



On the Private Provision of Contentious Public Characteristics

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Abstract

We extend the standard public good provision model to allow players to either like or dislike the public characteristic. Those who dislike it are able to take actions to reduce its level. We present conditions under which the existence of a unique noncooperative equilibrium is retained, and analyze its normative and comparative static properties.

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1 Introduction

Since the contributions of Cornes and Sandler [9] and Bergstrom et al. [4], the literature on the voluntary provision of public goods in a world of many players has mushroomed, and its growth shows no sign of abating. However, as their titles indicate, those papers confine themselves to situations in which all players regard the public characteristic as a good. Subsequent literature, with

few exceptions, has retained this emphasis. The explicit assumptions in these models that the public characteristic is a good for all agents can be relaxed without changing much, so long as players are unable to take actions to reduce the level implied by the actions of others¹. However, it is not hard to find settings in which some players regard the characteristic as a good, and can take actions to augment its provision, while others regard it as a bad and can deplete it - it is for this reason that we will refer to the public quantity as a characteristic, since this language leaves open whether, for a given individual, it is a good or a bad. Such situations raise new issues, and merit more careful formal analysis than they have hitherto received.

Two possible reasons may account for the failure to extend the basic model in this direction. First, the tendency to think of contribution as synonymous with subscription - "putting dollar bills in the hat" - makes it seem natural to prohibit individuals from depleting the level. It would, after all, amount to stealing the sums earmarked by others for public provision. Second, the best response approach has difficulty handling even some aspects of the basic model, let alone its various extensions. This observation doubtless explains why the few existing theoretical treatments confine attention to models with two players². However, a number of significant public good issues have recently attracted attention in which an important feature is the presence of many heterogeneous players, and in which it seems clear that both augmentation and depletion are feasible actions for individuals. Furthermore, by exploiting the aggregative structure of models with a public characteristic using the tools suggested recently by Cornes and Hartley [11], it is possible to provide a transparent analysis of such situations. The time therefore seems ripe for a preliminary analysis of such an extension of the basic model.

Before introducing our formal model, we should say rather more to justify the empirical claim that we have just made concerning the significance of situations in which one player's public good is another's public bad, and in which each can take actions that shift the total quantity towards her own preferred level. In several publications Barrett (e.g.[3]) argues that, although abrupt climate change is likely to impose costs on all countries, there are likely to be both winners and losers from gradual climate change. Global warming imposes costs on low-lying countries such as Bangladesh, the Maldives and various Pacific islands, but may raise agricultural productivity in others. Indeed, Barrett [2] explicitly envisages a situation in which the losers from climate change use geo-engineering to cool temperatures, while the winners use geo-engineering to enhance global warming. A number of prominent candidates as alternative sources of 'clean' energy have provoked bitter controversy, with significant resources being de-

¹Indeed, Cornes and Sandler ([10], p.113) make this point. Buchanan [6] had, much earlier, noted that situations arise in which "things which are goods for some persons may simultaneously be bads for others"

²Two notable theoretical contributions, Caplan et al. [8] and Martin et al. [14], construct 2-player models in which global warming benefits one player and harms the other. Theirs are dynamic models, for which the assumption of just two players is particularly crucial for tractability.

voted to lobbying both in favour of and in opposition to their implementation. For example, Warren et al [15] note the strongly conflicting attitudes towards the location of windfarms in the UK, not only between environmentalists and windfarm operators, but also between different groups of environmentalists³.

Bostedt [5] observes that, within Sweden, many consider the Swedish wolf as a public good. Yet, there is a notable exception: reindeer herders in the northern part of the country regard it as a public bad since the wolf preys on the migrating reindeers. One can cite many other examples of animals that are, for one section of the community, a public good, while another group regards their preservation as costly, perhaps because they are predators of domesticated animals [lynxes] or destroyers of plantations [wild boars], they compete with humans or domesticated animals for food, are perceived carriers of disease [badgers spreading bovine tuberculosis in the UK], or come directly into conflict with human settlements [elephants trampling on Indian villages]. Research into genetically modified crops provides a host of further examples of conflict between those who see such research as offering an attractive solution to the pressures generated by population and real income growth on food production, and those who see it as leading to irreversible contamination of the environment.

