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Price Differentiation and Menu Costs in Credit Card Payments¹

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Abstract: We build a model of credit card payments where the retailers are allowed to charge differential prices depending on the instrument of payment chosen by the consumer. We follow the approach in Rochet and Wright (2010) but assume a credit card system without any type of no-surcharge rule. In a Hotelling competition framework at the retailers level, the competitive equilibrium prices are computed assuming that the store credit provided by the retailer is less cost efficient than the one provided by the credit card. In accordance with the literature, we obtain that the interchange fee becomes neutral if we eliminate the no-surcharge rule, when the interchange fee loses its ability to distort the individual consumer's decisions and displace the aggregated consumers' welfare from its maximum level. We prove that the average price obtained under price differentiation is smaller than the single retail price under the no-surcharge rule, despite the retailer's margins being the same in both scenarios. In addition, we introduce menu costs to prove that there is a value for the interchange fee such that there is equilibrium with price differentiation if and only if that fee is above this value. It must be interpreted as an endogenous cap for the interchange fee fixed by the credit card industry. Finally, we also obtain that under price differentiation with menu costs there is a non cooperative Nash equilibrium as in the well known "prisoner's dilemma" game.

Keywords: Credit cards, Payments, Two-sided markets.

JEL Classification: L11; E42; G18.

1 The views expressed in this working paper are those of the author(s) and do not necessarily reflect those of the Banco Central do Brasil.

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1. Introduction

There is an intensive international debate involving industry members, regulators and representatives of the society about the structure of the credit card market, the behavior of its players, the consequences on competitiveness and, most importantly, the social welfare (Weiner and Wright (2005), Bradford and Hayashi (2008)). Actually, the social welfare maximization should be the ultimate goal of any regulator, a difficult challenge that encompasses the assessment of distributive aspects, like defining weights on welfare for each segment of the society.

Among the most instigating issues in this debate is the one concerning the effects of the presence, or absence, of no-surcharge rules in the credit card payments on consumers. We can find in the literature that surcharging may have positive effects for merchants and consumers (Chakravorti and Emmons (2003), Bolt and Chakravorti (2008)) as well as studies showing ambiguous or positive impacts on the system (Rochet (2003), Rochet and Tirole (2008)). A distinctive feature of the card payment system is that, despite the fact that the cardholders make the choice of their payment instruments, the significant part of the transaction cost is incurred by the merchant. In practice, the fee structure has triggered the use of merchant fees to reward the issuance (interchange fee) and the usage of cards (card rewards), which is a typical behavior in two-side market structures. The central question is if the credit card industry can exert market power by imposing scheme rules prohibiting surcharging of credit card purchases by merchants. In other words, no-surcharge rules could prevent price signaling to cardholders about the relative costs of different payment methods, reinforcing the pattern where the higher the merchant fee, the greater the capacity to reward cardholders, leading to a less efficient allocation of resources in the payment system (“excess” of card usage).

Another important issue is that, under the no-surcharge rule, merchants recover the average cost of different instruments of payment charging all consumers equally. Consequently, consumers who do not use credit cards pay more than they would otherwise, in other words, consumers who use credit cards are subsidized in their

purchases (Chakravorti and Emmons (2003), Chakravorti and To (2007)). There are empirical studies that measure these cross subsidies in some jurisdictions, in general indicating that these are not negligible⁴.

Because of its anti-competitive nature, the no-surcharge rule has been prohibited in some jurisdictions, for instance, in the United Kingdom since 1991, in the Netherlands since 1994, in Sweden since 1995 and in Australia since 2003. The authorities judged that merchant pricing freedom is essential for effective price competition and competition between payment systems.

In Australia the prohibition on no-surcharge rules is stated in the Standards as “Neither the rules of the Scheme nor any participant in the Scheme shall prohibit a merchant from charging a credit cardholder any fee or surcharge for a credit card transaction”. Further, an assessment of interchange-fee capping in this country showed that issuers had recovered part of the loss of interchange fees in the short run (Chang et. al. (2005)). Those authors also showed that merchants definitely had a benefit which was not substantially passed to the consumers. Despite all this, regulators still recognize that the surcharging reforms in Australia have been successful and have provided significant public benefits. Notwithstanding, they have become concerned about cases where surcharges seem to be higher than the acceptance costs. Since the evidence obtained shows that in some instances surcharging has developed in a way that potentially compromises price signals and reduces the effectiveness of the reforms, regulators are currently reviewing the no-surcharge standards in order to provide card schemes with the ability to constraint the level of surcharges to something close to the merchant acceptance costs, whose main component is the merchant fees⁵.

When analyzing the convenience of eliminating the no-surcharge rule for credit card payments, it is quite reasonable to affirm (as we prove in this paper) that some merchants will have incentives to unilaterally surcharge credit card transactions above the

4 Schuh, S. and Stavins, J., (2010) and Central Bank of Brazil (2011a) and (2011b) are examples of empirical studies that estimate cross subsidies in the United States and Brazil, respectively.

5 Consultation documents of the Reserve Bank of Australia (2011a) and (2011b).

single price level, without any reduction of prices of other types of transactions. Notwithstanding, it is far from being a valid argument against differentiation, because this assertion does not take into account the existence of new equilibrium prices in the absence of the no-surcharge rule nor its effects on consumer welfare in the scenario with the no-surcharge rule.

As we will illustrate in the next section, through our theoretical analysis of a simple model, the fact that each merchant has the possibility to obtain a profit increase when he individually deviates from the single price, is not a guarantee that this extra profit is sustainable. Actually, a complete and coherent analysis needs, first of all, to find the new equilibrium prices. This will depend on the specific competitive environments and their effects on profit possibilities, in order to measure the welfare gains, or losses, when comparing both equilibria.

In the literature we can find important articles focusing on the economic role of the credit card interchange fee, as well as on its determination and possible regulation. In a system of profit-seeking firms which are imperfectly competitive, Schmalensee (2002) concludes that the interchange fee shifts the costs between issuers and acquirers and as a consequence, the charges between merchants and consumers. This allows enhancing the value of the payment system as a whole to its owners, due to the network externality. Rochet and Tirole (2002) analyze the welfare implications of a cooperative determination of the interchange fee by member banks, in a framework in which banks and merchants may have market power and consumers and merchants decide rationally on whether to buy or accept a payment card. Wright (2003) evaluates the social optimality of privately set interchange fees under the no-surcharge rule in two extremes of merchant pricing, namely monopolistic pricing and perfect competition. In addition, the positive aspect of the no-surcharge rule in preventing the excessive merchant surcharging is assessed. Rochet and Tirole (2006) analyze the welfare effects of the externalities inherent in the card payment system and discuss whether consumer surplus or social welfare is the proper benchmark for the study of the regulation in the card payment industry. They bring all the theoretical analysis to unravel the recent antitrust actions taken by regulators

and merchants against card associations in Australia, the UK and US. Wang (2010) suggests that card networks demand higher interchange fees to maximize member issuers' profits as card payments become more efficient and convenient. He also discusses positive and negative features of policy interventions.

The main reference of our work is Rochet and Wright (2010). They model the credit card explicitly, allowing a separate role for the credit functionality of credit cards, which is modeled apart from other payment cards (i.e., debit cards). They assume impossibility (or lack of incentives) of retailers to differentiate prices according to the instrument of payment chosen by consumers. Under those assumptions, they showed how a monopoly card network could select an interchange fee high enough to promote the utilization of credit cards in a level that exceeds the level that maximizes the aggregated consumer surplus. They show how a regulatory cap for the interchange fee could be used to increase consumer surplus.

Our work aims to extend the Rochet and Wright (2010) model, giving a distinctive subsidy to the debate and helping to clarify, through a simple theoretical model, the implications of price differentiation and of menu costs incurred by merchants in credit card payments. With this extension we are able to illustrate how the absence of a no-surcharge rule could generate equilibrium prices capable of improving consumer welfare and reduce the market power of banks through the interchange fee⁶, even under the assumption that retailers face menu costs associated to price differentiation. In this case, we prove that retailers will differentiate prices as long as the menu costs are not high enough.

