of choosing more tempted alternatives. In the first period the dynamic programming problem is given by:

$$V_1 = \left\{ \max_{c_1, s} \{u(c_1, s) + v(c_1, s) \mid c_1 + s = w \text{ and } s \geq 0 \} - \max_{\widetilde{c_1}, \widetilde{s_1}} \{v(\widetilde{c_1}, \widetilde{s}) \mid \widetilde{c_1} + \widetilde{s} = w \text{ and } \widetilde{s} \geq 0\}, \right.$$

where $V_1$ is the first period's value function, $\widetilde{c_1}$ is the first period's hypothetical temptation consumption, $s$ is the first period's saving, $\widetilde{s}$ is the first period's hypothetical temptation saving. $u(c_1, s)$ is the momentary utility function and $v(c_1, s)$ is the temptation utility function. In the second period the dynamic programming problem is given by:

$$V_2(s) = \left\{ \max_{c_2, s_2} \{u(c_2, s_2) + v(c_2, s_2) \mid c_2 + s_2 = Rs\} - \max_{\widetilde{c_2}, \widetilde{s_2}} \{v(\widetilde{c_2}, \widetilde{s_2}) \mid \widetilde{c_2} + \widetilde{s_2} = Rs\} \right.$$

where $Rs$ is asset available at the beginning of period 2 due to savings in period 1; $(c_2, s_2)$ is the choice of commitment consumption and savings in period 2; $(\widetilde{c_2}, \widetilde{s_2})$ is the choice of temptation consumption and savings in period 2; $u(c_2, s_2)$ denotes the momentary utility; and $v(c_2, s_2)$ denotes the temptation utility in the second period

This optimization problem is similar to the one in Krusell, Kuruscu and Smith (2010). In their paper, Krusell and co-authors consider a more general class of self-control preferences in which there are two temptation parameters: one characterizes the nature of temptation (extensive) and one characterizes the strength of temptation (intensive). In our setting, that is equivalent to the following functional forms: $u(c_1, s) = \left(\frac{c_1^{1-\sigma}}{1-\sigma} + \beta V_2(s)\right)$, $v(c_1, s) = \lambda\left(\frac{c_1^{1-\sigma}}{1-\sigma} + \kappa\beta V_2(s)\right)$, $u(c_2) = \frac{c_2^{1-\sigma}}{1-\sigma}$, and $v(c_2, s_2) = \lambda\left(\frac{c_2^{1-\sigma}}{1-\sigma}\right)$, where $\lambda$ is the parameter that captures the strength of temptation and $\kappa$ is the parameter that governs the nature of temptation. When $\kappa < 1$, the individual is tempted toward current consumption while when $\kappa > 1$ the individual is tempted toward future consumption. In our analysis, we consider a special case in which the individual is assume to consume all available wealth i.e. $\kappa = 0$ (as in Gul and Pesendorfer (2004)). We make this assumption choice to simplify the solution method for the second sub-max problem so that the first sub-max problem can be solved analytically.[2]

# 3 Temptation, inter-temporal allocation, and welfare

## 3.1 Inter-temporal allocation

An individual's optimization problem is solved by backward induction. In particular, we start with the individual's optimization problem period 2, and then we use that solution to solve the individual's optimization problem in period 1. As in Krusell *et al.* (2010), solving the optimization problem in each period is implemented in two steps: step 1 is to find a temptation allocation and step 2 is to find a commitment allocation.

To solve the individual's second period problem we first solve the sub-problem of $\max_{\widetilde{c_2}, \widetilde{s_2}} \{v(\widetilde{c_2}, \widetilde{s_2})\}$. Then, we solve the remaining sub-problem of $\max_{c_2, s_2} \{u(c_2, s_2) + v(c_2, s_2)\}$ by using the maximized values of hypothetical temptation consumption and bequest obtained in the previous step. The individual's first period problem is solved similarly. The details of the solution are followed.

[2] Relaxing this assumption allows us to study more complex interactions between the nature and the strength of tempation, which has implications for inter-temporal allocation and welfare. However, it is not a focus in this paper.

Since the individual can live for only two periods and have no bequest motive, the optimal allocation in the last period is trivial. The individual consumes all available wealth, so $\widetilde{c_2} = c_2 = Rs$ and $\widetilde{s_2} = s_2 = 0$. Note that since the choice of commitment consumption is identical to the choice of temptation consumption, the individual faces no temptation and self-control problem and no self-control cost in period 2. The value function in period 2 is $V_2(s) = \frac{(Rs)^{1-\sigma}}{1-\sigma}$.

By assumption the individual is tempted to consume all available wealth, the solution for temptation consumption in the first period is $\widetilde{c_1} = w$. Thus the optimization problem in period 1 is simplified to:

$$L(.) = \max_{c_1, s, \mu} \left\{ (1+\lambda) \frac{c_1^{1-\sigma}}{1-\sigma} + \beta V_2(s) + \mu (w - c_1 - s) \right\} - \lambda \frac{w^{1-\sigma}}{1-\sigma},$$

where $\mu$ is the shadow price. Assuming an interior solution, we can derive the following system of F.O.Cs equations: $\frac{\partial L}{\partial c_1} : (1+\lambda) c_1^{-\sigma} = \mu$, $\frac{\partial L}{\partial s} : \beta \frac{\partial V_2(s)}{\partial s} = \mu$, and $\frac{\partial L}{\partial \mu} : w - c_1 - s = 0$, which results in the following optimal allocation rule:

$$c_1 = \left( \frac{1}{1 + \frac{1}{\left(\frac{1+\lambda}{\beta R}\right)^{\frac{1}{\sigma}} R}} \right) w; \; s = \left( \frac{1}{1 + \left(\frac{1+\lambda}{\beta R}\right)^{\frac{1}{\sigma}} R} \right) w; \text{ and } c_2 = Rs.$$

## 3.2 Temptation and inter-temporal allocation

In this section we study how the presence of temptation influences individuals' optimal allocation and utility. In our simple model, individuals basically divide their lifetime wealth between current and future consumption. Let $\theta = \frac{1}{\left(\frac{1+\lambda}{\beta R}\right)^{\frac{1}{\sigma}} R}$ denote a relative importance/weight of second period consumption relative to first period consumption. Individuals give a weight of $\frac{1}{1+\theta}$ for consumption in period 1 and a weight of $\frac{\theta}{1+\theta}$ for savings. Individuals follow that rule of thumb to smooth consumption over their lifetime, so that $c_1 = \frac{1}{1+\theta} w$ and $s = \frac{\theta}{1+\theta} w$.

The temptation parameter $\lambda$ governing the strength of temptation appears the optimal allocation rule $\theta$. The temptation parameter is non-negative, $\lambda \geq 0$. This encompasses two distinct cases: when the temptation parameter $\lambda = 0$, there is no self-control problem; when $\lambda > 0$, individuals suffer from the pain of temptation and self-control.

We begin our analysis with the first setting where there is no temptation and self-control problem. With $\lambda = 0$ the weight for savings, i.e. the relative importance of future consumption, is specified as $\theta^{\lambda=0} = \frac{1}{\left(\frac{1}{\beta R}\right)^{\frac{1}{\sigma}} R}$. Alternatively, the weight $\theta^{\lambda=0}$ is interpreted as the relative strength of consumption smoothing motive. This weight depends on three factors: time discount factor $\beta$, real rate of return $R$, and the parameter governing inter-temporal elasticity of substitution $\sigma$.

In the second setting where individuals have self-control preferences i.e. $\lambda > 0$, the weight for consumption in period 1 is given by

$$\theta^{\lambda>0}=\frac{1}{\left(\frac{1}{\beta R}\right)^{\frac{1}{\sigma}}R}\left(\frac{1}{(1+\lambda)^{\frac{1}{\sigma}}}\right)=\overbrace{\theta^{\lambda=0}}^{\text{Consumption smoothing}}\overbrace{\left(\frac{1}{(1+\lambda)^{\frac{1}{\sigma}}}\right)}^{\text{Temptation distortion}} \tag{1}$$

This consists of two components: the first one $\left(\theta^{\lambda=0}\right)$ measures the relative strength of consumption smoothing motive with no temptation; and the second one $\left(\frac{1}{(1+\lambda)^{\frac{1}{\sigma}}}\right)$ measures to what extent temptation distorts consumption smoothing motive. We interpret $\left(\frac{1}{(1+\lambda)^{\frac{1}{\sigma}}}\right)$ as a distortion to consumption smoothing motive due to the presence of temptation.

The parameter $\lambda$ governing the strength of temptation is in the second component $\theta^{\lambda>0}$. It is simple to show that $\frac{d\theta}{d\lambda}<0$. Note that the parameter governing elasticity of inter-temporal substitution $\sigma$ also plays a role in mapping the temptation distortion into inter-termporal distortions. It appears that the more elastic inter-temporal substitution is the more severe temptation distorts the inter-temporal allocation rule.

As tempted individuals give a bigger weight to current consumption, $\theta^{\lambda=0}>\theta^{\lambda>0}$, the presence of temptation decreases the strength of consumption smoothing motive. A simple comparison of these two allocation rules points to two inequalities $c_1^{\lambda>0}>c_1^{\lambda=0}$ and $s^{\lambda>0}<s^{\lambda=0}$. That is, the urge of temptation distorts an optimal inter-temporal allocation toward more consumption in period 1. The presence of temptation induces individuals to give relatively bigger weight on current consumption relative to future consumption. Equivalently, individuals with temptation and self-control problem save less, compared to the first setting in which there is no temptation and self-control problem. As the value of temptation parameter $\lambda$ increases the weight for future consumption $\theta$ becomes smaller, $\frac{\partial\theta}{\partial\lambda}=-\frac{1}{(1+\lambda)^{1+\frac{1}{\sigma}}}<0$. This implies that the more severe the pain of self-control is the lower level of optimal savings is.

### 3.3 Temptation and welfare

In this section we turn our analysis to welfare consequences. In our setting, there are two channels through which temptation influences welfare. First, the presence of temptation in preferences distorts individuals' decisions on allocating income between young and old consumption. Particularly it increases young consumption and reduces savings thus old consumption. Second, individuals suffering from the pain of self-control try to balance their urge for a higher level of consumption in young age with the long term commitment for consumption smoothing. Yet, the urge for temptation to present consumption distorts the inter-temporal allocation rule (inter-temporal allocation distortion) and resisting to temptation incurs a cost (self-control cost). We are interested in the question how much utility would be lower if individuals suffer from temptation and self-control problems.

**Welfare measure.** Followed the previous literature we simply use the value function or indirect utility of young individuals as a measure of welfare. Since newborn individuals are identical in our model the value function $V_1$ is a social welfare function. Given the optimal inter-temporal allocation we can derive a measure of the social welfare as follows:

$$V_1 = \left[(1+\lambda)\left(\frac{1}{1+\theta}\right)^{1-\sigma} + \beta\left(\frac{\theta R}{1+\theta}\right)^{1-\sigma} - \lambda\right]\frac{w^{1-\sigma}}{1-\sigma}.$$

When temptation is removed from preferences ($\lambda = 0$), the self-control cost vanishes. In an economy in which individuals have no temptation and self-control problem, the social welfare is given by

$$V_1^{\lambda=0} = \left[\left(\frac{1}{1+\theta^{\lambda=0}}\right)^{1-\sigma} + \beta\left(\frac{\theta^{\lambda=0}}{1+\theta^{\lambda=0}}R\right)^{1-\sigma}\right]\frac{w^{1-\sigma}}{1-\sigma}.$$

It is known that in a perfect foresight, partial equilibrium overlapping generations model, the optimal allocation taken by individuals yields a similar allocation by a social planner. In that sense, individuals' optimal allocation with no temptation in preferences is the first-best allocation and yields a maximum level of welfare.

**Two transmission channels.** To analyze welfare consequences when individuals face a temptation and self-control problem, we rearrange the value function to have this expression:

$$V_1^{\lambda>0} = \left[\overbrace{\left(\frac{1}{1+\theta^{\lambda>0}}\right)^{1-\sigma} + \beta\left(\frac{\theta^{\lambda>0}R}{1+\theta^{\lambda>0}}\right)^{1-\sigma}}^{=\upsilon^{cs}} - \overbrace{\lambda\left(1-\left(\frac{1}{1+\theta^{\lambda>0}}\right)^{1-\sigma}\right)}^{=\upsilon^{scc}}\right]\frac{w^{1-\sigma}}{1-\sigma}. \quad (2)$$

This simple expression isolates two channels through which individuals' wealth maps into their utility. To simplify our notation we define two new functions $\upsilon^{cs} = \left(\frac{1}{1+\theta^{\lambda>0}}\right)^{1-\sigma} + \beta\left(\frac{\theta^{\lambda>0}R}{1+\theta^{\lambda>0}}\right)^{1-\sigma}$ and $\upsilon^{scc} = \lambda\left(1-\left(\frac{1}{1+\theta^{\lambda>0}}\right)^{1-\sigma}\right)$. Our interpretation of these two functions is simple. Function $\upsilon^{cs}$ represents an optimal rule that maps wealth into utility space. $\upsilon^{cs}$ is formulated by a number of factors including temptation, rate of time discount and risk aversion, and market prices. Function $\upsilon^{scc}$ maps wealth available at beginning of period 1 into dis-utility cost of self-control. $\upsilon^{scc}$ is driven by the strength of temptation and individuals' efforts to smooth consumption over their lifetime. In short, we can write $V_1^{\lambda>0} = [\upsilon^{cs} - \upsilon^{scc}]\frac{w^{1-\sigma}}{1-\sigma}$. We also denote $SCC = -\upsilon^{scc}\frac{w^{1-\sigma}}{1-\sigma}$ as dis-utility of self-control cost

We are able to identify analytically two transmission channels through which the presence of temptation influences individuals' welfare. First, the urge of temptation to affects individuals' inter-temporal allocation directly as individuals are tempted to consume more in period 1, which potentially lowers welfare. We call it the *inter-temporal allocation channel*. Second, the presence of temptation triggers individuals' efforts to control their temptation, which is costly and directly lowers welfare. We call this the *self-control channel*. How these two channels work is summarized in two variables $\upsilon^{cs}$ and $\upsilon^{scc}$.