In addition to the wide variety of environmental settings in which conflict arises over public characteristics, the literature on tax-financed public good provision also generates examples of conflicting interests. Imagine a population of individuals, all of whom place a nonnegative value on additional units of a public good. The good will be financed wholly by a direct tax - the simplest case would be an equal flat tax on each individual, its level depending on the scale, and hence total cost, of public good provision. Nonnegative valuations of the public good vary across individuals, so that each has her own ideal level of public good provision. This setting is summarized by the kind of diagram used in median voter models. However, suppose that we imagine that the players in this model can devote resources to lobbying for a higher, or lower, level of the public good. In modelling the lobbying process, we can imagine each player's induced preferences over two objects. One is the player's own income net of her tax payment and her lobbying expenditure. The other is the public good. The resulting structure is one in which, at any allocation, there may be both players who would ideally prefer less of the good, and also those who would prefer more. Another simple example may involve lobbying for or against trade protection

³“However, in the case of wind power there are strong ‘green’ arguments on both sides of the debate. Some environmentalists advocate windfarms because of their ‘clean energy’ credentials, while others oppose them because of their landscape impacts. Still others are caught awkwardly in the middle, supporting renewable energy in principle but opposing specific windfarm proposals“ ([15], p. 854)

“The landscape impacts of windfarms are exacerbated by the fact that the locations with the highest wind resource are often precisely those exposed upland areas which are valued for their scenic qualities and which are often ecologically sensitive. Opponents not only highlight the scenic impact of the turbines themselves, but also emphasize the visual impacts of the associated construction and upgrades to the electricity transmission grid. Further, they assert that the landscape impacts of windfarms will damage tourism.”

([15], p. 857)

by domestic groups who may either compete with a traded good, or use it as an input. In both cases, we can think in terms of each player's induced preferences over two objects: the resources remaining available for private consumption by that player after lobbying for her preferred position with respect to the public variable, and the level of that variable itself.

The simplest model is one which partitions all players exogenously into two sets. Members of one set regard the public characteristic as a good, members of the other regard it always as a bad. But we will argue below that this can be seen as a special case of a more general formulation in which members of the two groups are determined endogenously. In the global warming context, considerations of agricultural productivity may lead a country to benefit from global warming, but only up to a point. Our lobbying examples also are clear examples in which each player may have a finite most preferred level of the public characteristic, so that the prevailing may be either too high or too low for that player. Since our formulation can readily handle the endogenous determination of those who benefit from, and those who are hurt by, an increase in a public characteristic, it seems worthwhile to work with this somewhat more general model.

2 A General Model

2.1 Individual Preferences

The set of players, of whom there are n , is denoted by I . Player i has preferences over nonnegative quantities of two commodities - a private good and a public characteristic. Their quantities are denoted respectively by x_i and G . Her utility function, $u_i(x_i, G)$, has the following properties:

- $u_i(x_i, G)$ is quasiconcave, reflecting convex preference over both arguments.
- $u_i(x_i, G)$ is everywhere twice differentiable.
- $u_{ix} \equiv \frac{\partial u_i(x_i, G)}{\partial x_i} > 0$ for all $x_i, G > 0$.
- For any given level of x_i , there is a unique value of G - denote it by the function $\Gamma_i(x_i)$ - such that

$$\text{For all } G < \Gamma_i(x_i), u_{iG} \equiv \frac{\partial u_i(x_i, G)}{\partial G} > 0$$

$$\text{If } G = \Gamma_i(x_i), u_{iG} = 0$$

$$\text{For all } G > \Gamma_i(x_i), u_{iG} < 0.$$

The existing standard model of a pure public good can be thought of a special case in which, for all i , $\Gamma_i(x_i)$ takes the value infinity - or, at least, is sufficiently large as to represent infeasible allocations.

The level of G that is consumed by all players is defined as

$$G = G^0 + \sum_{j=1}^n a_j - \sum_{j=1}^n b_j,$$

where G^0 is the value that the public good would take in the absence of activity by the players — think of G as an indicator of air or water quality, in which case G^0 is the level that would prevail in the absence of anthropogenic climate modifications. There is no free disposal, so the current level of G is the level consumed by all, whether or not they regard G as a good at the margin. We will suppose that G can only take nonnegative values. A player whose marginal valuation of G is negative can, subject to her resource constraint, take steps to reduce its total quantity, but cannot avoid experiencing the total quantity. The quantities a_j [b_j] are, respectively, the amounts by which player j adds to [subtracts from] the existing level of G .