The paper is divided into four sections. Section 2 describes the model, which is similar to that defined in Rochet and Wright (2010). In Section 3, we present the main results of the paper, first considering the absence of friction given by the menu costs of price differentiation and secondly including such costs to analyze the effects of that

⁶ See Gans and King (2003) about the neutrality of the interchange fee under price differentiation.

market imperfection. In section 5, we summarize the main conclusions. The Appendix contains the detailed proofs of all the propositions presented in the paper.

2. The model

Two distinctive changes in the model proposed by Rochet and Wright (2010) are introduced. In a first version of our model we only allow retailers to differentiate the price of credit card payments from the price of the other payment instruments (store credit, cash, debit cards and others). The second specification introduces menu costs in the former version of the model.

As in Rochet and Wright (2010), we assume here that there is a continuum of consumers, all distributed uniformly in a unitary length interval. All consumers have identical quasi-linear preferences, spending their income on retail goods costing γ to produce. There are two payment technologies. The first one corresponds to a group of “cash” payment technologies, which could include money, checks, debit cards or other instruments not involving any credit functionality. The second one corresponds, exclusively, to the credit cards’ payment technology. As an alternative to both technologies, each retailer can directly provide credit to the consumer, which is called “store credit”. Credit cards are held by a constant fraction x of consumers and assumed to be more costly than cash, and both costs are normalized assuming cash has zero cost. Credit cards allow consumers to purchase on credit and entail a cost (or benefit, if negative) f for the consumer (buyer), which is received (or paid) by the issuer, and a cost m (merchant fee) for the retailer (seller). The store credit is an alternative to the credit function of the credit card and entails a random transaction specific cost (or benefit, if negative) c_b for the consumer and cost c_s to the retailer.

Each consumer purchases one unit of the retail good, called “ordinary purchases”, providing him utility $u_0 > \gamma$, but, in addition, and with probability θ , he also receives utility $u_1 > \gamma$ from consuming another unit of the retail good called “credit purchases”. It

is assumed that merchants cannot bundle the two transactions nor distinguish between “ordinary” and “credit” purchases.

When making ordinary purchases, all consumers can choose between cash or store credit, but only a fraction x of them have possibility to choose credit cards. On the other hand, when making a credit purchase, cash is not an option for any consumer. Additionally, it is assumed that each consumer always has sufficient cash to pay for his ordinary purchases, but must rely on credit for credit purchases.

The transaction specific cost c_B of a store credit is observed by the consumer only when he is in the store, which is drawn from a continuous distribution with the cumulative distribution function H . We assume the distribution has full support over some range $(\underline{c}_B, \overline{c}_B)$, where \underline{c}_B is sufficiently negative, such that cardholders will sometimes choose to use store credit even if cash can be used instead, and \overline{c}_B is positive but not too high (in comparison with $u_1 - \gamma$), such that consumers will always prefer to make the credit purchase even if they have to pay with store credit rather than not buy at all. The draw c_B is the net cost of using store credit rather than credit cards or cash. A negative draw of c_B could represent a situation where a cardholder needs to preserve his cash or credit card balance for some other contingencies and thus values the use of store credit.

If the merchant fee m of a credit card purchase is smaller than the cost of store credit c_S ($m < c_S$), accepting credit cards is a potential mean for merchants to reduce their transaction costs of accepting credit purchases. But if $m > c_S$, acceptance of credit card increases merchant’s transaction costs.

In general, consumers will prefer credit cards to store credit when $c_B > f$, for both ordinary and credit purchases. In particular, when issuers give benefits ($f < 0$, i.e., cash back bonuses) to consumers in each credit card purchase, consumers will prefer to

use their credit cards rather than cash for ordinary purchases. It was proved in Rochet and Wright (2010) that, from the point of view of aggregated consumers, excessive incentives for credit card use could be socially wasteful.

The bank of the merchant, or acquirer of the transaction, incurs in an acquiring cost c_A and an interchange fee a (which is paid to the bank of the consumer) for each credit card transaction. It is assumed that acquirers are perfectly competitive, which implies that the merchant fee m is equal to the sum of the acquiring cost c_A and the interchange fee a ,

$$m = c_A + a. \quad (1)$$

The bank of the cardholder, or issuer of the card, incurs in an issuing cost c_I and receives the interchange fee a from the acquirer. It is assumed that issuers are imperfectly competitive, which implies that the cardholder fee f is equal to the net issuer cost c_I plus a constant profit margin π ,

$$f = c_I - a + \pi. \quad (2)$$

Thus, the total cost of a credit card transaction is

$$c = c_A + c_I \quad (3)$$

Denote by δ the excess cost of the store credit with respect to the total cost credit card transaction, which is defined by

$$\delta := c_S - c - \pi \quad (4)$$

We will restrict our analysis to the situation where, from the point of view of the suppliers of the credits (merchants or credit card industry), a credit card transaction is more cost efficient than the store credit, or, equivalently, $\delta > 0$.

Competition between retailers occurs as in the standard Hotelling model: consumers are uniformly distributed on an interval of unit length, with one retailer ($i=1,2$) located at each extremity of the interval. There is a transport cost t for consumers per unit of distance. Unlike Rochet and Wright (2010) we are interested here in the situation where retailers have the option to charge different retail prices according to the instruments of payment. To simplify, we restrict ourselves to the particular situation where the retailer is allowed to charge a price p^c for a credit card transaction which may be different from the price p^r charged for a cash or store credit transaction. We denote by Δ^c the spread between these two prices, namely:

$$\Delta^c := p^c - p^r \quad (5)$$

Figure 1 – Prices, costs and fees of instruments of payment

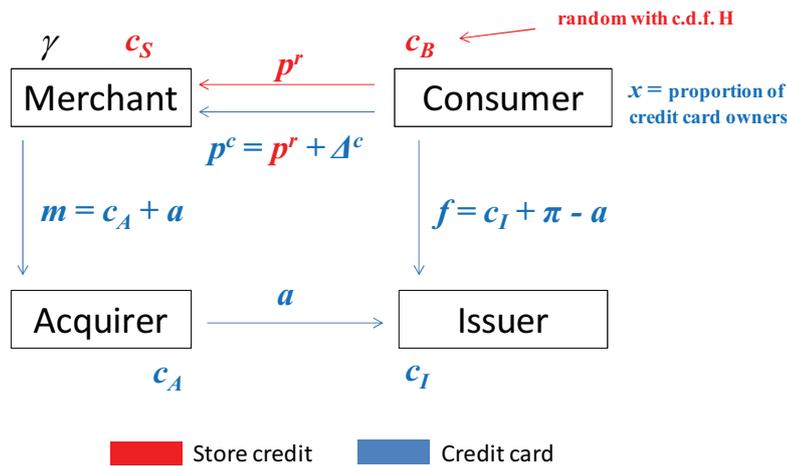


Figure 1 illustrates the interconnection between participants of the credit card market, as well as the respective prices, costs and fees charged by each one.

The timing of the decisions is as in the model of Rochet and Wright (2010), which can be divided into 9 steps, grouped in two periods: 5 steps before the arrival of the consumer to the store and 4 steps once the consumer is in the store.

Before arriving at the store:

1. The card network sets the interchange fee a ;
2. Banks set their fees: f for cardholders and m for retailers;
3. Retailers independently choose their card acceptance policies: $L_i^r = 1$ if retailer i accepts credit cards, 0 otherwise;
4. Retailers independently set retail prices p_i^r and $p_i^c = p_i^r + \Delta_i^c$;
5. Consumers select one retailer to patronize, after observing the observed retail prices, retail's acceptance policies, issuer's fee, the distribution of store credit cost and transport cost.