To identify direction of welfare effects through these two channels we analyze the signs of

$\upsilon^{cs}$ and $\upsilon^{scc}$ and the first derivatives. First, we focus on the effect through self-control channel. It is simple enough to show $\upsilon^{scc} > 0$ and $SCC = -\upsilon^{scc}\frac{w^{1-\sigma}}{1-\sigma} < 0$. Indeed, individuals suffer from balancing their urge for a higher level of consumption in young age with the long term commitment for consumption smoothing. We conclude that the presence of temptation in preferences results in negative welfare effects working through self-control channel. Moreover, we find that in this setting the size of choice set matters for welfare as it determines severity of self-control efforts. The more wealth available in young age the higher self-control cost individuals have to pay. Next, we pay attention to the welfare effect through inter-temporal allocation channel. However, we are not able to determine immediately the sign of $\upsilon^{cs}$. It is not clear whether or not $\upsilon^{cs} > 0$.

To understand exactly how the presence of temptation influences an individual's inter-temporal allocation and utility, we take the first derivative of value function $V_1^{\lambda>0}$ with respect to the temptation parameter $\lambda$ as

$$\frac{\partial V_1^{\lambda>0}}{\partial \lambda} = \left(\frac{\partial \upsilon^{cs}}{\partial \lambda} - \frac{\partial \upsilon^{scc}}{\partial \lambda}\right)\frac{w^{1-\sigma}}{1-\sigma}.$$

Intutitively, this derivative consists of two components that capture the effects of temptation on welfare working through inter-temporal allocation and self-control channels.

We begin with the mechanism operating through the first channel $\frac{\partial \upsilon^{cs}}{\partial \lambda}$. To ease our analysis, we define two new variables $g_{c_1} = \frac{1}{1+\theta}$ and $g_s = \frac{\theta}{1+\theta}$, which can be interpreted as the consumption rate and saving rate, respectively. and have the following expression

$$\frac{\partial \upsilon^{cs}}{\partial \lambda} = \left[\left(\frac{\partial g_{c_1}}{\partial \lambda}\right)^{-\sigma} + \beta R^{1-\sigma}\left(\frac{\partial g_s}{\partial \lambda}\right)^{-\sigma}\right].$$

Having used the condition $g_s = 1 - g_{c_1}$ we have $\frac{\partial g_{c_1}(\lambda)}{\partial \lambda} = -\frac{\partial g_s(\lambda)}{\partial \lambda}$. We can simplify this derivative to $\frac{\partial \upsilon^{cs}}{\partial \lambda} = \left[\left(\frac{\partial g_{c_1}(\lambda)}{\partial \lambda}\right)^{-\sigma}\left(1 - \frac{\beta R}{R^{\sigma}}\right)\right]$. Since $\left(\frac{\partial g_{c_1}(\lambda)}{\partial \lambda}\right)^{-\sigma} > 0$, the sign of this derivative is ambiguous and depends on the sign of $\left(1 - \frac{\beta R}{R^{\sigma}}\right)$. This implies that the inter-temporal allocation effect depends on interaction between the rate of time discount $\beta$ and interest rate $R$. In many cases, we can easily identify $R \geq 1$ but whether $\beta R$ is bigger or smaller than unity is not well defined. If we assume that $\beta R \leq 1$, we are able to determine analytically the sign of this derivative $\frac{\partial \upsilon^{cs}}{\partial \lambda} < 0$. That is, the mapping rule $\upsilon^{cs}$ is a decreasing function as the strength of temptation $\lambda$ increases. We conclude that the presence of temptation distorts the inter-temporal allocation rule and that the direction of welfare effect operating through the inter-temporal allocation channel is negative with respect to the strength of temptation i.e. $\upsilon^{cs \text{ with } \lambda>0} < \upsilon^{cs \text{ with } \lambda=0}$. On other hand, if we assume that $\beta R > 1$, the sign of $\frac{\partial \upsilon^{cs}}{\partial \lambda}$ is ambiguous.

Next, we examine the mechanism operating through the second channel. We consider the derivative

$$\frac{\partial \upsilon^{scc}}{\partial \lambda} = 1 - \left(\frac{1}{1+\theta^{\lambda>0}}\right)^{1-\sigma} - \lambda \frac{\partial \left(\frac{1}{1+\theta^{\lambda>0}}\right)^{1-\sigma}}{\partial \theta^{\lambda>0}} \frac{\partial \theta^{\lambda>0}}{\partial \lambda}.$$

It turns out that the sign of $\frac{\partial \upsilon^{scc}}{\partial \lambda}$ depends on inter-temporal elasticity of substitution. It appears that $\frac{\partial \upsilon^{scc}}{\partial \lambda} > 0$ for any $\sigma \geq 1$. This implies that the negative welfare effect operating through self-control cost is magnified as severity of temptation is increased.

**Proposition 1** *The final welfare effect caused by the presence of temptation in preferences is ambiguous because the sign of* $\frac{dV_1^\lambda}{d\lambda}$ *is not clearly defined in a general case. However, under certain conditions as* $R > 1$*,* $\beta R \leq 1$ *and* $\sigma \geq 1$*, the final welfare effect is negative,* $\frac{dV_1^\lambda}{d\lambda} < 0$.

The intuition for welfare losses is straightforward. As noted before, in a partial equilibrium economy with standard preference the inter-temporal allocation rule is the first best allocation rule. The presence of temptation in preferences induces individuals to use a less efficient rule to allocate their wealth over their life cycle. That is, temptation influences individuals' behavior and move decentralized allocation away from such a socially optimal allocation; and therefore, lowers welfare. In addition, the presence of temptation triggers individuals' efforts to resist temptation, which also results in dis-utility costs of self-control.

**A decomposition of welfare effects.** If we consider the difference between utility of an individual with standard preferences and that of an individual with temptation and self-control preferences. In our simple setting, we can show that the presence of temptation lowers utility, $dV_1^\lambda = V_1^{\lambda>0} - V_1^{\lambda=0} < 0$, under certain restrictions on parameter values. This result is that the presence of temptation in preferences leads to welfare losses not totally surprising. Indeed, this has been documented in the previous studies (e.g. see Gul and Pesendorfer (2001, 2004) and Krusell et al. (2010)). However, the previous studies do not explicitly break down the welfare effect according to two transmission channels.

To enhance our understanding of the mechanics behind the welfare loss result, we conduct a welfare decomposition exercise. We re-arrange changes in welfare as follows:

$$dV_1^\lambda = V_1^{\lambda>0} - V_1^{\lambda=0} = \left[ \overbrace{\left(\upsilon^{cs \text{ with } \lambda>0} - \upsilon^{cs \text{ with } \lambda=0}\right)}^{dV^{cs,\ \lambda}\text{: the inter-temporal allocation effect}} - \overbrace{\left(0 - \upsilon^{scc}\right)}^{=dV^{scc,\ \lambda}\text{: the self-control effect}} \right] \frac{w^{1-\sigma}}{1-\sigma}. \quad (3)$$

With this decomposition we are able to isolate analytically two transmission channels through which the presence of temptation lowers individuals' welfare: First, the urge of temptation induces individuals change their consumption-saving behavior, which distorts inter-temporal allocation directly and lowers welfare. We call it the *inter-temporal allocation effect.* Second, the presence of temptation triggers individuals' efforts to control their temptation, which is costly and also lowers welfare. We call it the *self-control effect.*

Equation (3) points out that welfare losses due to the presence of temptation have two components: one resulting from the inter-temporal allocation distortions and one resulting from the self-control cost. More precisely, the first component $(\upsilon^{cs \text{ with } \lambda>0} - \upsilon^{cs \text{ with } \lambda=0}) < 0$ measures the welfare losses coming from the adverse effect of temptation on inter-temporal allocation; and the second component $-(0 - \upsilon^{scc})$ measures the welfare losses resulting from exerting efforts to self-control.

**Proposition 2** *Welfare losses due to the presence of temptation in preferences are driven by two mechanims: one resulting from distorting inter-temporal allocation and one resulting from exerting efforts to self-control. Analytically, welfare losses are decomposed as*

$$dV_1^{\lambda} = V_1^{\lambda>0} - V_1^{\lambda=0} = \left[ \overbrace{\left(\upsilon^{cs\ with\ \lambda>0} - \upsilon^{cs\ with\ \lambda=0}\right)}^{the\ inter\text{-}temporal\ allocation\ effect} - \overbrace{(0 - \upsilon^{scc})}^{the\ self\text{-}control\ effect} \right] \frac{w^{1-\sigma}}{1-\sigma}. \quad (4)$$

Note that different from the previous studies the simplicity of our model allows us to decompose analytically the total welfare loss into two separate channels accordingly, $dV_1^{\lambda} = dV^{cs,\ \lambda} + dV^{scc,\ \lambda}$. Qualitatively, by examining equation (4), we can show how each channel contributes to the final welfare effect. It is clear that the size of each channel depends on the values of the temptation parameter, other fundamental parameters and real interest rate. However, we are not able to determine the quantitative role of each channel unless we plug in the actual parameter values. In the next section, we conduct a quantitative analysis.

### 3.4 A quantitative analysis

In this section we parameterize our model and quantitatively explore the effects of temptation on individuals' allocation and welfare.

**Parameterization.** Individuals have standard CRRA preferences in the form of $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. The empirical studies estimate the values of the relative risk aversion parameter $\sigma$ between 1 and 10 (Auerbach and Kotlikoff (1987)). We consider a range of the risk-aversion parameter values between 1 and 6, $\sigma = [2, 4, 6]$.

We also assume that the temptation function has the form of $v(c) = \lambda u(c)$. A higher values of $\lambda$ implies an increase in the strength of temptation. We assume that the temptation function is strictly increasing i.e. individuals are tempted to consume their entire wealth each period. Their is no unique estimated values of the temptation parameter. DeJong and Ripoll (2007) estimate that $\lambda = 0.0786$. Huang, Liu and Zhu (2007) report significantly higher values for the temptation parameter, $\lambda$. In the benchmark, they report that $\lambda$ is either equal to 0.2849 or 0.4795 depending on the data set used to construct the average wealth-consumption ratio. Note that while DeJong and Ripoll (2007) use aggregate time series data, Huang, Liu and Zhu (2007) use household-level pseudo-panel data set. In a recent study, in the benchmark analysis, Bucciol (forthcoming) finds that $\lambda = 0.1899$. In our analysis, we consider a range of the temptation parameter values $\lambda$ between 0 and 0.125, $\lambda = [0, 0.025, 0.05, \ldots, 0.125]$

Regarding endowments, we assume that in period 1 individuals are homogeneous in terms of productivity. Without loss of generality we normalize market wage to one ($w = 1$) and interest rate to zero ($r = 0$), and set time discount rate at 1 ($\beta = 1$). We also consider an example with 2 percent interest rate later.

**Experiments.** We solve the model numerically and conduct various experiments to identify quantitatively the welfare effect of temptation. More specifically, we explore how an increase in the values of temptation parameter leads to welfare losses and how interaction between temptation and risk-aversion influences welfare outcome. We report results in Table 1.

**Results.** We report changes in welfare due to the presence of temptation in column 2 of Table 1. We also report decomposition of welfare changes into the inter-temporal allocation effect and the self-control effect in columns 3 and 4, respectively.

| | $\lambda$ | $dV_1$ | $\mathrm{dV}^{cs}$ | $\mathrm{dV}^{scc}$ | $\mathrm{dV}^{cs}/dV_1$ | $\mathrm{dV}^{scc}/dV_1$ |
|---|---|---|---|---|---|---|
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 0.025 | -0.621 | -0.004 | -0.617 | 0.614 | 99.386 |
| $\sigma$=2 | 0.050 | -1.235 | -0.015 | -1.220 | 1.205 | 98.795 |
| | 0.075 | -1.841 | -0.033 | -1.808 | 1.776 | 98.224 |
| | 0.100 | -2.440 | -0.057 | -2.384 | 2.327 | 97.673 |
| | 0.125 | -3.033 | -0.087 | -2.946 | 2.860 | 97.140 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 0.025 | -0.932 | -0.005 | -0.927 | 0.545 | 99.455 |
| $\sigma$=4 | 0.050 | -1.855 | -0.020 | -1.835 | 1.070 | 98.930 |
| | 0.075 | -2.767 | -0.044 | -2.724 | 1.575 | 98.425 |
| | 0.100 | -3.671 | -0.076 | -3.595 | 2.063 | 97.937 |
| | 0.125 | -4.565 | -0.116 | -4.449 | 2.534 | 97.466 |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 0.025 | -1.088 | -0.006 | -1.082 | 0.525 | 99.475 |
| $\sigma$=6 | 0.050 | -2.165 | -0.022 | -2.142 | 1.031 | 98.969 |
| | 0.075 | -3.230 | -0.049 | -3.181 | 1.518 | 98.482 |
| | 0.100 | -4.286 | -0.085 | -4.200 | 1.988 | 98.012 |
| | 0.125 | -5.331 | -0.130 | -5.201 | 2.441 | 97.559 |

Table 1: Temptation and Welfare. Note that annual interest rate is set at 0, r = .0. $\lambda$ is temptation paramter; $dV_1$ is percentage changes in utility (column 2), compared to no temptation case; $dV^{cs}$ is changes in utility due to the allocation effect (column 3); $dV^{scc}$ is changes in utility due to the self-control effect (column 4).

Our quantitative results confirm that the presence of temptation significantly lowers welfare. As shown in column 2 of Table 1, increases in temptation parameter value decrease the agent's utility. Considering the case with $\sigma = 2$, an incremental increase in the value of temptation parameter by .025 decreases welfare by 0.6 percent. Compared the case with no temptation and self-control problem (first row with $\lambda = 0$), an agent with temptation parameter value around 0.1 has significant lower welfare by 2 percents.