This formulation, though accommodating idiosyncratic unit costs, confines attention to linear technologies. For simplicity, we will focus on the linear technology throughout this paper. Our model may be extended to incorporate increasing costs in the augmenting or depleting of the public characteristic, but this complicates matters, requiring as it does a careful restatement of our normality assumption, without delivering significant additional insights.

2.2 Individual Resource Constraints

Player i has an exogenously determined resource endowment, w_i . This may be used for private consumption, or for either augmenting or reducing the total public good level. Resource constraints take the form

$$x_i + c_i^a a_i + c_i^b b_i \leq w_i \quad \forall i \in I,$$

where c_i^a [c_i^b] are unit costs associated with augmenting [depleting] the level of public characteristic implied by the actions of all players other than player i .

Standard formulations of the public good model are special cases. The model associated with Cornes and Sandler [9] and Bergstrom et al [4] can be modified to allow individuals to regard the public characteristic as a bad without significantly changing the outcome, as long as players are not permitted to undo the contributions of others. Effectively, they impose the assumptions that $c_i^a = c^a$ and $c_i^b = \infty$ — depleting the level of public good is infeasibly costly. The models developed by Buchholz and Konrad [7], Ihori [13] and Cornes and Hartley [11] allow idiosyncratic costs associated with augmenting the public good, but can also be interpreted as making depletion of the public quantity prohibitively costly. Like all of these earlier contributions, Cornes and Hartley [12] effectively restrict attention to public goods, but allow the marginal costs associated with the generation of G from individual activities to be variable.

Having introduced both preferences and resource constraints, we now impose one final assumption on the model, which we label “Cost-contingent Normality”:

Cost-contingent normality For every player i , her preferences and unit costs are such that the private good and that player's public good are everywhere normal, given the unit costs that characterize the available augmenting and depletion technologies.

Figure 1 depicts the preferences and budget set of an individual player. For any given level of private good consumption, the player regards increases in G as beneficial up to some finite level. Thereafter, further increases in G are harmful to her. The locus of allocations at which further increases in G cease to be a good and become a bad – the locus of ‘satiation points’ – is the dotted line through the lowest point of each indifference curve. In the situation shown, the higher the level of private good consumption, the higher is the level of satiation with respect to the public characteristic.

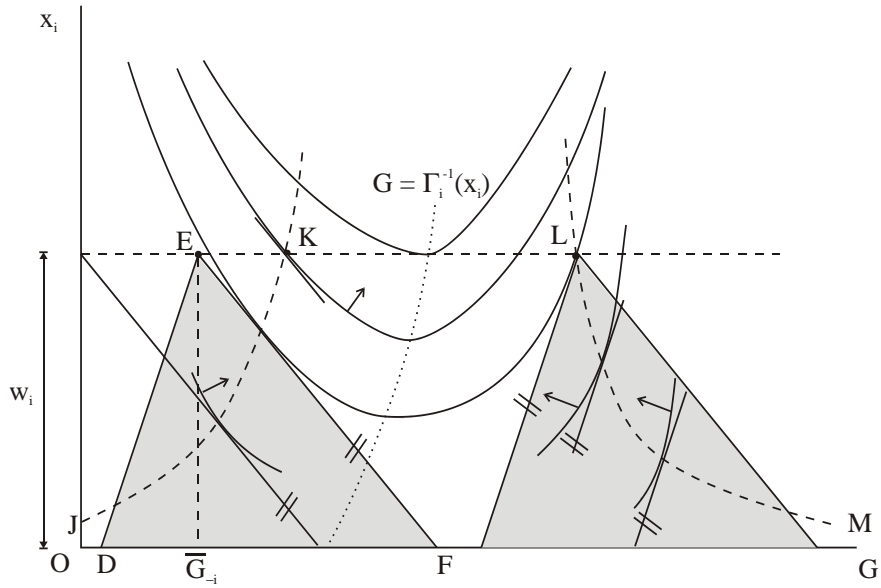


Figure 1: Individual preferences and constraints

The player's money income, or resource endowment, is measured by the vertical distance w_i , and, in the absence of any action by her that either raises or reduces the level of the public characteristic, the level of the public good is, say, the level \bar{G}_{-i} . Her endowment is a point such as E in the figure. Starting from E, her budget set is the shaded area in the figure. Starting at E, the player can, if desired, augment the existing level of G . By so doing, she can attain allocations along the line EF, the slope of which is $-c_i^a$. Alternatively, she may take steps to reduce the level of G , in which case she can move along the line DE, with slope c_i^b . Starting from the endowment point E, any allocation within the shaded area is attainable by the individual.