Once the consumer is in the store:

6. Consumer buys a first unit of the retail good (“ordinary purchase”), and pays for it using cash or credit card (if he has one);
7. Nature decides whether the consumer has an opportunity for an additional credit purchase, which will occur with probability θ ;
8. The cost c_B of using store credit for the buyer is drawn according to the c.d.f. H , with full support on $(\underline{c}_B, \overline{c}_B)$;
9. Cardholders then select their mode of payment. We set $L_i^c = 1$ if the consumer prefers credit cards over cash when buying at the retailer i , or 0 otherwise. In other words:

$$L_i^c := \begin{cases} 1 & \text{if } f + \Delta_i^c \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

3. Analysis and results

Assuming the merchant charges price p_i^r for cash and store credit, and charges an additional spread Δ_i^c specifically on credit card transactions, we obtain (see Appendix) that the expected margin of the retailer i is given by

$$M_i = (1 + \theta) \cdot (p_i^r - \gamma) - (H(0) + \theta) \cdot c_S - x \cdot L_i^r \cdot \bar{\Gamma}(a, \Delta_i^c) \quad (7)$$

where

$$\bar{\Gamma}(a, \Delta_i^c) := [1 - H(0)] L_i^c \cdot c_S + [1 - H(f + \Delta_i^c)] (L_i^c + \theta) \cdot (m - c_S - \Delta_i^c) \quad (8)$$

The first two terms at the right hand side of (7) correspond to the expected revenue of the retailer i , the net cost of the products and the net cost of the instruments of payment, if there are not credit card users ($x = 0$) or the retailer decides not to accept credit cards ($L_i^r = 0$). The third term corresponds to the expected margin reduction associated to the use of credit cards.

We obtain (see Appendix) that the utility of the consumer that chooses to purchase the good from the retailer i is given by

$$U_i = u_0 + \theta \cdot u_1 - (1 + \theta) \cdot p_i^r - \int_{c_B}^{\bar{c}_B} c_B \cdot dH(c_B) - \theta \cdot E(c_B) + x \cdot L_i^r \cdot \bar{S}(a, \Delta_i^c) \quad (9)$$

where

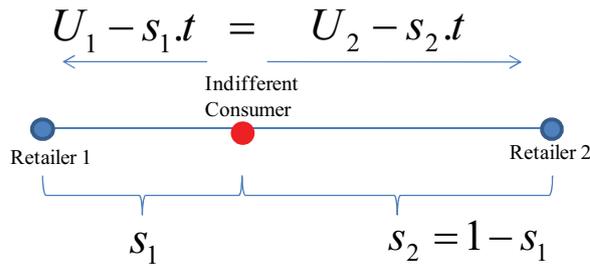
$$\bar{S}(a, \Delta_i^c) := (L_i^c + \theta) \cdot \left(\int_{f + \Delta_i^c}^{\bar{c}_B} (c_B - f - \Delta_i^c) \cdot dH(c_B) \right) - L_i^c \cdot \int_{\bar{c}_B}^{\bar{c}_B} c_B \cdot dH(c_B) \quad (10)$$

The first five terms at the right side of (9) correspond to the net expected utility of the consumption (taking into account the product cost and the credit cost of the store), when there are no credit card users or the retailer decides not to accept credit cards. The

sixth term corresponds to the additional welfare associated exclusively to the use of credit cards.

The market share of both retailers is determined by computing the position of the indifferent consumer in the region (interval of size one) where all consumers are uniformly distributed. Since there is a cost t for every unit of displacement, the utility minus the displacement cost of a consumer who decides to purchase the good from retailer i is $U_i - s_i \cdot t$. Therefore, the distance s_i between the indifferent consumer and the retailer i is equal to the proportion of consumers choosing retailer i . Figure 2 shows net utilities and the market shares of both retailers.

Figure 2 – Indifferent consumer in the Hotelling model



Then the market share s_i of the retailer i depends on the interchange fee, prices and spreads, and is given by the following expression

$$s_i = \frac{1}{2} + (1 + \theta) \cdot \left(\frac{p_j^r - p_i^r}{2t} \right) + x \cdot \left(\frac{L_i \cdot \bar{S}(a, \Delta_i^c) - L_j \cdot \bar{S}(a, \Delta_j^c)}{2t} \right) \quad (11)$$

Note that, for a fixed interchange fee a , the (Nash) equilibrium price \bar{p} when differentiation is not allowed, as defined by equation (5) in Rochet and Wright (2010), satisfies the following equation

$$(1+\theta).\bar{p} = t + (1+\theta).\gamma + (H(0)+\theta).c_S + \frac{x}{3}.(L_j^r - L_i^r).\bar{\phi}(a,0) + x.L_j^r.\bar{\Gamma}(a,0) \quad (12)$$

where

$$\bar{\phi}(a, \Delta_i^c) := \bar{S}(a, \Delta_i^c) - \bar{\Gamma}(a, \Delta_i^c) \quad (13)$$

is the difference between the additional welfare of the consumer and the additional cost of the retailer associated to each credit card transaction.

For each $\delta > 0$, as defined in (4), consider the following parameter definition

$$\bar{\phi}_\delta := (1+\theta).\left(\int_{\delta}^{\bar{c}_B} (c_B + \delta).dH(c_B)\right) - \int_0^{\bar{c}_B} (c_B + c_S).dH(c_B) \quad (14)$$

and note that $\bar{\phi}_\delta = \bar{\phi}(a, a + c_A - c_S)$.

Assuming $\bar{\phi}_\delta > 0$ means that, if the spread is equal to $m - c_S (= a + c_A - c_S)$, the benefit of a credit card transaction for consumers is greater than the cost of the same transaction for retailers. Note that if $\delta > 0$ is sufficiently small, the assumption $\bar{\phi}_\delta > 0$ is equivalent to

$$\theta.\left(\int_0^{\bar{c}_B} (c_B + c_S).dH(c_B)\right) > (1+\theta).\int_0^{\bar{c}_B} (c + \pi).dH(c_B) \quad (15)$$

where the left-hand side term above corresponds to the costs savings in using credit cards for extraordinary purchases and the right-hand side term corresponds to the cost of using credit cards in both types of purchases.

The results in this paper are obtained under the three basic assumptions below. The first one relaxes the non-surcharge rule allowing retailers to charge different prices for credit card transactions, and represents the main assumption.

Assumption 1: Price differentiation of credit cards transactions is allowed.

The second assumption is related to the cost efficiency of the credit card industry compared to the store credit instrument.

Assumption 2: The parameter δ , defined by (4), is strictly positive.

Assumption 2 above means that from the point of view of the lenders, the sum of the total bank costs and profits of credit card transactions $c_A + c_I + \pi$ is lower than the retailer's cost of providing store credit c_S . In this specific sense, the credit card industry is more cost efficient than retailers in providing credit.

As demonstrated in Rochet and Wright (2010), under the no-surcharge rule, if the consumer's benefit of a credit card transaction is equal to the cost savings of generating credit through a credit card transaction instead of a store credit ($f = \delta$), consumers obtain the maximum aggregated welfare. They proved that any other level of credit cards' costs/benefits f (which depends on the interchange fee, since $f = c_I + \pi - a$) will generate a loss in the consumers' welfare. In other words, despite consumers are individually deciding their instruments of payments in an optimal way, from the aggregated point of view, those decisions generate an inefficient level of credit card usage, if compared to the optimal situation when $f = \delta$.

The third assumption has a more sophisticated interpretation, and it essentially imposes restrictions on the retailers' average cost and the consumers' average benefits of a credit card transaction.

Assumption 3: The parameter $\bar{\phi}_\delta$, defined in (14), is strictly positive.

Assumption 3 above is equivalent to imposing that, when the spread charged by both merchants is equal to $m - c_S$, the consumers' average benefit from credit card transactions is greater than the retailers' average cost from the same credit card transactions. Note that, if retailers recover those costs through the average price paid by consumers, Assumption 3 implies that consumers have positive average benefits in using credit cards.