Considering other cases with $\sigma = 4$ or 6, we find a similar pattern in which higher temptation parameter values lead to lower welfare. Moreover, we find that reductions in welfare are bigger when inter-temporal elasticity of substitution is larger. An incremental increase in the value of temptation parameter by .025 decreases welfare by almost 1 percent with $\sigma = 4$; meanwhile, an incremental increase in the value of temptation parameter by .025 decreases welfare by almost 1.2 percent with $\sigma = 6$. Yet, the negative welfare effect of temptation is more severe when individuals are more risk-averse. This implies that the two opposing forces, long-term commitments for consumption smoothing and a short-term urge of temptation, magnify the negative welfare effect.

We decompose welfare losses into two transmission channels using equation 3: the inter-temporal allocation effect (column 3) and the self-control effect (column 4). To ease our analysis we also calculate percentage contributions by the inter-temporal allocation effect (column 5) and the self-control effect (column 6). We consider the case $\sigma = 2$ and $\lambda = .025$ (row 2). Compared to no temptation case, there is a drop in welfare by .6 percent (row 2 of column 2), of which .6 percent is contributed by the inter-temporal allocation effect (row 2 of column 5), while 99.4 percent is contributed by the self-control effect (row 2 of column 6). As temptation and self-control problems become more severe, the welfare loss contributed by the self-control cost declines a little bit but it still significantly dominates the inter-temporal allocation effect. For example, with $\lambda = .1$ almost 98 percent of welfare loss is due to the self-control effect (row 5 of column 6). This pattern is overall true for all other cases with bigger $\sigma$. This result indicates welfare losses are contributed mostly by the self-control effect rather than by the allocation effect.

Hence, individuals with temptation and self-control preferences have lower utility in an ex ante sense if tempting allocations are available in their choice set. In our simple setting, we are able to decompose such welfare losses according to two channels of effects. We find that the urge for temptation to over-consumption distorts individuals' inter-temporal allocation, but welfare losses resulting from that channel are relatively small. In fact, dis-utility costs of self-control efforts to resist temptation is a main channel contributing to negative welfare outcomes.

**Remark.** In such environment in which individuals face temptation and self-control problem, the size and shape of choice sets matter for welfare because the presence of temptation creates an urge for any potentially tempting alternative which is costly to control. Individuals would be better off if they would be able to eliminate all temptation alternative in their choice sets i.e. choose from a smaller choice set. This implies that individuals with temptation/self-control problems would be willing to pay in order to obtain full commitment to future consumption levels, and so that they are be able to choose these future consumption levels now, subject to the present-value budget constraint. Put it differently, there are induced preferences for commitment when individuals suffer from temptation and self-control problems. Such induced preferences for commitment devices reflects a wish both to get risk of succumbing to temptation on inter-temporal allocation, and to eliminate dis-utility costs of self-control. In our quantitative analysis using standard parameter values in the previous literature, we identify the main underlying force behind welfare losses is dis-utility cost of self-control efforts rather than the temptation distortion to inter-temporal allocation.

In the absence of market mechanism for commitment, temptation and self-control problem give a rise for fiscal intervention through tax-transfer programs. Basically, governments with tax and transfer instruments can provide a corrective tool that potentially improves welfare: $(i)$ correcting the inter-temporal allocation distortions, and $(ii)$ releasing severity of the self-control utility costs. In next section, we analyze how fiscal policy can correct individuals' behavior and improve welfare.

## 4 Fiscal policy as a temptation control device

In this section we study how fiscal policy potentially improves welfare in an economy where there is lack of market mechanism for commitment. We will extend our basic model to include a government, and characterize equilibrium and conditions for the existence of a government. We focus on two specific fiscal programs: savings subsidy and Pay-As-You-Go (PAYG) social security.

**Government.** In what follows we assume that there is an institution, called a government that runs these two fiscal programs. We also assume that government imposes two separate earmark taxes on income to finance such two fiscal programs. The corresponding budget constraints are given by:

$$\tau^w w = \tau^s s \text{ and } \tau^{ss} w = T, \tag{5}$$

where $T$ is the level of social security benefits, $\tau^s$ is a subsidy rate to savings, and $\tau^w$ and $\tau^{ss}$ are tax on labor incomes to finance the saving subsidy and social security programs, respectively. The household problem in the new model is given by:

$$V_1^{\lambda,\tau^{ss},\tau^s} = \begin{cases} \max_{c_1,s} \{(1+\lambda)\, u(c_1, s) \mid c_1 + (1-\tau^s)\, s = (1-\tau^w-\tau^{ss})\, w \text{ and } s \geq 0 \} \\ \quad - \max_{\widetilde{c}_1,\widetilde{s}_1} \{v(\widetilde{c}_1, \widetilde{s}) \mid \widetilde{c}_1 + \widetilde{s} = (1-\tau^w-\tau^{ss})\, w \text{ and } \widetilde{s} \geq 0\}, \end{cases} \tag{6}$$

and

$$V_2(s) = \begin{cases} \max_{c_2,s_2} \{(1+\lambda)\, u(c_2, s_2) \mid c_2 + s_2 = Rs + T\} \\ \quad - \max_{\widetilde{c}_2,\widetilde{s}_2} \{v(\widetilde{c}_2, \widetilde{s}_2) \mid \widetilde{c}_2 + \widetilde{s}_2 = Rs + T\}, \end{cases}$$

for young and old agents, respectively.

**Equilibrium.** Let $g_s$ be the agent's optimal saving rule

$$g_s = \begin{cases} \left( \dfrac{\left(1-\tau^w-\tau^{ss}-\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}}\tau^{ss}\right)}{(1-\tau^s)+\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}}R} \right) \text{ if } s > 0, \\ = 0 \text{ if otherwise.} \end{cases}$$

The household's optimal allocations:

$$\begin{aligned} s &= g_s w, && (7)\\ c_1 &= \overbrace{(1-\tau^w-\tau^{ss}-(1-\tau^s)\,g_s)}^{=g_{c_1}}w, && (8)\\ \text{and } c_2 &= \overbrace{(Rg_s+\tau^{ss})}^{=g_{c_2}}w, && (9)\end{aligned}$$

where, two new variables $g_{c_1}$ and $g_{c_2}$ are the optimal rules that map an agent's total wealth into consumption when young and consumption when old, respectively.

For given market factor prices $\{R, w\}$, and government policy $\{\tau^s, T\}$, a competitive equilibrium is an allocation $\{s, c_1, c_2\}$ that solves the household problem (6), and taxes $\{\tau^w, \tau^{ss}\}$ that clear the government budget (5).

**A condition for fiscal intervention.** In our environment, fiscal intervention distorts individuals's inter-temporal allocation which lowers welfare. On other hand, fiscal intervention potentially improves welfare as it mitigates inter-temporal distortions and releases self-control costs to resist temptation. There are two opposing effects on welfare: the positive one resulting from correcting inter-temporal distortion and releasing self-control cost and the negative one resulting from policy distortions. This trade-off raises questions: whether fiscal intervention is socially desired and to what extent a government should intervene. To derive the condition for fiscal intervention we construct a social welfare criteria. We denote $V_1^{\lambda,\tau^w,\tau^{ss}}$ the welfare measure associated with fiscal policies $\tau^w$ and $\tau^{ss}$

$$V_1^{\lambda,\tau^w,\tau^{ss}} = \left[\overbrace{\frac{(g_{c_1})^{1-\sigma}+\beta\,(g_{c_2})^{1-\sigma}}{1-\sigma}}^{v^{cs}} - \overbrace{\lambda\left(\frac{(1-\tau^w-\tau^{ss})^{1-\sigma}-(g_{c_1})^{1-\sigma}}{1-\sigma}\right)}^{v^{scc}}\right] w^{1-\sigma}. \tag{10}$$

Let $V_1^{\lambda>0,\tau^w=0,\tau^{ss}=0}$ denote the welfare when there is no fiscal intervention, $\tau^w=0$ and $\tau^{ss}=0$. Analytically, a fiscal intervention is socially desired if

$$dV_1^{\lambda,\tau^w,\tau^{ss}} = V_1^{\lambda \text{ with } \tau^w>0 \text{ or } \tau^s>0} - V_1^{\lambda \text{ with } \tau^w=0,\tau^{ss}=0} > 0. \tag{11}$$

This implies that fiscal intervention is socially justified if welfare gains can at least offset welfare losses.

## 4.1 Saving subsidy

In this section we analyze how a saving subsidy program can work as a temptation control device. To focus on the effects of a saving sudsidy program only we assume away a PAYG social security program ($\tau^{ss=0}$), so that government only runs a saving subsidy program with income tax as a financing instrument. The government budget constraint is given by $\tau^w w = \tau^s s$.

Let $V_1^{\lambda>0,\tau^s>0}$ denote the value function when there is a saving subsidy program

$$V_1^{\lambda>0,\tau^s>0} = \left[ \overbrace{\frac{(g_{c_1})^{1-\sigma} + \beta (g_{c_2})^{1-\sigma}}{1-\sigma}}^{\upsilon^{cs \text{ with } \tau^s>0}} - \lambda \overbrace{\left( \frac{(1-\tau^w)^{1-\sigma} - (g_{c_1})^{1-\sigma}}{1-\sigma} \right)}^{\upsilon^{scc \text{ with } \tau^s>0}} \right] w^{1-\sigma},$$

and let $V_1^{\lambda>0,\tau^s=0}$ denote the value function when there is no saving subsidy program. As stated in equation 11, the introduction of a saving subsidy program is socially desired if the condition $V_1^{\lambda>0,\tau^s>0} - V_1^{\lambda>0,\tau^s=0} > 0$ is satisfied.

Intuitively, welfare outcomes are driven by the corrective distortions created by a saving subsidy program. When the corrective distortions created by tax and subsidy dominate the distortions caused by temptation, positive welfare outcomes appear. Krusell et al. (2010)) study an optimal capital tax problem in a similar context and find that the welfare criteria is satisfied with negative capital tax rates i.e. investment subsidy. Our analysis in previous sections points out two driving mechanisms behind welfare gains: one operating through the inter-temporal allocation effect and one operating through the self-control cost effect. Notice that, Krusell et al. (2010) mention these channels, they do not analyze the role of each channel. Quantitatively, it is not clear which channel is relatively more important.

We conduct a welfare decomposition exercise to identify the mechanisms behind the welfare effect when a saving subsidy program is introduced. We argue that a saving subsidy program plays two fundamental roles: $(i)$ a device to mitigate the adverse effect of temptation on inter-temporal allocation and $(ii)$ a device to reduce self-control cost as tax-financing instrument restrains the tempted agents' choice set.

First, we argue that an introduction of a saving subsidy program can potentially neutralize the adverse effect on individuals' inter-temporal allocation (*the inter-temporal allocation effect*). To illustrate this point we examine the optimal saving rule

$$s = \frac{(1-\tau^w)}{(1-\tau^s) + \left( \frac{(1+\lambda)(1-\tau^s)}{\beta R} \right)^{\frac{1}{\sigma}} R} w. \tag{12}$$

It is clear that a saving subsidy program influences individuals' optimal saving decision in two opposing directions. On one hand, saving subsidy rate distorts relative prices between current consumption and future consumption. It makes real rates of return on investment relatively higher and it induces individuals to save more, $\frac{\partial s}{\partial \tau^s} > 0$ (price effect). This effect works against the adverse effect caused by temptation on inter-temporal allocation. On other hand, to finance a subsidy program government needs to collect more labor income tax, $\frac{\partial \tau^w}{\partial \tau^s} > 0$; and this lowers individuals' savings due to reduction in income, $\frac{\partial s}{\partial \tau^w} < 0$ (income effect). The saving effect of a saving subsidy program is not decisive. If price effect dominates income effect, individuals tend to save more and consume less and $\left(\upsilon^{cs \text{ with } \tau^s>0} - \upsilon^{cs \text{ with } \tau^s=0}\right) > 0$; otherwise, they save less.

Second, an introduction of a saving subsidy program has direct and indirect effects on self-control cost (*the self-control cost effect*). We examine changes in self-control costs

$$SCC = -\upsilon^{scc \text{ with } \tau^s>0}\frac{w^{1-\sigma}}{1-\sigma} = -\lambda\left(\frac{(1-\tau^w)^{1-\sigma}}{1-\sigma} - \frac{(g_{c_1})^{1-\sigma}}{1-\sigma}\right)\frac{w^{1-\sigma}}{1-\sigma}.$$

With a saving subsidy program in play, there are two opposing effects on self-control costs. First, income tax as a financing instrument reduces income available in period 1; and it therefore reduces severity of self-control cost. On other hand, the increase in savings due to subsidizing widens the gap between commitment consumption and temptation consumption, which increases the pain of self-control. For any given level of wealth, the gap $\left(\upsilon^{scc \text{ with } \tau^s>0} - \upsilon^{scc \text{ with } ,\tau^s=0}\right)$ measures how these two effects interplay. If the former is dominant $\left(\upsilon^{scc \text{ with } \tau^s>0} - \upsilon^{scc \text{ with } \tau^s=0}\right) <$ 0, a saving subsidy program leads to a reduction in self-control costs.