We are interested in identifying how the player's optimal response behaves as \bar{G}_{-i} varies parametrically. But first we use the figure to explain the meaning of our normality assumption. As drawn, the figure shows that, for sufficiently low levels of \bar{G}_{-i} , the player will choose to augment its level, and that the relevant income expansion path – the upward-sloping dashed line JK – is everywhere upward-sloping until she spends her entire budget on private good consumption – this is the point K. Thereafter, further increases in \bar{G}_{-i} lead to locations at which the player wishes neither to augment nor reduce the existing level, until the point L is reached. At that point, further increases in \bar{G}_{-i} lead her to prefer to reduce the quantity of the public characteristic, along the locus LM.

The meaning of the cost-contingent normality assumption can be stated as follows. It means that, in Figure 1, the segment JK of the individual's income expansion path associated with the unit cost c_i^a is everywhere upward-sloping, and that the segment LM, associated with the unit cost c_i^b , is everywhere downward-sloping. The significance of these observations for modeling individual behaviour and for the properties of Nash equilibrium will become clear as we proceed.

3 Pareto Efficiency in the General Model

Any Pareto efficient allocation may be characterized as an allocation satisfying the first-order conditions required for maximizing a weighted sum of individual utility levels subject to the overall resource constraint. The Lagrangean associated with this problem is

$$L = \sum_{i=1}^n \alpha_i u_i \left(x_i, \bar{G} + \sum_{j=1}^n a_j - \sum_{j=1}^n b_j \right) + \lambda \left[W - \sum_{i=1}^n x_i - \sum_{i=1}^n c_i^a a_i - \sum_{i=1}^n c_i^b b_i \right].$$

The first-order conditions are

$$\begin{aligned} \text{wrt } x_k & : & [\alpha_k u_{kx} - \lambda] x_k^* &= 0 \\ \text{wrt } a_k & : & \left[\sum_{i=1}^n \alpha_i u_{iG} - \lambda c_k^a \right] a_k^* &= 0 \\ \text{wrt } b_k & : & \left[- \sum_{i=1}^n \alpha_i u_{iG} - \lambda c_k^b \right] b_k^* &= 0 \\ \text{wrt } \lambda & : & W - \sum_{i=1}^n x_i^* - \sum_{i=1}^n c_i^a a_i^* - \sum_{i=1}^n c_i^b b_i^* &= 0, \end{aligned} \quad (1)$$

where the asterisks denote values of the variables at a Pareto efficient allocation. We will assume that preferences and technology are such that each player will choose a strictly nonzero level of consumption of the private good at any Pareto efficient allocation. Then, for each player,

$$\alpha_k = \frac{\lambda}{u_{kx}}.$$

Writing the player's marginal rate of substitution as $\mu_i(x_i, G) = \frac{u_{iG}}{u_{ix}}$, the second and third sets of FOCs can be written as

$$\begin{aligned} \left[\sum_{i=1}^n \mu_i(x_i, G) - c_i^a \right] a_k^* &= 0 \\ \left[\sum_{i=1}^n \mu_i(x_i, G) + c_k^b \right] b_k^* &= 0. \end{aligned}$$

Several immediate implications may be drawn:

- If any a_k^* is strictly positive, all b^* 's must be zero
- Conversely, if any b_k^* is strictly positive, all a^* 's must be zero.
- If the Pareto optimal level of the public characteristic exceeds [falls short of] G^0 , only the player(s) with the least unit cost of augmenting [depleting] the level of the public characteristic should do so. All others should spend their entire income endowment on private good consumption.
- Samuelson's rule becomes

Either $\sum_{i=1}^n \mu_i(x_i, G) = \overleftarrow{c}^a$, where \overleftarrow{c}^a is the common value of the marginal cost associated with each strict adder, and $G^{opt} > G^0$
or $\sum_{i=1}^n \mu_i(x_i, G) = -\overleftarrow{c}^b$, where \overleftarrow{c}^b is the common value of the marginal cost associated with each strict subtractor, and $G^{opt} < G^0$
or $-\overleftarrow{c}^b < \sum_{i=1}^n \mu_i(x_i, G) < \overleftarrow{c}^a$, and $G^{opt} = G^0$.