3.1. Equilibrium prices under price differentiation

In this subsection we provide some propositions that allow us to analyze the impacts of ruling out the no-surcharge rules in the credit card systems. All the propositions are obtained under Assumptions 1, 2 and 3.

Proposition 1: If both retailers charge the single price \bar{p} , as defined in equation (12), and the merchant fee is greater than the cost of the store credit ($m > c_S$), retailers have incentives to impose a surcharge over the single price.

Proof: See Appendix 1.

An immediate consequence of Proposition 1 above is that the single price \bar{p} charged for every instrument of payment is not the (Nash) equilibrium price under price differentiation.

The proof of Proposition 1 employs the fact that the profit function is strictly increasing in the price spread, meaning that it is desirable for the merchant to surcharge credit cards transactions above \bar{p} . However, it is worth noting that this result is only a static comparative assessment, whose utility is exclusively to prove that the single price strategy is no longer equilibrium under the price differentiation assumption.

Any policy assessment needs to address more relevant questions like: is there a competitive equilibrium with that price differentiation characteristic? And if the response is positive, how does the consumer welfare in that equilibrium fare when compared to that of a single price? The results in the following proposition help us to clarify these questions.

Proposition 2: For each interchange fee a defined by the banks, there exist a pair of prices (\bar{p}^r, \bar{p}^c) , the price charged for cash/store credit transactions and the one charged exclusively for credit card transactions, and a neighborhood of it, where this pair is a Nash equilibrium in that neighborhood. Specifically, if both retailers are charging those prices, none of them, has incentives to deviate from those prices in that neighborhood. The prices are given by

$$\bar{p}^r = \gamma + \frac{t + [H(0) + x \cdot (1 - H(0)) + \theta] c_S}{(1 + \theta)} \quad (16)$$

and

$$\bar{p}^c = \bar{p}^r + m - c_S \quad (17)$$

Proof: See Appendix.

Note that the price \bar{p}^r does not depend on interchange fee a . Actually, only the price of credit cards transactions \bar{p}^c depends on it.

From equations (17) and (1), we obtain that the equilibrium spread is given by

$$\bar{\Delta}^c = c_A - a - c_S \quad (18)$$

and, consequently, using (18) and (2), we conclude that $f + \bar{\Delta}^c = \delta$. In other words, the interchange fee loses its capability of affecting the consumers' net benefit of a credit card transaction ($f + \bar{\Delta}^c$), under differentiation, which becomes constant and equal to the optimal benefit value (δ) under the no-surcharge rule. This is a remarkable difference with respect to the Rochet and Wright (2010) findings.

Note that the average equilibrium price is

$$\bar{p}^m = (1 - \alpha_\Delta) \cdot \bar{p}^r + \alpha_\Delta \cdot \bar{p}^c \quad (19)$$

where $\alpha_\Delta := x \cdot [1 - H(f + \bar{\Delta}^c)]$ is the proportion of card owners that, under price differentiation, prefer to use credit cards rather than store credit or cash.

Proposition 3: The price \bar{p} is a convex combination of the prices \bar{p}^r and \bar{p}^c . More specifically,

$$\bar{p} = (1 - \alpha_0) \cdot \bar{p}^r + \alpha_0 \cdot \bar{p}^c \quad (20)$$

where $\alpha_0 := x \cdot [1 - H(f)]$ corresponds to the proportion of credit card owners that prefer credit cards to any other instruments.

Proof: See appendix.

An important and immediate consequence of Proposition 3 is that, when $\bar{\Delta}^c > 0$, we have $\alpha_\Delta = x \cdot [1 - H(f + \bar{\Delta}^c)] < x \cdot [1 - H(f)] = \alpha_0$, and using (19) and (20) we obtain that the average price \bar{p}^m under price differentiation is lower than the single price \bar{p} under the no-surcharge rule.

Figure 3 below illustrates how the price \bar{p} can be decomposed in order to make the subsidy components that are eliminated in the new scenario with differentiation more explicit.

Figure 3 – Equilibrium prices decomposition

1) Cash:

$$\bar{p} = \underbrace{\gamma}_{\text{Product cost}} + \underbrace{\frac{t}{1+\theta}}_{\text{Transportation markup}} + \underbrace{\left(1 - \frac{(1-x) \cdot [1-H(0)]}{1+\theta}\right) \cdot c_S}_{\text{Subsidy paid}} + \underbrace{x \cdot [1-H(f)] \cdot (m-c_S)}_{\text{Subsidy paid}}$$

2) Store credit:

$$\bar{p} = \underbrace{\gamma}_{\text{Product cost}} + \underbrace{\frac{t}{1+\theta}}_{\text{Transportation markup}} + \underbrace{c_S}_{\text{Cost of the store credit}} - \underbrace{\frac{(1-x) \cdot [1-H(0)]}{1+\theta} \cdot c_S}_{\text{Subsidy received}} + \underbrace{x \cdot [1-H(f)] \cdot (m-c_S)}_{\text{Subsidy paid}}$$

3) Credit card:

$$\bar{p} = \underbrace{\gamma}_{\text{Product cost}} + \underbrace{\frac{t}{1+\theta}}_{\text{Transportation markup}} + \underbrace{m}_{\text{Merchant fee of the credit card}} - \underbrace{\frac{(1-x) \cdot [1-H(0)]}{1+\theta} \cdot c_S}_{\text{Subsidy received}} - \underbrace{x \cdot [1-H(f)] \cdot (m-c_S)}_{\text{Subsidy received}}$$

Eliminated with differentiation of credit cards prices

Note that part of the total subsidy is eliminated with the price differentiation of credit card transactions, but one component of the subsidy remains. This component corresponds to the one associated with the group of consumers that does not have credit cards (fixed proportion $1-x$) and, at the same time does not see any benefit in using store credit. Thus, they use cash. This particular group of consumers pays a subsidy to the other consumers. The subsidy occurs because they have fewer options of payment instruments and, as a consequence, less market power than the others.

Notice that the remaining subsidy from cash users to the others is eliminated if we suppose that every consumer has a credit card ($x=1$). However, since we are not allowing for price differentiation between cash and store credit transactions, a subsidy

between them persists. This is because the price \bar{p}^r , as an average price, does not reflect the different costs of each instrument (cash has zero cost and store credit has cost c_S).

The following proposition shows that there is a positive impact on consumer welfare as a consequence of the elimination of barriers to price differentiation of credit card transactions.

Proposition 4: The consumer welfare in the equilibrium under price differentiation is greater than the corresponding under the no-surcharge rule. They are equal only if the interchange rate a is equal to $c_S - c_A$.

Proof: See appendix.

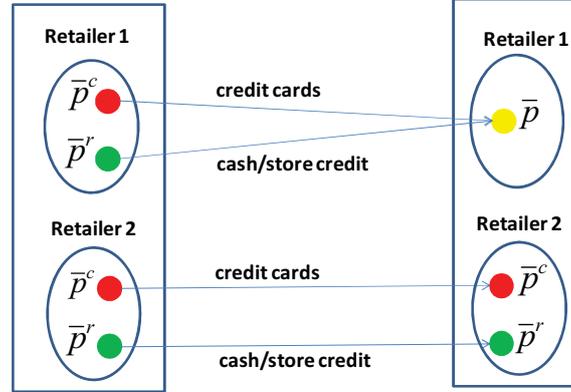
With respect to the merchants' profits the following proposition shows that retailers are indifferent with respect to the no-surcharge rule. In a model which consider both convenience users and interest-paying users of credit cards, Chakravorti and Emmons (2003) showed that retailers prefers to charge different prices.

Proposition 5: The equilibrium merchant profit is the same under price differentiation and under the no-surcharge rule.

Proof: See appendix.

To analyze the effect of the interchange fee a , fixed by banks, on the incentives of retailers to deviate from a single price, it will be useful to compute the profit of a retailer resulting from this deviation whilst the other retailer continues to charge the differentiated prices \bar{p}^r and \bar{p}^c . Figure 4 illustrates the situation where Retailer 1 moves unilaterally to the single price strategy, whilst Retailer 2 continues to charge two prices.