Hence, the final welfare outcome depends on how the inter-temporal allocation effect and the self-control cost effect play out. Our core goal is to identify which one is the main channel of welfare gains when a saving subsidy program is introduced. To that end we conduct a welfare decomposition analysis. We follow a similar approach in the previous section to decompose the welfare effect as follows: $dV_1^{\lambda,\tau^s} = V_1^{\lambda>0,\tau^s>0} - V_1^{\lambda>0,\tau^s=0}$, where $V_1^{\lambda>0,\tau^s=0}$ is the social welfare when there is no a saving subsidy program. Changes in welfare after a saving subsidy program is introduced can be broken down into

$$dV_1^{\lambda,\tau^s} = V_1^{\lambda,\tau^s>0} - V_1^{\lambda,\tau^s=0} = \left[\begin{array}{c} \overbrace{\left[\upsilon^{cs \text{ with } \tau^s>0} - \upsilon^{cs \text{ with } \tau^s=0}\right]}^{\text{Change in allocation due to saving subsidy}} - \\ \overbrace{\left[\upsilon^{scc \text{ with } \tau^s=0} - \upsilon^{scc \text{ with } ,\tau^s=0}\right]}^{\text{Change in self-control effort due to saving subsidy}} \end{array}\right] w^{1-\sigma}. \qquad (13)$$

There are two components of the final welfare outcome: change in welfare due to change in inter-temporal allocation, and change in welfare due to change in self-control cost. As discussed, subsidizing savings improves welfare via two channels: First, it potentially eliminates some inter-temporal distortions caused by temptation (the inter-temporal allocation effect). Second, the tax financing instrument mitigates severity of self control costs (the self-control effect).

The final change in welfare can be summarized as $dV_1^{\lambda,\tau^s} = dV^{cs \text{ with } \lambda,\tau^s} + dV^{scc \text{ with } \lambda,\tau^s}$. A saving subsidy program is socially desired if welfare gains from correcting individuals' saving incentive and welfare gains from releasing self-control cost together are positive i.e. $dV_1^{\lambda,\tau^s} > 0$.

**A quantitative analysis.** We consider a specific example with $\sigma = 2$, $\lambda = .1$, $\beta = 1$ and $r = 0$. We conduct a policy experiment in which government collects income tax at rate $\tau^w$ to subsidize savings at rate $\tau^s$. We report results in Table 2.

An introduction of a saving subsidy program results in positive welfare outcome in an environment where temptation and self-control problem are present. The tempted individuals are willing to pay income tax up to $\tau^w = 7.5\%$ to subsidize saving. The positive welfare gains result from eliminating the adverse effect of temptation on inter-temporal allocation,

| $\tau^w$ | $\tau^s$ | s | $c_1$ | $c_2$ | $dV_1$ | $dV^{cs}$ | $dV^{scc}$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.4881 | 0.5119 | 0.4881 | 0.0000 | 0.0000 | 0.0000 |
| 0.0250 | 0.0506 | 0.4946 | 0.5054 | 0.4946 | 0.0019 | 0.0018 | 0.0001 |
| 0.0500 | 0.0998 | 0.5012 | 0.4988 | 0.5012 | 0.0024 | 0.0022 | 0.0001 |
| 0.0750 | 0.1476 | 0.5081 | 0.4919 | 0.5081 | 0.0014 | 0.0012 | 0.0002 |
| 0.1000 | 0.1942 | 0.5151 | 0.4849 | 0.5151 | -0.0011 | -0.0014 | 0.0002 |
| 0.1250 | 0.2393 | 0.5223 | 0.4777 | 0.5223 | -0.0054 | -0.0057 | 0.0003 |
| 0.1500 | 0.2832 | 0.5297 | 0.4703 | 0.5297 | -0.0115 | -0.0119 | 0.0004 |
| 0.1750 | 0.3257 | 0.5373 | 0.4627 | 0.5373 | -0.0196 | -0.0201 | 0.0004 |
| 0.2000 | 0.3669 | 0.5451 | 0.4549 | 0.5451 | -0.0300 | -0.0305 | 0.0005 |
| 0.2250 | 0.4068 | 0.5532 | 0.4468 | 0.5532 | -0.0429 | -0.0435 | 0.0006 |
| 0.2500 | 0.4453 | 0.5614 | 0.4386 | 0.5614 | -0.0584 | -0.0590 | 0.0007 |
| 0.2750 | 0.4825 | 0.5700 | 0.4300 | 0.5700 | -0.0769 | -0.0776 | 0.0007 |

Table 2: Saving Subsidy, Allocation and Welfare with nil annual interest rate r=0. Note that $\tau^w$ is income tax rate; $\tau^s$ is saving subsidy rate; $s$ is savings; $c_1$ is consumption when young; $c_2$ is consumption when old; $dV_1$ is changes in utility, compared to no saving subsidy program; $dV^{cs}$ is changes in utility due to the allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

$dV^{cs \text{ with } \lambda,\tau^s}$, and from releasing self-control cost, $dV^{scc \text{ with } \lambda,\tau^s}$. Considering the case $\tau^w = 5\%$ and $\tau^s = 10\%$ (row 3 of Table 2), we find that more than 91 percent of welfare gain $dV_1^{\lambda,\tau^s}$ is due to the increase in $dV^{cs \text{ with } \lambda,\tau^s}$. This is quite different from zero percentage point gain due to $dV^{cs \text{ with } \lambda,\tau^s}$ when a PAYG social security program with similar scale $\tau^{ss} = 5\%$ (see row 2 Table 6) is introduced. Yet, the major source of welfare gains comes from correcting inter-temporal allocation rather than releasing self-control cost. We find a shift in quantitative role of the transmission mechanisms. Income tax as a financing instrument plays a limited role in releasing self-control cost. The intuition is clear. As argued before, saving stimulating comes at a cost as it widens the gap between commitment consumption and temptation consumption, and magnifies severity of self-control problem. This consequently offsets the role of income tax in releasing self-control cost.

**Remark.** The role of a saving subsidy program in an environment when agents have temptation and self-control preferences has been analyzed in Krusell et al. (2010). They characterize an optimal taxation problem and find that negative capital tax is socially optimal. Notably, they emphasize the role of negative capital tax to correct the inter-temporal distortion caused by temptation. However, in their analysis they do not explicitly decompose the welfare effect of a saving subsidy program into two separate driving mechanisms as we do in this analysis. Our approach enhances our understanding of the underlying mechanisms and highlights which channel is relatively more important.

## 4.2 Alternative options to subsidize saving

In this section, we analyze the role of alternative tax financing instruments and an interest payment subsidy program.

**Saving subsidy program with consumption tax.** We consider an alternative case in which government collects consumption tax from young individuals to cover fiscal cost of a saving subsidy program. The government budget constraint with consumption tax is given by $\tau^c c_1 = \tau^s s$, where $\tau^c$ is a consumption tax rate. The household budget constraint in period 1 becomes $(1+\tau^c)\,c_1 + (1-\tau^s)\,s = w$. The optimal saving is

$$s = \left[ \frac{1}{(1-\tau^s) + \left( \frac{(1-\tau^s)}{(1+\tau^c)} \frac{(1+\lambda)}{\beta R} \right)^{\frac{1}{\sigma}} R} \right] w.$$

Let us examine the saving rule with consumption tax as a financing instrument. It is easy to see that consumption tax affects individuals' inter-temporal allocation in a similar fashion that subsidizing saving works. That is, consumption tax decreases price of future consumption relative to current consumption i.e. the slope of the budget constraint. This basically induces individuals to save more for their future consumption. Differently, consumption tax has no direct effect on the size of the choice set in period 1 when individuals experience the pain of self-control. This subsequently does not provide a tool to release severity of self-control cost.

Since tax-financing instruments potentially play a key role in releasing self-control cost, they have important implications for welfare. We decompose welfare effects according to the two channels of welfare effects and conduct a quantitative analysis in which we vary consumption tax rates between 0 and .1. We report results in Table 3.

| $\tau^c$ | $\tau^s$ | s | $c_1$ | $c_2$ | $dV_1$ | $dV^{cs}$ | $dV^{scc}$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.4881 | 0.5119 | 0.4881 | 0.0000 | 0.0000 | 0.0000 |
| 0.0100 | 0.0103 | 0.4931 | 0.5069 | 0.4931 | -0.2004 | 0.0015 | -0.2019 |
| 0.0200 | 0.0202 | 0.4981 | 0.5019 | 0.4981 | -0.2017 | 0.0022 | -0.2039 |
| 0.0300 | 0.0297 | 0.5029 | 0.4971 | 0.5029 | -0.2037 | 0.0021 | -0.2058 |
| 0.0400 | 0.0388 | 0.5077 | 0.4923 | 0.5077 | -0.2065 | 0.0013 | -0.2078 |
| 0.0500 | 0.0476 | 0.5125 | 0.4875 | 0.5125 | -0.2100 | -0.0002 | -0.2098 |
| 0.0600 | 0.0560 | 0.5171 | 0.4829 | 0.5171 | -0.2142 | -0.0024 | -0.2118 |
| 0.0700 | 0.0642 | 0.5217 | 0.4783 | 0.5217 | -0.2190 | -0.0053 | -0.2137 |
| 0.0800 | 0.0720 | 0.5263 | 0.4737 | 0.5263 | -0.2245 | -0.0088 | -0.2157 |
| 0.0900 | 0.0796 | 0.5307 | 0.4693 | 0.5307 | -0.2306 | -0.0129 | -0.2178 |
| 0.1000 | 0.0869 | 0.5351 | 0.4649 | 0.5351 | -0.2373 | -0.0176 | -0.2198 |

Table 3: Saving Subsidy, Allocation and Welfare with $r = 0$. Note that $\tau^c$ is consumption tax applied to the young; $\tau^s$ is saving subsidy; $s$ is savings; $c_1$ is consumption when young; $c_2$ is consumption when old; $dV_1$ is changes in utility; $dV^{cs}$ is changes in utility due to the allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

We find that a saving subsidy program financed by the consumption tax revenue induces individuals to save more and consume less in young age. This eliminates inter-temporal allocation distortions caused by temptation and leads to the increases in welfare. As seen in column 7 of Table 3, the changes in welfare due to the inter-temporal allocation correcting ($dV^{cs}$) is positive for subsidy rates up to around 4%.

As income tax $\tau^w$ is not used as a financing instrument, welfare gains from releasing self-control costs are completely removed. In fact, a saving subsidy program makes individuals suffer more from the pain of self-control as it widens the gap between the temptation consumption level and the commitment consumption level. It subsequently increases the self-control costs. This mechanism can be seen directly from the self-control cost equation

$$SCC = -\lambda \left( \frac{1}{1-\sigma} - \frac{(g_{c_1})^{1-\sigma}}{1-\sigma} \right) \frac{w^{1-\sigma}}{1-\sigma}.$$

An increase in consumption tax decreases $g_{c_1}$, which in turn increases dis-utility cost of self control.

In a sum, when government relies on consumption tax to finance a saving subsidy program, a trade off between inter-temporal distortion and self-control cost releasing is excluded. Since the negative effect from tax distortions and increased severity of self-control cost tend to dominate the positive effect from mitigating inter-temporal allocation distortions, the final welfare effect is negative. Overall, as stated in column 6 of Table 3, this saving subsidy program is not socially desired, so that when $\tau^s = 0$, it results in highest social welfare.

This result is opposite to our previous result in which when government uses income tax as a financing instrument. The opposing results highlights that both sides of a saving subsidy program are equally important, at least in our partial equilibrium setting. We conclude that just subsidizing saving itself plays a limiting role and tax financing instruments are relatively important in determining the final welfare effect.

**Interest payment subsidy with income tax.** Government can use different fiscal instruments to encourage individual to save. We consider the case that government subsidizes interest payment rather saving. Note that the key difference between a interest payment subsidy program and a saving subsidy program is the timing of fiscal intervention. The former is a late intervention program while the latter is an early intervention program. First, we consider a program in which government collects income tax ($\tau^w$) in period 1 to finance a interest payment subsidy program ($\tau^r$):

$$c_1 + s = (1 - \tau^w)\, w \text{ and } c_2 = (1 + \tau^r)\, Rs.$$

We consider a similar example with $\sigma = 2$, $\lambda = .1$, $\beta = 1$ and $r = 0$. We run a number of policy experiments in which government varies income tax rate between 0 and .1 and report results in Table 4

An introduction of an interest payment subsidy program results in positive welfare outcome in an environment where individuals suffer from the pain of temptation and self-control problem. Our results indicate that individuals are willing to pay income tax up to $\tau^w = 9\%$ to subsidize interest payment. The positive welfare outcome is driven by two forces: ($i$) correcting the adverse effect of temptation on inter-temporal allocation, $dV^{cs}$, and ($ii$) releasing self-control cost, $dV^{scc}$. Comparing two channels we find that the major source of welfare gains come from

| $\tau^w$ | $\tau^r$ | s | $c_1$ | $c_2$ | $dV_1$ | $dV^{cs}$ | $dV^{scc}$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.4881 | 0.5119 | 0.4881 | 0.0000 | 0.0000 | 0.0000 |
| 0.0100 | 0.0208 | 0.4807 | 0.5093 | 0.4907 | 0.0009 | 0.0009 | 0.0000 |
| 0.0200 | 0.0423 | 0.4733 | 0.5067 | 0.4933 | 0.0016 | 0.0015 | 0.0000 |
| 0.0300 | 0.0644 | 0.4659 | 0.5041 | 0.4959 | 0.0021 | 0.0020 | 0.0001 |
| 0.0400 | 0.0872 | 0.4585 | 0.5015 | 0.4985 | 0.0023 | 0.0022 | 0.0001 |
| 0.0500 | 0.1108 | 0.4512 | 0.4988 | 0.5012 | 0.0024 | 0.0022 | 0.0001 |
| 0.0600 | 0.1352 | 0.4439 | 0.4961 | 0.5039 | 0.0022 | 0.0020 | 0.0001 |
| 0.0700 | 0.1603 | 0.4367 | 0.4933 | 0.5067 | 0.0017 | 0.0016 | 0.0002 |
| 0.0800 | 0.1863 | 0.4294 | 0.4906 | 0.5094 | 0.0010 | 0.0008 | 0.0002 |
| 0.0900 | 0.2132 | 0.4222 | 0.4878 | 0.5122 | 0.0001 | -0.0001 | 0.0002 |
| 0.1000 | 0.2409 | 0.4151 | 0.4849 | 0.5151 | -0.0011 | -0.0014 | 0.0002 |

Table 4: Interest Payment Subsidy with Income Tax. Annual interest rate is set at .0, r = 0.0 Note that $\tau^w$ is income tax; $\tau^r$ is interest payment subsidy; $s$ is savings; $c_1$ is consumption when young; $c_2$ is consumption when old; $dV$ is change in utility; $dV^{cs}$ is changes in utility due to the allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

the former rather than the latter. Hence, we obtain a similar result when government uses income tax revenue to finance a saving subsidy program. That is subsidizing savings or interest payment with income tax leads to a similar outcome in a perfect foresight model. Notably, a interest payment subsidy program appears to create less distortion.