4 Noncooperative Individual Behaviour in the General Model

We now show that, under our stated assumptions, player i 's contribution to the public characteristic, which in the present model may be positive, zero or negative, may be represented by a single-valued function of her money income and the total prevailing level of the public characteristic. Following Cornes and Hartley [11], we call this her replacement function and we denote it by $r_i(w_i, G_{-i} : c_i^a, c_i^b)$.

A player who chooses to augment provision of the public good at a unit cost of c_i^a will, if at an interior solution, choose an allocation that lies on the 'unrestricted income expansion path' that passes through all allocations at which her marginal rates of substitution and of transformation are equal. This is represented in Figure 1 by the dashed line that passes through the points J, K and beyond K. Such allocations may be described by the function $\Gamma_i(w_i, G_{-i}, c_i^a)$, which describes the most preferred level of G given that the player's budget line passes through the point (w_i, G_{-i}) and she has a unit cost of c_i^a . Note that this definition of $\Gamma_i(w_i, G_{-i}, c_i^a)$ ignores the restriction that forbids her consumption

of the private good to exceed her money income. Hence, if in Figure 1, if she enjoys the indicated level of money income, and if G_{-i} implies an endowment point to the right of K, $\Gamma_i(w_i, G_{-i}, c_i^a)$ would pick out a point above and to the left of her endowment point, which is not feasible. We define $\Gamma_i(w_i, G_{-i}, c_i^b)$ in a similar way for a player who chooses to deplete G and note that, again, for certain values of G_{-i} , it implies an infeasible outcome.

Now define the following sets T_i :

$$\begin{aligned} T_i &= \{G_{-i} \mid \Gamma_i(w_i, G_{-i}, c_i^a) \geq G_{-i}\} \\ U_i &= \{G_{-i} \mid \Gamma_i(w_i, G_{-i}, c_i^a) < G_{-i}\} \text{ and } \Gamma_i(w_i, G_{-i}, c_i^b) > G_{-i} \\ V_i &= \{G_{-i} \mid \Gamma_i(w_i, G_{-i}, c_i^b) \leq G_{-i}\}. \end{aligned}$$

The lines of argument set out in Cornes and Hartley may be followed to establish that these three sets are intervals. Furthermore the chosen level of the augmenting activity monotonically and continuously falls as the observed level of G rises until, at the allocation K, it becomes zero. As G rises further, the player chooses neither to augment nor to reduce the prevailing level until G reaches the level indicated at M. Thereafter, further rises in G are associated with a monotonic and continuous increase in the depleting activity.

Panel (a) of Figure 2 reproduces the restricted income expansion path JKLM shown in Figure 1. Panel (b) graphs the player's implied level of addition to, or subtraction from, the prevailing level of the public characteristic as a function of that level. The function that is graphed here is the player's replacement function:

$$\hat{g}_i = r_i(G, w_i, c_i^a, c_i^b).$$

Under our assumptions, the replacement function uniquely identifies the most preferred level of contribution by player i (which in the present model may be positive, zero, or negative) that is consistent with a given level of the total level of the public characteristic. Its relevant properties are summarized by the following proposition.

Proposition 1 *Under our assumptions, player i has a replacement function $r_i(G, w_i, c_i^a, c_i^b)$ with the following properties:*

1. *The value of $r_i(G, w_i, c_i^a, c_i^b)$ is bounded above and below by, respectively, $\frac{w_i}{c_i^a}$ and $-\frac{w_i}{c_i^b}$.*
2. *$r_i(G, w_i, c_i^a, c_i^b)$ is continuous.*
3. *$r_i(G, w_i, c_i^a, c_i^b)$ is everywhere nonincreasing in G , and is strictly decreasing wherever it is nonzero.*

It is convenient to consider the sum of all individual replacement functions:

$$R(G, \mathbf{w}, \mathbf{c}^a, \mathbf{c}^b) \equiv \sum_{j=1}^n r_j(G, w_j, c_j^a, c_j^b),$$

where boldface denotes a vector of the relevant parameter. This relationship plays a key role in our characterization of Nash equilibrium and in its analysis.