Figure 4 – Retailer unilateral movement to single price



The following proposition shows that the merchant has no incentive to individually decide to charge the single price \bar{p} . In the next subsection (Proposition 7), the same type of analysis is revisited, but in a context of existence of menu costs when considering the possibility of differentiated prices.

Proposition 6: Suppose that both retailers are initially charging differentiated prices \bar{p}^r and \bar{p}^c , with a positive spread $\bar{\Delta}^c$. If one of them decides to charge the single price \bar{p} , the profit of this retailer will decrease, whereas the profit of the competitor will augment in the same amount. The size of the reduction/increase $\varepsilon(a)$ is given by:

$$\varepsilon(a) = \frac{x \cdot (1 + \theta)}{2} \int_{-\delta + c_S - c_A - a}^{-\delta} (-\delta - c_B) \cdot dH(c_B) \quad (21)$$

which depends on the value of the interchange fee a .

Proof: See appendix.

A consequence of Propositions 5 and 6 is that, despite both retailers having the same profit under price differentiation and under a single price, no one has, without cooperation, incentives to move individually to the single price.

Figure 5 illustrates the variations on profits under price differentiation assumption when retailers change from the differential prices equilibrium to the single price equilibrium. The gray cell indicates the profits of Retailer 1 and Retailer 2, which are equal to $t/2$.

Figure 5 – Profits under price differentiation assumption

Retailers' profits under price differentiation		Retailer 2	
		two prices p^r and p^c	single price p
Retailer 1	two prices p^r and p^c	$t/2 ; t/2$	$t/2 + \varepsilon(a) ; t/2 - \varepsilon(a)$
	single price p	$t/2 - \varepsilon(a) ; t/2 + \varepsilon(a)$	$t/2 ; t/2$

In Figure 5, the cell below the gray one indicates that if Retailer 1 individually decides to charge the single price, his profit reduces from $t/2$ to $t/2 - \varepsilon(a)$. The same occurs with Retailer 2, as indicated in the cell at the opposite side of the gray one. The cell at the opposed side of the diagonal indicates the fact, if both decides to charge the single price \bar{p} , both profits are the same and equal to $t/2$. We can observe here that the single price decision is not actually equilibrium.

3.2. Menu costs

In this last subsection we analyze the effects of the introduction of the menu costs in the model of price differentiation that we are considering. In this framework, we will call a menu cost to any cost faced by the retailer as a consequence of charging different prices according to the instrument of payment used by the consumer. As usual, a reason for this type of friction is the cost of hiring consultants to develop pricing strategies for possible differentiation. The costs of implementing and updating the systems with the information of differentiated price are also a menu costs.

Another reason for including menu costs can be related to the legal insecurity regarding eventual penalties that the retailer could suffer as a result of applying price differentiation. In jurisdictions where there is not a clear rule with respect to the price differentiation or even where there are institutional conflicts regarding to an existing rule, the sellers attribute high costs to price differentiation.⁷

Suppose that the margin of the Retailer i is given by

$$M_i^\mu := (1 + \theta)(p_i^r - \gamma) - (H(0) + \theta)c_s - x.L_i.\bar{\Gamma}(\Delta_i^c) - \mu_i.I(\Delta_i^c) \quad (22)$$

where $I(\Delta_i^c) := \begin{cases} 0 & ; \text{if } \Delta_i^c = 0 \\ 1 & ; \text{if } \Delta_i^c \neq 0 \end{cases}$

Proposition 7: Suppose that retailers face menu costs per transaction μ_1 and μ_2 , so both margins are given by (22). Then, the differential prices \bar{p}^r and \bar{p}^c remain being local equilibrium; namely, none of the retailers is willing to deviate independently of those prices in a neighborhood of them.

Proof: See appendix.

Proposition 7 asserts that, even in the presence of menu costs, the price differentiation strategy remains as a local Nash equilibrium. Notwithstanding of this local result, it is important to analyze if it remains true in more global terms. As we showed in Proposition 5, the profits of the retailers are the same under differential pricing and under single pricing. If we introduce the menu costs, we would expect that the retailers prefer to set a single price. However, depending on the size of the extra profit $\varepsilon(a)$ defined in (21)

⁷ The Syndicate of Retailers of Belo Horizonte, the capital of the State of Minas Gerais (Brazil), appealed to the Court of Justice against fines applied by the Institute of Consumer Protection of Minas Gerais (Procon/MG) to retailers who differentiate prices of credit cards transactions. In July 9th, 2012 the 6th Chamber of Court of Justice announced the sentencing in favor of the Syndicate. In fact, there is not explicit legal basis for the applied fines; however the Procon/MG argues that the prohibition is supported in a regulatory act issued by the Ministry of Finance (No. 118 of March 11th, 1998). Indeed, that regulatory act refers to the implementation of the transition rules for a new currency unit of that era. Since it is not a final sentence, the Procon/MG is appealing to the Court of Justice of the State of Minas Gerais.

compared to the size of menu cost of each retailer, it might be more profitable to adopt differential prices. Figure 6 shows the profits of both retailers depending on their individual decisions of pricing strategies, when we assume the existence of menu costs.

Figure 6 – Profits under price differentiation and menu costs assumptions

Retailers' profits under price differentiation and menu costs		Retailer 2	
		two prices p^r and p^c	single price p
Retailer 1	two prices p^r and p^c	$t/2 - \mu_1$; $t/2 - \mu_2$	$t/2 - \mu_1 + \varepsilon(a)$; $t/2 - \varepsilon(a)$
	single price p	$t/2 - \varepsilon(a)$; $t/2 - \mu_2 + \varepsilon(a)$	$t/2$; $t/2$

Thus, we can conclude the following proposition.

Proposition 8: Suppose that retailers face menu costs per transaction μ_1 and μ_2 . Then, regarding the game given in Figure 6, we have:

- (i) If both menu costs are lower than the extra profit $\varepsilon(a)$ defined in (21), the Nash equilibrium is both retailers charge differential prices \bar{p}^r and \bar{p}^c .
- (ii) If both menu costs are greater than the extra profit $\varepsilon(a)$ defined in (21), the Nash equilibrium is both retailers charge the single price \bar{p} .
- (iii) If $\mu_i < \varepsilon(a) < \mu_j$, the Nash equilibrium is the retailer i charge differential prices \bar{p}^r and \bar{p}^c and the retailer j set a single price \bar{p} .

Proof: It is a straightforward result from Figure 6.

Proposition 8 dissects the effect of the menu costs on the retailers' decision about surcharging or not. Essentially, the greater the menu costs, the higher the incentive to not surcharge. Therefore, if the cost of applying surcharge is high (whatever the reason) compared with the extra profit, the seller will prefer to set a single price. In this vein, Rochet (2003) argued that surcharging is seldom used even when the system does not prohibit it, probably because of transaction costs. This reveals an interesting topic of

applied research, to explain why in some economies where there is no a non-surcharge rule, the retailers do not charge differential prices. Reciprocally, if the extra profit resulting from surcharging is greater than the menu costs, both retailers will charge differentiated prices, resulting the Nash equilibrium of the “prisoner’s dilemma” game.

Let us make one final comment linking the findings of Rochet and Wright (2010) with ours and showing how the abolishment of the non-surcharge rule will reduce the market power of the banks. Let $\tilde{a} := \min\{\varepsilon^{-1}(\mu_1); \varepsilon^{-1}(\mu_2)\}$. If the banks decide to set a high interchange fee (say $a > \tilde{a}$), the retailers would be willing to set the differential prices \bar{p}^r and \bar{p}^c . In particular, suppose that the threshold value \tilde{a} is lower than \bar{a} , which is the interchange fee that maximizes the banks profit under the no-surcharge rule, as obtained in Rochet and Wright (2010). In this situation, the possibility and desire of retailers to differentiate prices can destabilize the former equilibrium price \bar{p} , enforcing banks to set the interchange fee \tilde{a} . In this way, the market power and the profits of the credit card issuers are reduced; consequently, the consumers surplus is augmented.