**Interest payment subsidy with consumption tax.** To further highlight the role of tax financing instrument we consider another case in which government uses consumption tax to finance an interest payment subsidy program. The household budget constraints are defined as

$$(1 + \tau^c) c_1 + s = w \text{ and } c_2 = (1 + \tau^r) Rs.$$

We conduct similar policy experiments as before and report the results in Table 5.

| $\tau^C$ | $\tau^r$ | s | $c_1$ | $c_2$ | $dV_1$ | $dV^{cs}$ | $dV^{scc}$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.4881 | 0.5119 | 0.4881 | 0.0000 | 0.0000 | 0.0000 |
| 0.0100 | 0.0106 | 0.4829 | 0.5120 | 0.4880 | -0.0000 | -0.0000 | 0.0000 |
| 0.0200 | 0.0214 | 0.4777 | 0.5121 | 0.4879 | -0.0000 | -0.0001 | 0.0001 |
| 0.0300 | 0.0325 | 0.4724 | 0.5122 | 0.4878 | -0.0000 | -0.0001 | 0.0001 |
| 0.0400 | 0.0439 | 0.4671 | 0.5124 | 0.4876 | -0.0000 | -0.0002 | 0.0002 |
| 0.0500 | 0.0555 | 0.4618 | 0.5126 | 0.4874 | -0.0000 | -0.0003 | 0.0002 |
| 0.0600 | 0.0674 | 0.4565 | 0.5128 | 0.4872 | -0.0000 | -0.0003 | 0.0003 |
| 0.0700 | 0.0796 | 0.4511 | 0.5130 | 0.4870 | -0.0000 | -0.0004 | 0.0004 |
| 0.0800 | 0.0921 | 0.4456 | 0.5133 | 0.4867 | -0.0000 | -0.0006 | 0.0005 |
| 0.0900 | 0.1050 | 0.4402 | 0.5136 | 0.4864 | -0.0000 | -0.0007 | 0.0007 |
| 0.1000 | 0.1183 | 0.4346 | 0.5140 | 0.4860 | -0.0001 | -0.0009 | 0.0008 |

Table 5: Interest Payment Subsidy with Consumption Tax. Annual interest rate is set at .0, r = 0.0 Note that $\tau^c$ is income tax; $\tau^r$ is interest payment subsidy; $s$ is savings; $c_1$ is consumption when young; $c_2$ is consumption when old; $dV$ is change in utility; $dV^{cs}$ is changes in utility due to the allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

We again find that an interest payment subsidy program with consumption tax as a financing instrument results in negative welfare outcome; and when $\tau^s = 0$, it is the highest social welfare. This result is similar to the experiment where government collects consumption tax to finance a saving subsidy program. It is not surprising because the main driving mechanisms are similar. This result again emphasizes the role of a tax financing instrument. More importantly, it highlights the importance of considering the effects from both sides of a tax-transfers program rather than just focusing on one side story.

Hence, our analysis points out the importance of tax-financing instrument to ease additional self-control costs created by subsidizing saving, which is understated in the previous literature.

### 4.3 Social security

In this section we analyze how a PAYG social security program assists individuals to deal with temptation and self-control problems. We close a saving subsidy programs, $\tau^s = \tau^w = 0$, and assume that the government runs a PAYG social security program only. The government budget constraint is given by $T = \tau^{ss} w$.

**Welfare implications.** With an existence of a PAYG social security system, the interior optimal saving is given by:

$$s = \frac{\overbrace{\left(1 - \tau^{ss} - \left(\frac{(1+\lambda)}{\beta R}\right)^{\frac{1}{\sigma}} \tau^{ss}\right)}^{=g_s}}{1 + \left(\frac{(1+\lambda)}{\beta R}\right)^{\frac{1}{\sigma}} R} w. \tag{14}$$

It is straightforward to show that $\frac{\partial s}{\partial \tau^{ss}} < 0$ from equation 14. That is, PAYG social security crowds private savings. This classic result is well documented in the previous literature. The intuition is that PAYG social security redistributes income from the young who have marginal propensity to save to the old who have low marginal propensity to save.

In an environment where individuals face an oversaving problem, PAYG social security systems offer a mechanism to correct such inefficient allocation of wealth over lifetime. The crowding out effects potentially result in welfare gains. Unfortunately, in the environment we consider here individuals face an undersaving problem instead. To establish the welfare criteria to justify for an introduction of a PAYG social security program we consider the following welfare function:

$$V_1^{\lambda>0,\tau^{ss}>0} = \left[\overbrace{\frac{(g_{c_1})^{1-\sigma}}{1-\sigma} + \beta\frac{(g_{c_2})^{1-\sigma}}{1-\sigma}}^{=\upsilon^{cs \text{ with } \lambda>0,\tau^{ss}>0}} - \lambda\overbrace{\left(\frac{(1-\tau^{ss})^{1-\sigma}}{1-\sigma} - \frac{(g_{c_1})^{1-\sigma}}{1-\sigma}\right)}^{=\upsilon^{scc \text{ with } \lambda>0,\tau^{ss}>0}}\right] w^{1-\sigma}. \tag{15}$$

Note that there are two new functions: $\upsilon^{cs \text{ with } \lambda>0,\tau^{ss}>0} = \frac{(g_{c_1})^{1-\sigma}}{1-\sigma} + \beta\frac{(g_{c_2})^{1-\sigma}}{1-\sigma}$ and $\upsilon^{scc \text{ with } \lambda>0,\tau^{ss}>0}$

$= \lambda \left( \frac{(1-\tau^{ss})^{1-\sigma}}{1-\sigma} - \frac{(g_{c_1})^{1-\sigma}}{1-\sigma} \right)$. The former is a mapping rule that projects wealth into an individual's utility; the latter is a measure of an individual's dis-utility cost. Social security tax rate $\tau^{ss}$ appears in these two measures. When $\tau^{ss} = 0$, this is an economy without a PAYG social security program. Let $V_1^{\lambda>0,\tau^{ss}=0}$ and $V_1^{\lambda>0,\tau^{ss}>0}$ be the welfare function without and with a PAYG social security program, respectively.

Since the presence of temptation causes an undersaving problem, PAYG social security is not needed to be a policy tool that corrects inter-temporal allocation inefficiency. In fact, PAYG social security decreases individuals' savings even further and worsens the inter-temporal distortion caused by temptation, which results in negative welfare effects (*the inter-temporal allocation effect*). On other hand, a PAYG social security program, however, provides a tool to mitigate severity of self-control efforts (*the self-control effect).* More specifically, social security tax reduces the size of the choice set in period 1 as $c_1 + s = (1 - \tau^{ss})\, w$. In our setting where individuals only face self-control cost when young rather when old, the less wealth available when young the less dis-utility cost they have to pay to eliminate temptation. This is equivalent to welfare gain. The final welfare outcome is ambiguous and depends on interplay between these two opposing effects. The condition to justify whether a society desires a PAYG social security program is given by

$$dV_1^{\lambda,\tau^{ss}} = V_1^{\lambda>0,\tau^{ss}>0} - V_1^{\lambda>0,\tau^{ss}=0} > 0.$$

Yet, a PAYG social security system is socially desired when welfare gains resulting from releasing severity of self-control efforts dominate welfare losses due to dampening inter-temporal allocation distortion.

**Welfare decomposition.** To have a deeper understanding of how a PAYG social security program works to improve welfare we conduct a welfare decomposition exercise:

$$dV_1^{\lambda,\tau^{ss}} = \begin{bmatrix} \overbrace{\left[\upsilon^{cs \text{ with } \tau^{ss}>0} - \upsilon^{cs \text{ with } \tau^{ss}=0}\right]}^{\text{Change in allocation due to social security}} \\ - \overbrace{\left[\upsilon^{scc \text{ with } \tau^{ss}>0} - \upsilon^{scc \text{ with } \tau^{ss}=0}\right]}^{\text{Change in self-control efforts due to social security}} \end{bmatrix} w^{1-\sigma}. \tag{16}$$

An introduction of PAYG social security program creates two opposing effects on individuals' welfare: First, it distorts the inter-temporal allocation rule in $\upsilon^{cs \text{ with } \lambda>0,\tau^{ss}>0}$; Second, it reduces dis-utility cost of self-control via payroll tax $\tau^{ss}$ in $\upsilon^{scc \text{ with } \tau^{ss}>0}$. Changes in inter-temporal allocation and self-control efforts after introducing a PAYG social security program are characterized in two separate components. The first component $\left(\upsilon^{cs \text{ with } \tau^{ss}>0} - \upsilon^{cs \text{ with } \tau^{ss}=0}\right)$ measures the effect of a PAYG social security program on individuals' inter-temporal allocation. The second component $-\left(\upsilon^{scc \text{ with } \tau^{ss}>0} - \upsilon^{scc \text{ with } \tau^{ss}=0}\right)$ measures the effect of PAYG social security on individuals' self-control efforts.

Arguably, when borrowing constraint is binding an introduction of a PAYG social security program magnifies the inter-temporal allocation distortion, the first component is clearly negative, $\left(\upsilon^{cs \text{ with } \tau^{ss}>0} - \upsilon^{cs \text{ with } \tau^{ss}=0}\right) < 0$; otherwise 0. On other hand, with $\tau^{ss} > 0$ a PAYG social security program reduces the size of budget constraint in young age, which releases severity of self-control cost and $\upsilon^{scc}$ decreases. The second component is positive $-\left(\upsilon^{scc \text{ with } \tau^{ss}>0} - \upsilon^{scc \text{ with } \tau^{ss}=0}\right) > 0$. Let $dV^{cs \text{ with } \lambda,\tau^{ss}} = \left(\upsilon^{cs \text{ with } \tau^{ss}>0} - \upsilon^{cs \text{ with } \tau^{ss}=0}\right) w^{1-\sigma}$ and $dV^{scc \text{ with } \lambda,\tau^{ss}} = -\left(\upsilon^{scc \text{ with } \tau^{ss}>0} - \upsilon^{scc \text{ with } \tau^{ss}=0}\right) w^{1-\sigma}$ be changes in welfare due to the inter-temporal allocation effect and the self-control cost effect, respectively. The final welfare outcome depends on a trade-off between these two opposing forces. Mathematically, this is summarized as

$$dV_1^{\lambda,\tau^{ss}} = dV^{cs \text{ with } \lambda,\tau^{ss}} + dV^{scc \text{ with } \lambda,\tau^{ss}}.$$

When the latter $dV^{scc \text{ with } \lambda,\tau^{ss}}$ dominates the former $dV^{cs \text{ with } \lambda,\tau^{ss}}$, a PAYG social security program is socially justified as it leads to welfare gain $dV_1^{\lambda,\tau^{ss}} > 0$.

**A quantitative analysis.** We consider a numerical example to supplement our theoretical analysis above. We consider a case when $\sigma = 2$, $\lambda = .1$, $\beta = 1$ and $r = 0$. Note that if $r > 0$, social security is not an efficient saving technology as it has lower rate of return that leads to efficiency loss and negative welfare effect. We assume away that efficiency loss by setting $r = 0$. That is, a PAYG social security program and financial market yield an identical real rate of return in this experiment. We make this assumption so that we can focus on between trade-offs between inter-temporal distortion and self-control cost releasing. [3]

We solve the model numerically and conduct a quantitative analysis in which we vary social security tax rate $\tau^{ss}$ between 0 and .65. We report results in Table 6.

| $\tau^{ss}$ | s | $c_1$ | $c_2$ | $dV_1$ | $dV^{cs}$ | $dV^{scc}$ |
|---|---|---|---|---|---|---|
| 0.0000 | 0.4881 | 0.5119 | 0.4881 | 0.0000 | 0.0000 | -0.0000 |
| 0.0500 | 0.4381 | 0.5119 | 0.4881 | 0.0053 | 0.0000 | 0.0053 |
| 0.1000 | 0.3881 | 0.5119 | 0.4881 | 0.0111 | 0.0000 | 0.0111 |
| 0.1500 | 0.3381 | 0.5119 | 0.4881 | 0.0176 | 0.0000 | 0.0176 |
| 0.2000 | 0.2881 | 0.5119 | 0.4881 | 0.0250 | 0.0000 | 0.0250 |
| 0.2500 | 0.2381 | 0.5119 | 0.4881 | 0.0333 | 0.0000 | 0.0333 |
| 0.3000 | 0.1881 | 0.5119 | 0.4881 | 0.0429 | 0.0000 | 0.0429 |
| 0.3500 | 0.1381 | 0.5119 | 0.4881 | 0.0538 | 0.0000 | 0.0538 |
| 0.4000 | 0.0881 | 0.5119 | 0.4881 | 0.0667 | 0.0000 | 0.0667 |
| 0.4500 | 0.0381 | 0.5119 | 0.4881 | 0.0818 | 0.0000 | 0.0818 |
| 0.5000 | 0.0000 | 0.5000 | 0.5000 | 0.0976 | 0.0023 | 0.0953 |
| 0.5500 | 0.0000 | 0.4500 | 0.5500 | 0.0572 | -0.0381 | 0.0953 |
| 0.6000 | 0.0000 | 0.4000 | 0.6000 | -0.0690 | -0.1644 | 0.0953 |
| 0.6500 | 0.0000 | 0.3500 | 0.6500 | -0.2980 | -0.3933 | 0.0953 |

Table 6: PAYG Social Security, Allocation and Welfare. Note that annual interest rate is r = .0; $\tau^{ss}$ is social security tax rate; $s$ is savings; $c_1$ is consumption when young; $c_2$ is consumption when old; $dV_1$ is changes in utility, compared to no social security; $dV^{cs}$ is changes in utility due to the allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

[3] Note that in our model we do not explicitly model economic growth and population growth. However, we implicitly include these factors in real interest rate.