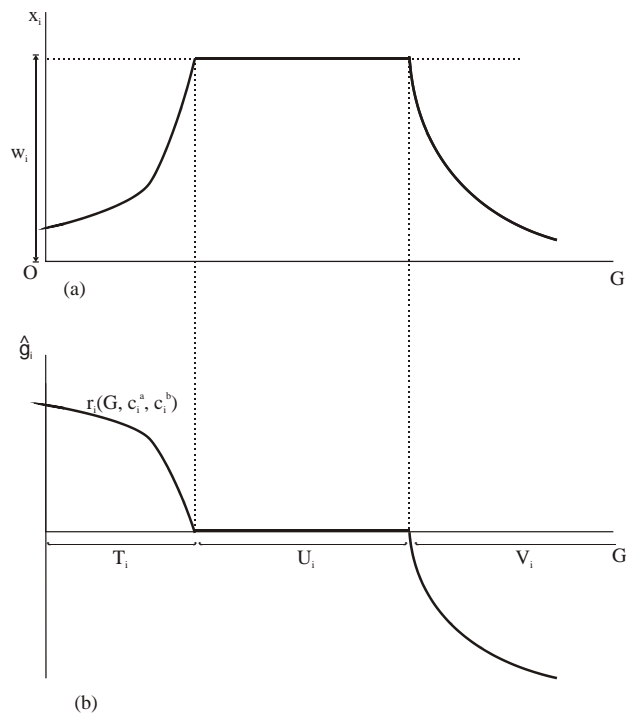


Figure 2: From income expansion path to replacement function

5 Noncooperative Equilibrium in the General Model: Existence and Uniqueness

Our interest in deriving the aggregate replacement function is motivated by the fact that it provides a convenient way of describing a Nash equilibrium. In the basic model of private public good provision, every unit of the public good enjoyed by the players is the result of some individual's contribution. In the absence of any such actions by players, the level of public good is naturally zero. In the present model, this may not be such a natural assumption. In the absence of human activity, the level of carbon dioxide in the atmosphere is some positive amount. In the absence of conservation or culling measures, the population of some type of flora or fauna may also be positive. We could define our public characteristic in such a way as to make its initial level, in the absence of human intervention, zero. Instead, we define the characteristic in such a way that, in the absence of human activity, it takes some exogenous positive value G^0 , and cannot, in any feasible allocation, be negative.

A Nash equilibrium is an allocation at which the difference between the prevailing level of the public good and its initial level G^0 is consistent with the most preferred actions of all players. More precisely, we require that the value of the aggregate replacement function equals the difference between prevailing and initial values of G :

$$R(G, \mathbf{w}, \mathbf{c}^a, \mathbf{c}^b) = G - G^0.$$

The common value of the two sides may, in the present context, be of either sign. The most significant feature of this condition is that it is a single equation in a single unknown, G . Moreover, since the aggregate replacement value on the left hand side is an unweighted sum of the individual replacement functions, it is a simple matter to infer relevant properties of the aggregate function from those of the individual replacement functions.

The following proposition summarizes salient properties of the aggregate replacement function $R(\cdot)$:

Proposition 2 *Under our assumptions,*

1. $R(\cdot)$ is bounded above and below by, respectively, $\sum_{j=1}^n \frac{w_j}{c_j^a}$ and $-\sum_{j=1}^n \frac{w_j}{c_j^b}$.
2. $R(\cdot)$ is continuous.
3. $R(\cdot)$ is everywhere nonincreasing in G , and is strictly decreasing wherever it is strictly nonzero.

Figure 3 shows the graphs of individual replacement functions in a 3-player example, and also that of the aggregate function. Once the initial value of G is determined, a Nash equilibrium is a point of intersection between the 45° ray through the point G^0 and the graph of the latter.