4. Conclusions

In this paper, we adapt the framework of Rochet and Wright (2010), to the absence of no-surcharge rules for prices of credit cards transactions. In this setting we prove that the equilibrium prices for the purchases using credit cards and using cash or store credit are not the same. In particular, we obtain that the equilibrium surcharge spread is the difference between the merchant fee and the cost he has to provide the store credit.

The result regarding the equilibrium price spread is remarkable, especially in jurisdictions where the debate agenda is the necessity of defining a merchant’s surcharge cap (as in Australia). Our results assert that the surcharge cap should not exceed the competitive equilibrium price spread that we found, namely, that the surcharge cap must be lower or equal to the difference between the merchant fee and the store credit cost faced by merchants. In particular, this result implies that only if the cost of the store

credit is equal to or greater than the merchant fee, the no-surcharge rule should be acceptable from the point of view of those who seek to preserve the welfare of the consumers. This is a contrasting result with the model analyzed in Rochet (2003).

Initially, we prove that the single price is not equilibrium. It is a consequence of each retailer being willing to unilaterally surcharge the credit card payments and deviate from the single price equilibrium stated by the no-surcharge rules.

The result given above leads us to the following question: if price differentiation is allowed, might it provide some degree of market power to the merchants so that they would be able to keep a surcharge on credit card transactions with an average price of all transactions greater than the single price found under the no-surcharge rule? The answer is definitely no. In order to show that, we computed the new equilibrium prices when price differentiation of credit card payments is allowed, and proved that the average price is lower than the single price under the no-surcharge rule. Moreover, the new aggregated consumer welfare in the price differentiation equilibrium is, in general, greater than the corresponding one in the single price equilibrium. Equality would only take place if the interchange fee was at the level that maximizes the consumers' surplus under the no-surcharge rule framework.

We also obtain that the merchants' profits under price differentiation are equal to those under the single price equilibrium. This result raised the following question: would the retailer have incentives to unilaterally deviate towards the single price? We found that he would not. In fact, we proved that none of the retailers have incentives to unilaterally deviate from the equilibrium with price differentiation to the single price.

In the last exercise we introduce menu costs due to the price differentiation to analyze whether this sort of friction may inhibit retailers' incentives to differentiate prices. In this new context, it is simple to conclude that the strategy of a single price may turn out to be equilibrium, which is independent of the existence of a no-surcharge rule. In fact, if both retailers charge differential prices, their profits will fall below the levels

attained under a single price. Furthermore, we obtain that, for high interchange fee values, the strategies of price differentiation remain as equilibrium of the “prisoner’s dilemma” game type. Actually, we prove that if both retailers are charging differential prices none of them has incentives to unilaterally change to the single price strategy. The intuition behind this is the following: if only one merchant charges the single price, he loses market share and, consequently, reduces his profit. If this reduction of profit is greater than the menu cost savings, there will be a net cost associated to the decision of moving toward the single price strategy.

In summary, by using a simple credit card market model we were able to illustrate how the absence of a no-surcharge rule could generate equilibrium prices capable of improving consumer welfare. This may reduce the market power that banks have using the interchange fee, even in the presence of menu costs associated to price differentiation that the retailers may face.

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APPENDIX

To prove that, under price differentiation, the single price \bar{p} is not equilibrium and, to find the new equilibrium prices, we first derive the merchant margin, the consumer utility and merchant market share, and use them to find the profit function. Subsequently, we compute the derivatives of the profit function with respect to the base price \bar{p}_i^r and the spread $\bar{\Delta}_i^r$.

Figure A1 shows a decomposition of the merchant's expected margin into several components. The first group of components corresponds to the margin obtained by merchants from consumers that cannot use credit cards, either by not having them, or because the retailer did not adhere to the credit card system. The second group of components corresponds to the retailer margins the consumers who have a credit card conditioned to his adherence to the credit card system.

Figure A1 – Merchant's expected margin

$$\begin{aligned}
 M_i = & \underbrace{(1 - x.L_i^r)}_{\text{Cannot use credit cards}} \cdot \left[\underbrace{H(0).(p_i^r - \gamma - c_s)}_{\text{store credit}} + \underbrace{(1 - H(0).(p_i^r - \gamma))}_{\text{cash}} \right] \\
 & + \underbrace{\theta.(p_i^r - \gamma - c_s)}_{\text{ordinary purchase}} \\
 & + x.L_i^r \cdot \left[\underbrace{H(L_i^c.(f + \Delta_i^c))(p_i^r - \gamma - c_s)}_{\text{store credit}} + \underbrace{(1 - H(L_i^c.(f + \Delta_i^c)))(p_i^r - \gamma + L_i^c.(\Delta_i^c - m))}_{\text{credit card and cash}} \right] \\
 & + \theta \cdot \left[\underbrace{H(f + \Delta_i^c).(p_i^r - \gamma - c_s)}_{\text{store credit}} + \underbrace{(1 - H(f + \Delta_i^c))(p_i^r - \gamma + \Delta_i^c - m)}_{\text{credit card}} \right] \\
 & \underbrace{\hspace{10em}}_{\text{credit purchase}}
 \end{aligned}$$

The formula in Figure A1 can be simplified to derive equation (7). Then the partial derivatives with respect to the base price and the spread are given by

$$\frac{\partial M_i}{\partial p_i^r} = (1 + \theta) \quad (\text{A1})$$

and

$$\frac{\partial M_i}{\partial \Delta_i^c} = x.L_i^r.(L_i^c + \theta).\{h(f + \Delta_i^c)(m - c_s - \Delta_i^c) + [1 - H(f + \Delta_i^c)]\} \quad (\text{A2})$$

Consumers decide which retailer to patronize computing its utility and subtracting the transportation costs of each choice. To calculate the market share we need to identify the indifferent consumer in the interval.

Analogously, Figure A2 below shows a decomposition of the consumers expected utility. The first group of components corresponds to the utility obtained by consumers that cannot use credit cards, either by not having them, or because the retailer did not adhere to the credit card system. The second group of components corresponds to the utility of consumers that have a credit card and the chosen retailer adhered to the system.

Figure A2 – Consumer's utility

$$\begin{aligned}
 U_i = & \underbrace{(1-x.L_i^r)}_{\text{cannot use credit card}} \left(\underbrace{u_0 + \theta u_1}_{\text{utility of consumption}} - \underbrace{(1+\theta).p_i^r}_{\text{cost of product}} - \underbrace{\int_{c_B}^0 c_B.dH(c_B)}_{\text{cost of store credit (ordinary and credit purchases)}} - \underbrace{\theta.E(c_B)}_{\text{cost of store credit (ordinary and credit purchases)}} \right) \\
 & + \underbrace{x.L_i^r}_{\text{can use credit card}} \left(\underbrace{u_0 + \theta u_1}_{\text{utility of consumption}} - \underbrace{(1+\theta).p_i^r}_{\text{cost of product}} - \underbrace{\int_{c_B}^{f+\Delta_i^c} c_B.dH(c_B)}_{\text{cost of store credit (ordinary purchases)}} - \underbrace{\int_{L_i^c(f+\Delta_i^c)}^{\bar{c}_B} L_i^c.(f + \Delta_i^c).dH(c_B)}_{\text{cost of credit card (ordinary purchase)}} \right) \\
 & - \theta \cdot \left(\underbrace{\int_{c_B}^{f+\Delta_i^c} c_B.dH(c_B)}_{\text{cost of store credit (credit purchase)}} + \underbrace{\int_{f+\Delta_i^c}^{\bar{c}_B} (f + \Delta_i^c).dH(c_B)}_{\text{cost of credit card (credit purchase)}} \right)
 \end{aligned}$$

The expression in Figure A2 can be simplified to derive equation (9), which is used to obtain the market share equation (11). The derivatives of the merchant's market share (11), with respect to the base price and the spread are given by

$$\frac{\partial s_i}{\partial p_i^r} = -\frac{(1+\theta)}{2.t} \quad (\text{A3})$$

and

$$\frac{\partial s_i}{\partial \Delta_i^c} = -(L_i^c + \theta) \cdot \frac{x.L_i^r}{2.t} \cdot [1 - H(f + \Delta_i^c)] \quad (\text{A4})$$

The merchant's expected profit is the product of the margin and the market share. As we can see below, the expressions (A5) to (A8) are useful to compute the derivatives of the profit function.