As seen in Table 6, an introduction of a PAYG social security program is socially justified when individuals suffer temptation and lack of self-control problem. More specifically, there are positive welfare effects until social security tax rate is 55 percent, $\tau^{ss} = .55$. As seen in columns $5, 6$ and 7 of Table 6, welfare gains are driven mainly by the self-control cost effect rather the inter-temporal allocation effect. With social security tax rates between 0 and .45, the increase in welfare is 100 percent contributed by releasing the self-control cost as it tightens individuals' choice set in period 1. On other hand, the welfare effect resulting from inter-temporal allocation distortion is zero because individuals just optimally re-adjust their savings accordingly in response to the increase in forced savings. There is no real change in inter-temporal allocation of consumption. However, when no-borrowing constraint is binding, $\tau^{ss} \geq .5$, PAYG social security has real effects on individuals' life-cycle consumption behavior; and therefore, the inter-temporal allocation effect starts operating, $dV^{cs} < 0$.

Particularly, when $\tau^{ss} < 0.5$, both the inter-temporal allocation effect and the self-control cost effect are positive, and so is the final welfare effect. When $\tau^{ss} = 0.55$, there is no welfare gain from releasing self-control cost while there is welfare loss due to the tax distortion to inter-temporal allocation. Since welfare losses due to that distortion are relatively small while welfare gains due to self-control cost releasing from a forced savings are relatively big, PAYG social security still results in a positive welfare outcome. However, when $\tau^{ss} > .55$, the negative effect dominates the positive effect and the negative final welfare outcomes are realized. Yet, the final welfare outcome depends on to what extent welfare gains from releasing self-control cost can offset the welfare losses from distorting inter-temporal allocation.

A PAYG social security program is an inter-generational transfer mechanism in which government collects revenue from young individuals and redistributes that revenue to old individuals. In an economy model with standard preferences and no income and longevity risk, introducing a PAYG social security program results in welfare losses unless the economy is dynamic inefficient. This classic result dates back to Aaron (1966) and Samuelson (1975). Our quantitative analysis points out a number of social security designs in which welfare gains resulting from releasing self-control costs dominates welfare losses resulting from inter-temporal distortions. We conclude that there are induced preferences for a PAYG social security program in a dynamically efficient economy populated with individuals suffering from temptation and self-control problems. That is, the presence of temptation and self-control problems gives rise for PAYG social security as a device to control temptation.

This finding is not totally surprising as it has been documented in previous studies (e.g. see Kumru and Thanopoulos (2008) and Kumru and Tran (2012)). Notice that, these authors conduct analysis in dynamic general equilibrium overlapping generations models with heterogenous agents. In such frameworks, PAYG social security plays a mixed role: a form of social insurance against interruption or loss of earnings and a self-control cost reducing device. That approach is more comprehensive and realistic for policy analysis. However, as a result of model complexity, it is impossible to separate out analytically the two transmission channels of welfare effects, and to isolate analytically the role of social security as a corrective tool, which we focus on in this analysis. Nevertheless, our work enhances understanding of the driving mechanisms behind the welfare implications of a PAYG social security program.

**The efficiency effect and welfare.** A central point that has been discussed in the previous literature is that PAYG social security might not be an efficient saving technology (e.g. see Aaron (1966)). In the previous quantitative analysis, we completely abstract from such efficiency loss. We now consider that issue. The most simple way to include that efficiency loss caused by a PAYG social security program in our analysis is to allow positive real interest rate.

We consider an economy with 2 percent annual real interest rate, $r = .02$. In this setting, PAYG social security is an inefficient saving channel as it yields nil rate of return, compared to 2 percent in the financial markets. This inefficient saving instrument decreases individuals' lifetime wealth (*the efficiency effect*), which then leads to lower welfare. To explore quantitatively how important this efficient effect is we repeat our previous policy experiments. We report results in Table 7.

| $\tau^{ss}$ | s | $c_1$ | $c_2$ | $dV_1$ | $dV^{cs}$ | $dV^{scc}$ |
|---|---|---|---|---|---|---|
| 0.0000 | 0.4147 | 0.5853 | 0.7511 | 0.0000 | 0.0000 | -0.0000 |
| 0.0500 | 0.3778 | 0.5722 | 0.7343 | -0.0683 | -0.0696 | 0.0013 |
| 0.1000 | 0.3409 | 0.5591 | 0.7175 | -0.1394 | -0.1425 | 0.0031 |
| 0.1500 | 0.3040 | 0.5460 | 0.7006 | -0.2136 | -0.2190 | 0.0053 |
| 0.2000 | 0.2671 | 0.5329 | 0.6838 | -0.2909 | -0.2991 | 0.0082 |
| 0.2500 | 0.2302 | 0.5198 | 0.6670 | -0.3715 | -0.3833 | 0.0118 |
| 0.3000 | 0.1933 | 0.5067 | 0.6502 | -0.4556 | -0.4719 | 0.0163 |
| 0.3500 | 0.1564 | 0.4936 | 0.6334 | -0.5431 | -0.5652 | 0.0221 |
| 0.4000 | 0.1195 | 0.4805 | 0.6165 | -0.6342 | -0.6635 | 0.0294 |
| 0.4500 | 0.0827 | 0.4673 | 0.5997 | -0.7287 | -0.7674 | 0.0387 |
| 0.5000 | 0.0458 | 0.4542 | 0.5829 | -0.8266 | -0.8773 | 0.0507 |
| 0.5500 | 0.0089 | 0.4411 | 0.5661 | -0.9273 | -0.9937 | 0.0664 |
| 0.6000 | 0.0000 | 0.4000 | 1.0868 | -0.3095 | -0.3803 | 0.0708 |
| 0.6500 | 0.0000 | 0.3500 | 1.1774 | -0.5959 | -0.6667 | 0.0708 |

Table 7: Social Security, Allocation and Welfare with Efficiency Loss. Note that annual interest rate 2 percent, r = .02; $\tau^{ss}$ is social security tax rate; $s$ is savings; $c_1$ is consumption when young; $c_2$ is consumption when old; $dV_1$ is changes in utility, compared to no social security; $dV^{cs}$ is changes in utility due to the allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

The efficiency effect complicates the trade-off between inter-temporal allocation distortion and self-control cost releasing. First, a PAYG social security program still provides a tool to control temptation, so that there are welfare gains operating through the self-control cost releasing channel. As stated in column 7 of Table 7, $dV^{scc \text{ with } \lambda, \tau^{ss}}$ is positive. On other hand, the efficiency losses caused by PAYG social security negatively effect inter-temporal allocation and welfare. As seen in column 6 of Table 7, $dV^{cs \text{ with } \lambda, \tau^{ss}}$ is negative. The higher social security tax rate is the more negative $dV^{cs \text{ with } \lambda, \tau^{ss}}$ is. Overall, an expansion of a PAYG social security system effects welfare negatively.

Hence, there is a new trade-off between welfare losses due to the efficiency effect and welfare gains due to the self-control cost effect, in addition to trade-off between inter-temporal allocation distortion and self-control cost releasing. Whether a PAYG social security program is socially desired or not depends on how these trade-offs play out. In our example here, the negative welfare effect due to inter-temporal distortion and efficiency losses dominates the positive effect from releasing severity of self-control cost. The final welfare effect is always negative. We conclude that social security is not socially desired in a partial general equilibrium model with 2 percent real interest rate.

**Fully funded (FF) social security.** The efficiency effect is an important factor that determines whether a society desires a PAYG social security program as a temptation control device or not. To elaborate this point further we consider another social security system with fair benefit payment $T = Rw\tau^{ss}$. This is referred in the literature as a fully funded (FF) social security system. In this design, a FF social security program works purely as a forced savings mechanism that yields a market rate of return. Yet, FF social security and private savings are two equivalent saving mechanisms. With this institutional restriction we completely remove potential efficiency loss associated with a PAYG social security system.

| $\tau^{ss}$ | s | $c_1$ | $c_2$ | $dV_1$ | $dV^{cs}$ | $dV^{scc}$ |
|---|---|---|---|---|---|---|
| 0.0000 | 0.4147 | 0.5853 | 0.7511 | 0.0000 | 0.0000 | -0.0000 |
| 0.0500 | 0.3647 | 0.5853 | 0.7511 | 0.0053 | 0.0000 | 0.0053 |
| 0.1000 | 0.3147 | 0.5853 | 0.7511 | 0.0111 | 0.0000 | 0.0111 |
| 0.1500 | 0.2647 | 0.5853 | 0.7511 | 0.0176 | 0.0000 | 0.0176 |
| 0.2000 | 0.2147 | 0.5853 | 0.7511 | 0.0250 | 0.0000 | 0.0250 |
| 0.2500 | 0.1647 | 0.5853 | 0.7511 | 0.0333 | 0.0000 | 0.0333 |
| 0.3000 | 0.1147 | 0.5853 | 0.7511 | 0.0429 | 0.0000 | 0.0429 |
| 0.3500 | 0.0647 | 0.5853 | 0.7511 | 0.0538 | 0.0000 | 0.0538 |
| 0.4000 | 0.0147 | 0.5853 | 0.7511 | 0.0667 | 0.0000 | 0.0667 |
| 0.4500 | 0.0000 | 0.5500 | 0.8151 | 0.0656 | -0.0052 | 0.0708 |
| 0.5000 | 0.0000 | 0.5000 | 0.9057 | 0.0065 | -0.0644 | 0.0708 |
| 0.5500 | 0.0000 | 0.4500 | 0.9962 | -0.1154 | -0.1862 | 0.0708 |
| 0.6000 | 0.0000 | 0.4000 | 1.0868 | -0.3095 | -0.3803 | 0.0708 |
| 0.6500 | 0.0000 | 0.3500 | 1.1774 | -0.5959 | -0.6667 | 0.0708 |

Table 8: Fully Funded Social Security, Allocation and Welfare. Note that annual interest rate is r = .02; $\tau^{ss}$ is social security tax rate; $s$ is savings; $c_1$ is consumption when young; $c_2$ is consumption when old; $dV_1$ is changes in utility, compared to no social security; $dV^{cs}$ is changes in utility due to the allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

As efficiency loss is eliminated, a FF social security program is purely a temptation control device while it creates no wealth effect. Basically, in a FF social security system the government completely eliminates the dynamic trade-off between welfare loess due to efficiency loss and welfare gain due to self-control cost releasing. Note that the only mechanism at work is the trade-off between inter-termporal allocation distortion and self-control cost releasing. To determine the full effects of a fully funded social security program, we conduct a quantitative analysis with $r = .02$. We report results of this experiment in Table 8.

We obtain very similar results as in Table 6 for the PAYG social security program with no efficiency effect. More specifically, when borrowing constraint is not binding FF social security has no effect on inter-temporal decisions. Individuals with perfect foresight just adjust their savings accordingly in response to the increase in social security payments in old age. Once borrowing constraint is binding, an introduction of a FF social security system results in a real effect on individuals' inter-temporal allocation, which then further dampens the negative welfare effect of the inter-temporal allocation effect.

Overall, as stated in Table 8, a FF social security program is still socially desired (up to $\tau^{ss} = .5$). Since no-borrowing constraint is binding earlier at $\tau^{ss} = .45$, the preference for FF social security is lower than that for PAYG social security because the inter-temporal allocation distortion is operative earlier.

### 4.4 Saving subsidy vs. social security

Our previous result indicates that both saving subsidy and PAYG social security programs potentially result in welfare gains but the driving forces are different. The former relies mainly on the transmission channel that mitigates inter-temporal allocation distortions while the latter relies mostly on the transmission channel that releases self-control efforts. In this section we conduct a comparison between these two programs to highlight which channel/program is more effective in assisting individuals to deal with temptation and self-control problems.

We basically compare the welfare effects of a saving subsidy program in Table 5 to that of a PAYG social security program in Table 2. We start with a two similar size programs financed by 5 percent income tax rate. We find that the saving subsidy program results in a much smaller welfare gain, compared to the social security program in similar size. More specifically, this saving subsidy program results an increase in welfare by .0024 while the social security program leads to an increase in welfare by .0053. This implies that welfare gains from releasing self-control costs dominate welfare gains from mitigating inter-temporal allocation distortions. The key reason behind is that the saving subsidy program induces individuals to save more, which increases severity of self-control problem. Consequently, some welfare gains from eliminating inter-temporal allocation distortion are taken away by that negative force.

Overall, we find that society's preference for a saving subsidy program is much smaller than that for a PAYG social security program. Individuals are willing to give up at most 7.5 percent of their income to finance a saving subsidy program, while they are willing to give up at most 50 percent of their income to finance a PAYG social security program. We conclude that a PAYG social security program dominates a saving subsidy program in terms of welfare. Note that in the above analysis, we set $r = 0.$ so that we assume away the efficiency losses caused by a PAYG social security program.