We can use the equations (7), (11), (A1) and (A3) to derive the expressions below:

$$\frac{2.t}{(1+\theta)} \cdot \frac{\partial M_i}{\partial p_i^r} \cdot s_i = t + (1+\theta) \cdot (p_j^r - p_i^r) + x \cdot (L_i^r \cdot \bar{S}(\Delta_i^c) - L_j^r \cdot \bar{S}(\Delta_j^c)) \quad (\text{A5})$$

and

$$\frac{2.t}{(1+\theta)} M_i \cdot \frac{\partial s_i}{\partial p_i^r} = -(1+\theta) \cdot (p_i^r - \gamma) + (H(0) + \theta) c_s + x \cdot L_i^r \cdot \bar{\Gamma}(\Delta_i^c) \quad (\text{A6})$$

We can use the equations (7), (11), (A2) and (A4) to derive the expressions below:

$$\begin{aligned} \frac{2.t}{(L_i^c + \theta)} \frac{\partial s_i}{\partial \Delta_i^c} \cdot M_i = & -x \cdot L_i^r \cdot [1 - H(f + \Delta_i^c)] \\ & \cdot [(1+\theta) \cdot (p_i^r - \gamma) - (H(0) + \theta) c_s - x \cdot L_i^r \cdot \bar{\Gamma}(\Delta_i^c)] \end{aligned} \quad (\text{A7})$$

and

$$\frac{2t}{(L_i^c + \theta)} \cdot \frac{\partial M_i}{\partial \Delta_i^c} \cdot s_i = x \cdot L_i^r \cdot \left\{ h(f + \Delta_i^c) \cdot (m - c_s - \Delta_i^c) + [1 - H(f + \Delta_i^c)] \right\} \quad (\text{A8})$$

$$\left\{ t + (1 + \theta) \cdot (p_j^r - p_i^r) + x \cdot (L_i^r \cdot \bar{S}(\Delta_i^c) - L_j^r \cdot \bar{S}(\Delta_j^c)) \right\}$$

Summing up (A5) and (A6), we obtain the derivative of the profit function with respect to the base price

$$\frac{2t}{(1 + \theta)} \cdot \frac{\partial \pi_i}{\partial p_i^r} = \left\{ \begin{aligned} & t + (1 + \theta) \cdot (p_j^r - p_i^r) + x \cdot (L_i^r \cdot \bar{S}(a, \Delta_i^c) - L_j^r \cdot \bar{S}(a, \Delta_j^c)) - (1 + \theta) \cdot (p_i^r - \gamma) \\ & + (H(0) + \theta) \cdot c_s + x \cdot L_i^r \cdot \bar{\Gamma}(a, \Delta_i^c) \end{aligned} \right. \quad (\text{A9})$$

Analogously, we find the derivatives of the profit function with respect to the spread from (A7) and (A8)

$$\frac{2t}{(L_i^c + \theta)} \cdot \frac{\partial \pi_i}{\partial \Delta_i^c} = \left\{ \begin{aligned} & -x \cdot L_i^r \cdot [1 - H(f + \Delta_i^c)] \cdot [(1 + \theta) \cdot (p_i^r - \gamma) - (H(0) + \theta) \cdot c_s - x \cdot L_i^r \cdot \bar{\Gamma}(a, \Delta_i^c)] \\ & + x \cdot L_i^r \cdot \left\{ h(f + \Delta_i^c) \cdot (m - c_s - \Delta_i^c) + [1 - H(f + \Delta_i^c)] \right\} \\ & \left\{ t + (1 + \theta) \cdot (p_j^r - p_i^r) + x \cdot (L_i^r \cdot \bar{S}(a, \Delta_i^c) - L_j^r \cdot \bar{S}(a, \Delta_j^c)) \right\} \end{aligned} \right. \quad (\text{A10})$$

Proof of Proposition 1:

Substituting (12) in (A10), with $p_i^r = p_j^r = \bar{p}$, $\Delta_i^c = 0$, $L_i^c = 1$ and $L_i^r = 1$, we obtain that the derivative of the merchant's expected profit with respect to the spread satisfy the following equation

$$\frac{2t}{(1 + \theta)} \cdot \frac{\partial \pi_i}{\partial \Delta_i^c} = t \cdot x \cdot h(f) \cdot (m - c_s) \quad (\text{A11})$$

where h represents the density function of the cumulative distribution function H .

Notice that the right hand side of (A11) is positive if all the following conditions are satisfied:

- There are transportation costs ($t > 0$);
- There are card users ($x > 0$);
- The density of consumers that are indifferent to the cost of a store credit or a credit card ($c_b = f$) is positive ($h(f) > 0$);
- The merchant fee is greater than the cost of the store credit ($m > c_s$).

We conclude that, if price differentiation is allowed, merchants have incentives to surcharge and, consequently, the single price \bar{p} is not equilibrium.

Proof of Proposition 2:

The first order conditions with respect to the base price and the spread, using equations (A9) and (A10), are given, respectively, by

$$\begin{aligned} & t + (1 + \theta) \cdot (p_j^r - p_i^r) + x \cdot (L_i^r \cdot \bar{S}(a, \Delta_i^c) - L_j^r \cdot \bar{S}(a, \Delta_j^c)) \\ & = (1 + \theta) \cdot (p_i^r - \gamma) - (H(0) + \theta) \cdot c_s - x \cdot L_i^r \cdot \bar{\Gamma}(a, \Delta_i^c) \end{aligned} \quad (\text{A12})$$

and

$$\begin{aligned} & x \cdot L_i^r \cdot [1 - H(f + \Delta_i^c)] \cdot \left\{ (1 + \theta) \cdot (p_i^r - \gamma) - (H(0) + \theta) \cdot c_s - x \cdot L_i^r \cdot \bar{\Gamma}(a, \Delta_i^c) \right\} \\ & = x \cdot L_i^r \cdot \left\{ h(f + \Delta_i^c) \cdot (m - c_s - \Delta_i^c) + [1 - H(f + \Delta_i^c)] \right\} \\ & \quad \cdot \left\{ t + (1 + \theta) \cdot (p_j^r - p_i^r) + x \cdot (L_i^r \cdot \bar{S}(\Delta_i^c) - L_j^r \cdot \bar{S}(a, \Delta_j^c)) \right\} \end{aligned} \quad (\text{A13})$$

Substituting (A12) in (A13) we obtain $h(f + \Delta_i^c) \cdot (\Delta_i^c - m + c_s) = 0$ and the spread in equilibrium is $\bar{\Delta}_i^c = m - c_s$.

Using (A12) we obtain the following expressions containing the base prices of both retailers

$$(1 + \theta).(2.p_i^r - p_j^r) = t + (1 + \theta).\gamma + (H(0) + \theta).c_s \\ + x.\bar{S}(a, \Delta_j^c).(L_i^r - L_j^r) + x.L_i^r.\bar{\Gamma}(a, \Delta_j^c)$$

and

$$(1 + \theta).(4.p_j^r - 2.p_i^r) = 2.t + 2.(1 + \theta).\gamma + 2.(H(0) + \theta).c_s \\ + 2.x.\bar{S}(a, \Delta_j^c).(L_j^r - L_i^r) + 2.x.L_j^r.\bar{\Gamma}(a, \Delta_j^c)$$

Summing up the equations above and rearranging we obtain that the equilibrium price satisfies the following equation

$$(1 + \theta).p_j^r = t + (1 + \theta).\gamma + (H(0) + \theta).c_s + \frac{x}{3}.(L_j^r - L_i^r).\bar{\phi}(a, \Delta_j^c) + x.L_j^r.\bar{\Gamma}(a, \Delta_j^c) \quad (A14)$$

From equation (A14) above, we obtain

$$(1 + \theta).(p_j^r - p_i^r) = -\frac{2.x}{3}.(L_i^r - L_j^r).\bar{\phi}(a, \Delta_j^c) - x.(L_i^r - L_j^r).\bar{\Gamma}(a, \Delta_j^c)$$

which can be included in the market share equation (11) to obtain

$$s_i = \frac{1}{2} + \frac{x.\bar{\phi}(a, \Delta_j^c).(L_i^r - L_j^r)}{6.t} \quad (A15)$$

If $\bar{\phi}(a, \bar{\Delta}^c) = \bar{\phi}_\delta > 0$, we conclude from formula (A15) that in equilibrium both retailers adhere to the credit card system, $L_i^r = 1$. This occurs because if one of them decides the contrary, he will lose market share. Consequently, the market share of both retailers are the same, $s_i = s_j = \bar{s} = \frac{1}{2}$.

Therefore, using $L_i^r = L_j^r = 1$ and $\Delta_j^c = \bar{\Delta}^c = m - c_s$, we conclude from (A14) and (8) that the equilibrium value for the base price is given by equation (16).

Proof of Proposition 3:

From (12) and (A13), since $L_i^r = L_j^r = 1$ and $\Delta_j^c = \bar{\Delta}^c = m - c_S$, we conclude that

$$\bar{p} - \bar{p}^r = x.[1 - H(f)].(m - c_S) \quad (\text{A16})$$

which can be rearranged, using equation (17), to obtain equation (20).

Proof of Proposition 4:

We can use equation (9) to show that the difference between the consumers' aggregated utilities (surplus) in both equilibria is given by

$$\Delta U_i = (1 + \theta).(\bar{p} - \bar{p}^r) - x.(1 + \theta). \left[\int_f^{f + \bar{\Delta}^c} (c_B - f).dH(c_B) + \bar{\Delta}^c.[1 - H(f + \bar{\Delta}^c)] \right]$$

Substituting equation (A16) into the equation above we obtain

$$\Delta U_i = x.[H(f + \bar{\Delta}^c) - H(f)].(1 + \theta).\bar{\Delta}^c - x.(1 + \theta). \left[\int_f^{f + \bar{\Delta}^c} (c_B - f).dH(c_B) \right]$$

Rewriting the equation above we obtain that

$$\Delta U_i = x.(1 + \theta). \left[\int_f^{f + \bar{\Delta}^c} (f + \bar{\Delta}^c - c_B).dH(c_B) \right] \rightarrow \begin{cases} = 0 & \text{se } \bar{\Delta}^c = 0 \\ > 0 & \text{se } \bar{\Delta}^c > 0 \end{cases}$$

or

$$\Delta U_i = x.(1 + \theta). \left[\int_{\delta}^{\delta + c_S - c_A - a} (c_B + \delta).dH(c_B) \right] \rightarrow \begin{cases} = 0 & \text{se } a = c_S - c_A \\ > 0 & \text{se } a < c_S - c_A \end{cases}$$

Proof of Proposition 5:

We use equation (7) to show that the difference between the retailers's margins in both equilibria is given by

$$\Delta M_i = (1 + \theta) \cdot \{x \cdot [1 - H(f)] \cdot \bar{\Delta}^c - (\bar{p} - \bar{p}^r)\}$$

which is equal to zero by equation (A16). Since the market shares are equal in both equilibria ($\bar{s} = 1/2$), we conclude that the merchants' profits are also the same. In other words, merchants are indifferent between both equilibria.

Proof of Proposition 6:

Supposing that retailer i decides to charge the single price \bar{p} , retailer j decides to charge differential prices \bar{p}^r and \bar{p}^c , and that $\bar{\Delta}_j = m - c_s > 0$, we will compute both market shares and margins to compare their profits.

Using equations (10), (11) and (A16), we obtain the market share of the retailer i

$$s_i = \frac{1}{2} + (1 + \theta) \cdot \left(\frac{\bar{p}_j^r - \bar{p}}{2t} \right) + x \cdot \left(\frac{\bar{S}(a, 0) - \bar{S}(a, \bar{\Delta}_j^c)}{2t} \right)$$

where

$$\bar{p}_j^r - \bar{p} = -x \cdot [1 - H(f)] \cdot \bar{\Delta}_j^c$$

and

$$\bar{S}(a, 0) - \bar{S}(a, \bar{\Delta}_j^c) = (1 + \theta) \cdot \left\{ \int_f^{f + \bar{\Delta}_j^c} (c_B - f) \cdot dH(c_B) + \left[1 - H(f + \bar{\Delta}_j^c) \right] \cdot \bar{\Delta}_j^c \right\}$$

Consequently, we obtain that

$$s_i = \frac{1}{2} - \frac{x \cdot (1 + \theta)}{2t} \cdot \left\{ \int_f^{f + \bar{\Delta}_j^c} (f + \bar{\Delta}_j^c - c_B) \cdot dH(c_B) \right\} < \frac{1}{2}$$

since the term inside the integral is positive. In other words, the retailer who decides to charge the single price will lose market share to the one charging differential prices.

Using equations (7), (8) and (A16) we compare both margins

$$M_i - M_j = (1 + \theta) \cdot (\bar{p} - \bar{p}_j^r) - x \cdot \left\{ \bar{\Gamma}(a, 0) - \bar{\Gamma}(a, \bar{\Delta}_j^c) \right\}$$

where

$$\bar{p} - \bar{p}_j^r = x \cdot [1 - H(f)] \cdot \bar{\Delta}_j^c$$

and

$$\bar{\Gamma}(a, 0) - \bar{\Gamma}(a, \bar{\Delta}_j^c) = (1 + \theta) \cdot \left\{ [H(f + \bar{\Delta}_j^c) - H(f)] \cdot (m - c_S) + [1 - H(f + \bar{\Delta}_j^c)] \cdot \bar{\Delta}_j^c \right\}$$

Consequently, we obtain that both margins are equal, since

$$M_i - M_j = -x \cdot (1 + \theta) \cdot \left\{ [H(f + \bar{\Delta}_j^c) - H(f)] \cdot (m - c_S - \bar{\Delta}_j^c) \right\} = 0$$

Indeed, it is easy to compute both the margins using (7). They are equal to the transportation cost t . Then comparing the profits we obtain that

$$\pi_i(\bar{\Delta}_i^c) - \pi_i(0) = \frac{t}{2} - \left(\frac{t}{2} - \varepsilon(a) \right) = \varepsilon(a)$$

where the decrease of the market share is

$$\varepsilon(a) := \frac{x \cdot (1 + \theta)}{2} \cdot \int_f^{f + \bar{\Delta}_j^c} (f + \bar{\Delta}_j^c - c_B) \cdot dH(c_B)$$

which is equivalent to equation (21).

Notice that the decrease of the market share can be rewritten as equation (21) and that

$$\varepsilon(a) \begin{cases} > 0 & \text{if } \bar{\Delta}_j^c = c_A + a - c_S > 0 \\ = 0 & \text{if } \bar{\Delta}_j^c = c_A + a - c_S = 0 \\ < 0 & \text{if } \bar{\Delta}_j^c = c_A + a - c_S < 0 \end{cases}$$

Proof of Proposition 7:

From equations (22) and (23), we obtain that if both retailers charge differential prices, the menu costs (μ_i) will decrease the margins. However, since the reduction in margin is constant with respect to small perturbations, they do not affect the first order conditions used to find equilibrium prices.