As pointed out in our previous argument, whether or not a PAYG social security as a temptation control device is socially justified depends on how the efficiency effect plays out. To see whether the welfare dominance of a PAYG social security program is robust when accounting for efficiency loss, we conduct a welfare comparison between two fiscal programs in an economy with 2 percent annual interest rate, $r = .02$. The policy experiments and results for a PAYG social security program is reported in Table 7. We implement a similar policy experiment for a saving subsidy program and report results in Table 9.

| $\tau^w$ | $\tau^s$ | s | $c_1$ | $c_2$ | $dV_1$ | $dV^{cs}$ | $dV^{scc}$ |
|---|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.4147 | 0.5853 | 0.7511 | 0.0000 | 0.0000 | 0.0000 |
| 0.0250 | 0.0592 | 0.4221 | 0.5779 | 0.7646 | 0.0018 | 0.0015 | 0.0004 |
| 0.0500 | 0.1163 | 0.4298 | 0.5702 | 0.7784 | 0.0023 | 0.0015 | 0.0007 |
| 0.0750 | 0.1714 | 0.4377 | 0.5623 | 0.7927 | 0.0012 | 0.0001 | 0.0011 |
| 0.1000 | 0.2243 | 0.4458 | 0.5542 | 0.8075 | -0.0015 | -0.0030 | 0.0015 |
| 0.1250 | 0.2752 | 0.4542 | 0.5458 | 0.8227 | -0.0059 | -0.0079 | 0.0019 |
| 0.1500 | 0.3241 | 0.4629 | 0.5371 | 0.8384 | -0.0123 | -0.0147 | 0.0023 |
| 0.1750 | 0.3709 | 0.4718 | 0.5282 | 0.8546 | -0.0208 | -0.0236 | 0.0027 |
| 0.2000 | 0.4158 | 0.4810 | 0.5190 | 0.8713 | -0.0316 | -0.0348 | 0.0032 |
| 0.2250 | 0.4587 | 0.4905 | 0.5095 | 0.8886 | -0.0449 | -0.0485 | 0.0036 |
| 0.2500 | 0.4996 | 0.5004 | 0.4996 | 0.9064 | -0.0610 | -0.0650 | 0.0040 |
| 0.2750 | 0.5387 | 0.5105 | 0.4895 | 0.9247 | -0.0801 | -0.0846 | 0.0045 |

Table 9: Saving Subsidy, Allocation and Welfare with 2 percent annual interest rate r = 0.02. Note that $\tau^w$ is income tax rate; $\tau^s$ is saving subsidy rate; $s$ is savings; $c_1$ is consumption when young; $c_2$ is consumption when old; $dV_1$ is changes in utility, compared to no saving subsidy program; $dV^{cs}$ is changes in utility due to the allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

As seen in column 6 of Table 9, an introduction of a saving subsidy program results in positive welfare outcome up to $\tau^w = 7.5\%$, which is similar to the case when $r = .0$. We conclude the welfare effects of subsidizing saving are quite robust to changes in real interest rate. The logic is that a saving subsidy program distorts relative prices between present and future consumption but does not introduce any inefficiency saving instrument. More importantly, we find that a saving subsidy program is still socially desired in an economy with 2 percent annual interest rate, while there is no social preference for a PAYG social security program (see Table 7). Yet, our result indicates that a PAYG social security program no longer dominates a saving subsidy program when the efficiency effect is taken into account.

### 4.5 General equilibrium channel

We have demonstrated that the presence of temptation distorts inter-temporal allocation and creates welfare losses, which gives rise for fiscal intervention to correct such distortion in a simple partial equilibrium model. It has been argued in the literature that the adverse effects of fiscal policies on allocation and welfare are further magnified when accounting for general equilibrium price adjustments (see e.g. Auerbach and Kotlikoff (1987)). In this section, we extend our analysis to a general equilibrium model.

**Model.** We formulate a dynamic general equilibrium economy that consists of a household sector and a government sector as described in the previous section and also a firm sector represented by a perfect competitive representative firm. The production technology is given by a constant returns to scale production function $Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$, where $K$ is the input of capital, $L$ is the input of effective labor services and $A$ is the total factor productivity. The firm rents capital and labour inputs from competitive factor markets to maximize its profit according to $\max_{K,L} \{AK^{\alpha}L^{1-\alpha} - qK - wL\}$, given rental rate, $q = \alpha AK^{\alpha-1}L^{1-\alpha}$, and wage rate, $w = (1-\alpha) AK^{\alpha}L^{-\alpha}$. Capital depreciates at rate $\delta$.

**Equilibrium.** Given government policy settings for fiscal policy, a steady state competitive equilibrium is such that: $(i)$ a collection of household decisions $\{c_1, s_1, c_2\}$ solve the household problem; $(ii)$ the firm chooses labour and capital inputs to solve the profit maximization problem; $(iii)$ the markets for capital and labor clear $K = s$ and $L = 1$; $(iv)$ factor prices are determined competitively, i.e., $w = (1-\alpha) AK^{\alpha}L^{-\alpha}$, $q = \alpha AK^{\alpha-1}L^{1-\alpha}$ and $r = q - \delta$; and $(v)$ the government budget constraint is satisfied.

**Calibration.** We parameterize the model and solve it numerically. We follow the previous literature to choose the values of fundamental parameter. Precisely, we set $A = 1$, $\alpha = .36$, and annual depreciation rate $\delta = .05$. We calibrate discount factor $\beta$ to match annual interest rate is 2 percent. Since one period is equivalent to 30 years we adjust annual values of parameters and variables accordingly. Notice that, in this general equilibrium model, all market prices, capital accumulation and income are endogenously determined by equilibrium conditions.

**Temptation, capital accumulation and welfare.** We begin with the effects of temptation on inter-temporal allocation and welfare. We explore numerically how an increase in the values of temptation parameter affects capital accumulation, income and social welfare. To construct a benchmark case for a comparison we consider an economy with no government sector. We re-calibrate the time discount factor to match annual interest rate is 2 percent and conduct a number of experiments with different values of the temptation parameter. We report results in Table 10.

In our general equilibrium setting, a "small" distortion to individuals' inter-temporal allocation appears to result in a "big" distortion to allocation of resources and efficiency at aggregate level. The reason is that general equilibrium price adjustments amplify the effects of inter-termporal allocation distortion on aggregate efficiency and welfare. As stated in column 3 of Table 10, increases in the strength of temptation induce agents to save less; and therefore, lower capital accumulation. Examining the case with $\sigma = 2$, we find that an increase of temptation parameter value from $\lambda = 0$ to $\lambda = .1$ decreases capital stock by 6.6 percent. Yet, a "small" deviation from a standard preferences results in a relative "big" impact on capital accumulation, and subsequently leads to a decline in output by 2.5 percent.

We also examine how the urge of temptation interacts with the preference for consumption smoothing. We find that the adverse effect of temptation on capital accumulation and output is mitigated as individuals are more risk-averse i.e. stronger consumption commitment. More precisely, with $\sigma = 4$ or $\sigma = 6$, there are smaller detrimental effects on capital stock and output as the urge of temptation rises.

To quantify the contribution of two underlying mechanisms to final welfare outcomes we

| | $\lambda$ | dK | dY | $dV_1$ | $dV^{cs}$ | $dV^{scc}$ |
|---|---|---|---|---|---|---|
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 0.025 | -1.744 | -0.631 | -0.566 | -0.003 | -0.562 |
| $\sigma$=2 | 0.050 | -3.431 | -1.249 | -1.123 | -0.014 | -1.109 |
| | 0.075 | -5.064 | -1.853 | -1.672 | -0.030 | -1.642 |
| | 0.100 | -6.646 | -2.445 | -2.213 | -0.051 | -2.161 |
| | 0.125 | -8.180 | -3.025 | -2.746 | -0.079 | -2.668 |
| | | | | | | |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 0.025 | -1.090 | -0.394 | -1.174 | -0.005 | -1.169 |
| $\sigma$=4 | 0.050 | -2.150 | -0.779 | -2.337 | -0.020 | -2.316 |
| | 0.075 | -3.181 | -1.157 | -3.489 | -0.045 | -3.445 |
| | 0.100 | -4.185 | -1.527 | -4.632 | -0.078 | -4.554 |
| | 0.125 | -5.163 | -1.890 | -5.765 | -0.119 | -5.646 |
| | | | | | | |
| | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| | 0.025 | -0.794 | -0.286 | -1.429 | -0.006 | -1.424 |
| $\sigma$=6 | 0.050 | -1.566 | -0.567 | -2.849 | -0.023 | -2.826 |
| | 0.075 | -2.319 | -0.841 | -4.258 | -0.050 | -4.208 |
| | 0.100 | -3.053 | -1.110 | -5.658 | -0.087 | -5.572 |
| | 0.125 | -3.770 | -1.374 | -7.049 | -0.132 | -6.917 |

Table 10: Temptation, Capital Accumulation and Welfare in a Dynamic General Equilibrium Economy. Notice that we report percentage changes in capital stock in column 3, percentage changes in income welfare in column 4, percentage changes in welfare in column 5 and decomposition of welfare losses due to the inter-temporal allocation effect and the self-control effect in columns 6 and 7, respectively.

conduct a welfare decomposition. We report results in columns 5 and 6 of Table 10. A quick comparison of columns 5 and 6 of Table 10 and that of Table 1 we shows a similar pattern. That is, the welfare losses contributed by inter-temporal allocation distortion are quantitatively small while the welfare losses contributed by efforts to resist temptation stand out as a main source. Note that since final welfare outcomes contain general equilibrium efficiency effects we are not be able to obtain a clean welfare decomposition as in a partial equilibrium model.

In the next section we analyze the corrective role of two fiscal policies as a temptation control device when accounting for the general equilibrium effects. We first focus on a saving subsidy program and then move to a PAYG social security program.

**Saving subsidy.** We run similar policy experiments in which the government collects income tax at rate $\tau^w$ to subsidize individuals' savings at rate $\tau^s$ and report results in Table 11.

In this general equilibrium model, a saving subsidy program results in significant welfare gains so that there is a strong social desire for a saving subsidy program. As reported in column 6 of Table 11, individuals are willing to give up to 45 percent of their income, $\tau^w = 45\%$, to support a saving subsidy program, which is much higher than that in a partial equilibrium model.

| $\tau^w$ | $\tau^s$ | dK | dY | dV$_1$ | dV$^{cs}$ | dV$^{scc}$ |
|---|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0500 | 0.1333 | 10.3762 | 3.6180 | 0.1434 | 0.1380 | 0.0054 |
| 0.1000 | 0.2508 | 21.4078 | 7.2331 | 0.2568 | 0.2464 | 0.0104 |
| 0.1500 | 0.3547 | 33.1315 | 10.8514 | 0.3403 | 0.3252 | 0.0150 |
| 0.2000 | 0.4466 | 45.5888 | 14.4791 | 0.3929 | 0.3736 | 0.0193 |
| 0.2500 | 0.5280 | 58.8252 | 18.1221 | 0.4131 | 0.3898 | 0.0233 |
| 0.3000 | 0.6001 | 72.8921 | 21.7865 | 0.3977 | 0.3707 | 0.0270 |
| 0.3500 | 0.6640 | 87.8466 | 25.4785 | 0.3424 | 0.3120 | 0.0304 |
| 0.4000 | 0.7203 | 103.7532 | 29.2045 | 0.2409 | 0.2073 | 0.0336 |
| 0.4500 | 0.7700 | 120.6846 | 32.9713 | 0.0839 | 0.0473 | 0.0366 |
| 0.5000 | 0.8136 | 138.7232 | 36.7861 | -0.1420 | -0.1814 | 0.0394 |
| 0.5500 | 0.8517 | 157.9629 | 40.6568 | -0.4563 | -0.4983 | 0.0420 |

Table 11: Saving Subsidy, Capital Accumulation and Welfare in in a Dynamic General Equilibrium Economy. Note that $\tau^w$ is income tax rate; $\tau^s$ is saving subsidy rate; $dY$ is changes in income; $dK$ is changes in capital; $dV_1$ is changes in utility, compared to no social security; $dV^{cs}$ is changes in utility due to the inter-temporal allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

The positive welfare outcome is driven by two mechanisms. First, saving subsidy/negative tax induces individuals to change their saving behaviors, which in return mitigates the adverse effect of temptation on inter-temporal allocation. As shown in column 3 of Table 11, aggregate capital increases substantially as saving subsidy rate increases. Technically, a savings subsidy program influences the slope and size of the budget constraint so that it corrects the incentive distortion caused by the urge of temptation. This subsequently induces agents to save more and results in higher aggregate capital stock and aggregate income. That is, the distortions created by subsidizing saving mitigates the efficiency losses caused by the detrimental effect of temptation and improves welfare. Second, the tax financing instrument also plays a role in releasing self-control cost. As income tax rate increases, it reduces the size of the choice sets and so do the self-control costs. However, the welfare gains through this channel is quite small. Compared column 7 to column 8 of Table 11, we find that the effect through inter-temporal allocation distortions is the main driving force.

**Social security.** We now turn our analysis to the effect of saving subsidy in a general equilibrium model. It has been argued in the previous literature that an introduction of a PAYG social security program result in efficiency and welfare losses in various types of general equilibrium overlapping generations models: deterministic model (e.g. see Auerbach and Kotlikoff (1987)) or a stochastic model with incomplete insurance markets and heterogenous agents (e.g. see Imrohoroglu *et al.* (1995)). The recent development in that literature also includes Gul-Pesendofer type of preferences (Kumru and Thanopoulos (2008), Bucciol (2011), and Kumru and Tran (2012)). These studies show that temptation and self-control problems give further rise for a PAYG social security as a device to reduce self-control. Notice that, these studies base their analysis on stochastic dynamic general equilibrium models with heterogenous agents. In such framework, PAYG social security plays two roles: a form of social insurance against interruption or loss of earnings and a device to control temptation and to release self-control cost. In our analysis, we abstract from stochastic factors and agent heterogeneity and assume away the redistributive role of a PAYG social security. We therefore are able to isolate the mechanics behind the corrective role of PAYG social security.

| $\tau^{ss}$ | dK | dY | dV$_1$ | dV$^{cs}$ | dV$^{scc}$ |
|---|---|---|---|---|---|
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.0500 | -20.2589 | -7.8266 | -0.5661 | -0.5677 | 0.0016 |
| 0.1000 | -37.1986 | -15.4197 | -1.2769 | -1.2789 | 0.0020 |
| 0.1500 | -51.1844 | -22.7531 | -2.1643 | -2.1655 | 0.0012 |
| 0.2000 | -62.5813 | -29.8041 | -3.2711 | -3.2702 | -0.0009 |
| 0.2500 | -71.7435 | -36.5542 | -4.6561 | -4.6518 | -0.0044 |
| 0.3000 | -79.0062 | -42.9898 | -6.4013 | -6.3918 | -0.0095 |

Table 12: PAYG Social Security, Capital Accumulation and Welfare in a Dynamic General Equilibrium Economy. Note that $\tau^{ss}$ is social security tax rate; $dK$ is changes in capital; $dY$ is changes in income; $dV_1$ is changes in utility, compared to no social security; $dV^{cs}$ is changes in utility due to the inter-temporal allocation effect; $dV^{scc}$ is changes in utility due to the self-control effect.

We repeat the previous experiments and report results in Table 12. Our results indicate that economic distortions caused by a PAYG social security program are further amplified when accounting for general equilibrium price adjustments. A seen in Table 12, capital, and output declines significantly as the government expands its PAYG social security program. Our results also show that the introduction of a PAYG social security program always leads to lower welfare. This implies that the welfare gain resulting from releasing self-control costs is not enough to compensate the welfare loss resulting from the adverse effects of PAYG social security on capital accumulation and aggregate efficiency. Hence, the introduction of a PAYG social security program is not socially justified.

This result is consistent with the results in previous studies including Kumru and Thanopoulos (2008) and Kumru and Tran (2012). Note that in this setting the preference for PAYG social security further weakened as the redistributive and insurance role of PAYG social security is eliminated. In addition, we assume a conservative form of temptation, $v(c) = \lambda u(c)$. The multiplicative factor $\lambda$ is a key parameter to determine the strength of temptation and severity of self-control problem. Clearly, as $\lambda$ increases the dis-utility cost of self-control intensifies. However, the urge for temptation is not as strong as in the cases in which the temptation function is in the form of $v(c) = \lambda \frac{c^{\rho}}{\rho}$ with $\rho > 0$. We believe that for the higher levels of temptation, the induced preference for the PAYG social security as a temptation control device is more pronounced. Once we relax this functional form restriction to have a more power function of temptation, positive welfare outcomes can be realized. For an example, Kumru and Thanopoulos (2008) find that the higher the strength of temptation the less distortive the PAYG social security system is. Nevertheless, it is not a central point of this paper. Our general equilibrium analysis here is to demonstrate that accounting the efficiency effect is essential to justify whether or not a society desires a PAYG social security program as a temptation control device.

**Saving subsidy vs. social security.** We compare the welfare effects of a saving subsidy program (Table 11) and the welfare effects of a PAYG social security program (Table 12), we find that a saving subsidy program results in welfare gains while a PAYG social security program results in welfare losses in a general equilibrium framework. The positive welfare effects of a saving subsidy program are more pronounced. Meanwhile, the negative efficiency effect and welfare losses caused by a social security program are amplified when accounting for the general equilibrium price adjustments. This is line with our previous results in a partial equilibrium model when the efficiency effect is in play.

Hence, the efficiency effects generated in a general equilibrium model plays an important role in determining whether or not a society desire a saving subsidy program or a PAYG social security program. This result stresses the importance of accounting for the feedback effects from general equilibrium channels.

# 5 Conclusion

In this paper we study the role of fiscal policies in an environment in which agents suffer from temptation and self-control problems but there is no market mechanism for commitment. In such environment, individuals value fiscal programs as a commitment device so that they can avoid the distortions in preferences. We consider two specific fiscal programs: social security and saving subsidy. We find that the incentive distortions created by such fiscal programs work as a corrective tool that mitigates the adverse effect of succumbing to temptation on inter-temporal allocation (*first transmission channel*) and releases severity of self-control problem (*second transmission channel*). More specifically, our results indicate that both fiscal programs potentially lead to welfare gains; however, the driving mechanisms are quite different. A social security program operates mainly through the first channel while a saving subsidy program operates mainly through the second channel. Moreover, we also find that the direction and size of welfare effects vary substantially when allowing for different tax-financing instruments as well as when accounting for general equilibrium channels. Particularly, we find that the welfare effect of a social security program dominates that of a saving subsidy program in a partial equilibrium model. However, that result reverses when the general equilibrium effects are taken into account. The key intuition is as follows. In a partial equilibrium model where all prices are fixed, the driving mechanism behind the positive welfare outcome is the first channel. In a general equilibrium model, as general equilibrium price adjustments amplify the efficiency losses caused by inter-temporal allocation distortions, the second channel becomes a dominant force.

The main framework for our analysis is a simple deterministic inter-temporal choice model with temptation and self-control preferences. In such deterministic framework, it is always better to commit individuals to a particular consumption path and remove/reduce alternative tempting choices. The simplicity of that approach allows us to isolate the underlying mechanisms; however, we completely abstract from a number of other fundamental factors. First, income uncertainty caused by either idiosyncratic or aggregate shocks is assumed away. In such stochastic framework, Gul-Pesendofer type agents face trade off between commitment to eliminate alternative tempting choices and flexibility to smooth consumption paths. Second, labor is inelastic in the current analysis. Inclusion of endogenous labor-leisure choice introduces interactions between temptation and intra- and inter- temporal allocation, which clearly has implications for welfare analysis. For more realistic and interesting policy analysis, one should take these factors into account. We leave these issues for future research.

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# 6 Appendix

## 6.1 Solving the basic model

Thus the young's agent problem is given by:

$$L(.) = \max_{c_1,s,\mu} \left\{ (1+\lambda) \frac{c_1^{1-\sigma}}{1-\sigma} + \beta V_2(s) \ + \ \mu (w - c_1 - s) \right\} - \lambda \frac{\widetilde{c_1}^{1-\sigma}}{1-\sigma}$$

where $\mu$ is the shadow price. The corresponding F.O.Cs yield the following system of equations:

$$\begin{aligned}
\frac{\partial L}{\partial c_1} &: \ (1+\lambda) c_1^{-\sigma} = \mu, \\
\frac{\partial L}{\partial s} &: \ \beta \frac{\partial V_2(s)}{\partial s} = \mu, \\
\frac{\partial L}{\partial \mu} &: \ w - c_1 - s = 0.
\end{aligned}$$

Combining yields:

$$\begin{aligned}
(1+\lambda) c_1^{-\sigma} &= \beta \frac{\partial V_2(s)}{\partial s}, \\
(1+\lambda) c_1^{-\sigma} &= \beta R (Rs)^{-\sigma}, \\
c_1 &= \left( \frac{(1+\lambda)}{\beta R} \right)^{\frac{1}{\sigma}} Rs.
\end{aligned}$$

Using $w - c_1 - s = 0$ leads to

$$\begin{aligned}
\left( \frac{(1+\lambda)}{\beta R} \right)^{\frac{1}{\sigma}} Rs + s &= w, \\
s &= \frac{1}{1 + \left( \frac{(1+\lambda)}{\beta R} \right)^{\frac{1}{\sigma}} R} w.
\end{aligned}$$

The optimal allocation are given by:

$$\begin{aligned}
s &= \left( \frac{1}{1 + \left( \frac{(1+\lambda)}{\beta R} \right)^{\frac{1}{\sigma}} R} \right) w = \overbrace{\frac{\theta}{1+\theta}}^{=g_s} w, \\
c_1 &= \left( \frac{1}{1 + \frac{1}{\left( \frac{(1+\lambda)}{\beta R} \right)^{\frac{1}{\sigma}} R}} \right) w = \frac{1}{1+\theta} w, \text{and } c_2 = Rs; \\
\widetilde{s_2} &= 0, \ \widetilde{c_2} = c_2 \text{ and } \widetilde{c_1} = w.
\end{aligned}$$

Given the optimal allocations we can rewrite the maximal value function in the second period as follows:

$$V_2 = \frac{\left(\frac{\theta R}{1+\theta}\right)^{1-\sigma}}{1-\sigma} w^{1-\sigma},$$

and the maximal value function is in the first period as follows:

$$V_1 = \left[ (1+\lambda) \left(\frac{1}{1+\theta}\right)^{1-\sigma} + \beta \left(\frac{\theta R}{1+\theta}\right)^{1-\sigma} - \lambda \right] \frac{w^{1-\sigma}}{1-\sigma}.$$

## 6.2 Solving the extended model with fiscal policy

The young's agent problem is given by:

$$L\left(.\right) = \max_{c_1, s, \mu} \left\{ (1+\lambda) \frac{c_1^{1-\sigma}}{1-\sigma} + \beta V_2\left(s\right) \ + \ \mu \left( \left(1 - \tau^w - \tau^{ss}\right) w - c_1 - \left(1-\tau^s\right) s \right) \right\} - \lambda \frac{\widetilde{c_1}^{1-\sigma}}{1-\sigma}$$

where $\mu$ is the shadow price. The corresponding F.O.Cs yield the following system of equations:

$$\begin{aligned}
\frac{\partial L}{\partial c_1} &: \ (1+\lambda) c_1^{-\sigma} = \mu, \\
\frac{\partial L}{\partial s} &: \ \beta \frac{\partial V_2\left(s\right)}{\partial s} = \left(1-\tau^s\right) \mu, \\
\frac{\partial L}{\partial \mu} &: \ \left(1 - \tau^w - \tau^{ss}\right) w - c_1 - \left(1-\tau^s\right) s = 0.
\end{aligned}$$

Combining yields:

$$\begin{aligned}
(1+\lambda) c_1^{-\sigma} &= \frac{1}{\left(1-\tau^s\right)} \beta \frac{\partial V_2\left(s\right)}{\partial s}, \\
(1+\lambda) c_1^{-\sigma} &= \frac{1}{\left(1-\tau^s\right)} \beta R \left(Rs + \tau^{ss} w\right)^{-\sigma}, \\
c_1 &= \left( \frac{(1+\lambda)}{\beta R} \left(1-\tau^s\right) \right)^{\frac{1}{\sigma}} \left(Rs + \tau^{ss} w\right).
\end{aligned}$$

Using $\left(1 - \tau^w - \tau^{ss}\right) w - c_1 - \left(1-\tau^s\right) s = 0$ leads to

$$\begin{aligned}
\left( \frac{(1+\lambda)}{\beta R} \left(1-\tau^s\right) \right)^{\frac{1}{\sigma}} \left(Rs + \tau^{ss} w\right) + \left(1-\tau^s\right) s &= \left(1 - \tau^w - \tau^{ss}\right) w, \\
s &= \frac{\left(1 - \tau^w - \tau^{ss} - \frac{(1+\lambda)}{\beta R} \left(1-\tau^s\right) \tau^{ss}\right)}{\left(1-\tau^s\right) + \left(\frac{(1+\lambda)}{\beta R} \left(1-\tau^s\right)\right)^{\frac{1}{\sigma}} R} w.
\end{aligned}$$

Following the same strategy as in previous section we obtain the following optimal allocations:

$$s = \frac{\left(1 - \tau^w - \tau^{ss} - \left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}} R \tau^{ss}\right)}{\left(1-\tau^s\right) + \left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}} R} w = g_s(\lambda; \tau^{ss}, \tau^w) w.$$

$$
\begin{aligned}
c_1 &= \left(\frac{(1+\lambda)}{\beta R}(1-\tau^s)\right)^{\frac{1}{\sigma}} (Rs + \tau^{ss} w), \\
&= \left(\frac{(1+\lambda)}{\beta R}(1-\tau^s)\right)^{\frac{1}{\sigma}} (R\overbrace{g_s}^{g_s(\lambda,\tau^{ss})} + \tau^{ss}) w \\
&= \left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}} \left(\frac{\left(1-\tau^w-\tau^{ss}-\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}}\tau^{ss}\right)}{\frac{(1-\tau^s)}{R}+\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}}} + \tau^{ss}\right) w, \\
&= \left(\frac{\left(1-\tau^w-\tau^{ss}-\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}}\tau^{ss}\right)}{1+\frac{(1-\tau^s)}{\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}} R}} + \tau^{ss}\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}}\right) w, \\
&= \left(\frac{1}{1+\frac{(1-\tau^s)}{\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}} R}} - \frac{\tau^w+\tau^{ss}+\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}}\tau^{ss}}{1+\frac{(1-\tau^s)}{\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}} R}} + \tau^{ss}\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}}\right) w,
\end{aligned}
$$

$$
\begin{aligned}
c_2 &= (Rg_s + \tau^{ss}) w, \\
\widetilde{c_2} &= g_s(\lambda; \tau^{ss}, \tau^w) w \\
\widetilde{s} &= 0, \text{ and } \widetilde{c_1} = (1-\tau^w-\tau^{ss}) w.
\end{aligned}
$$

The saving subsidy program is self-financed $\tau^w w = \tau^{ss} s$. We use the optimal saving rule to solve for an equilibrium labor tax rate that clears the government budget as

$$
\tau^w = \frac{\left(1-\tau^{ss}-\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}}\tau^{ss}\right)}{1+\frac{\left(1-\tau^s+\left(\frac{(1+\lambda)(1-\tau^s)}{\beta R}\right)^{\frac{1}{\sigma}} R\right)}{\tau^s}}.
$$

We derive the agent's value function as

$$
V_1^{\lambda,\tau^w,\tau^{ss}} = \left[\overbrace{\frac{(g_{c_1})^{1-\sigma}+\beta(g_{c_2})^{1-\sigma}}{1-\sigma}}^{\upsilon^{\lambda>0}(\beta,\sigma,R,\lambda,\tau^w,\tau^{ss})} - \lambda\overbrace{\left(\frac{(1-\tau^w-\tau^{ss})^{1-\sigma}-(g_{c_1})^{1-\sigma}}{1-\sigma}\right)}^{\upsilon^{scc}(\beta,\sigma,R,\lambda,\tau^w,\tau^{ss})}\right] w^{1-\sigma}.
$$