Property (3) of $R(\cdot)$, monotonicity, ensures that there cannot exist more than one equilibrium allocation. Furthermore, if one can show that two allocations exist such that at one of them $R(G, \mathbf{w}, \mathbf{c}^a, \mathbf{c}^b) \geq G - G^0$ and at the

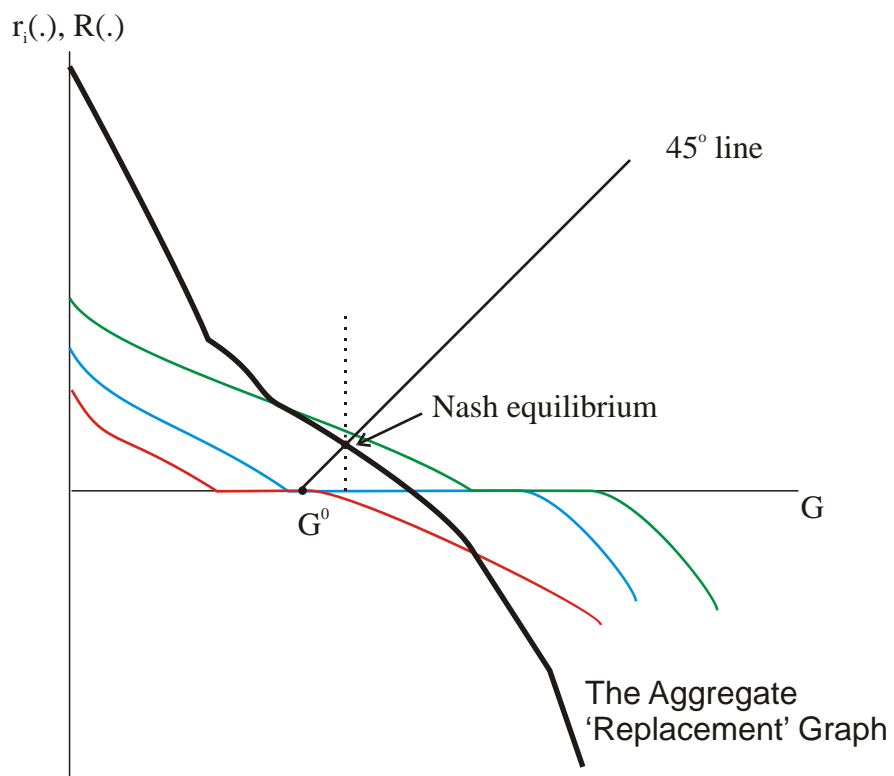


Figure 3: Nash equilibrium: A 3-player example

other $R(G, \mathbf{w}, \mathbf{c}^a, \mathbf{c}^b) \leq G - G^0$, then property (2), continuity, ensures that an equilibrium does indeed exist. In the basic model, it can be shown that $R(\cdot)$ is defined over some interval $G \in [\underline{G}, \infty]$ and that $R(\underline{G}) \geq \underline{G}$ and, for sufficiently large G , $R(G) < G$.

6 The Normality Assumption

6.1 Beyond Normality

Here, as in the basic model, we adopt the normality assumption essentially because it conveniently guarantees uniqueness of equilibrium. Of course, the reasonableness of normality is, at heart, an empirical issue. But theoretical reasons make it rather less innocent in the present context than in the basic model, for a reason that we now discuss. This also seems a good opportunity to show, with the help of a specific numerical example, what can happen if normality is violated.

Recall that in Figure 1, the locus of satiation points at which $\frac{\partial u_i}{\partial G} = 0$ happened to be everywhere upward-sloping. It could equally well have been downward-sloping. But for the moment let us maintain the assumption reflected in the figure. The continuity properties of our model imply that, in this case, if the unit cost associated with reducing the level of public characteristic were sufficiently small, the graph of the player's replacement relationship - which will now no longer be a single-valued function - will slope the 'wrong way'⁴. This is why we specifically imposed normality for the specific unit costs faced by the player.

6.2 A Numerical Example

Suppose that a player's preferences may be represented by the utility function

$$u(x, G) = 10 - \left(\left(\frac{x}{4} - 1 \right)^2 + \left(\frac{G}{2} - \frac{x}{8} - 1 \right)^2 \right)$$

and that the player's money income is $w = 1$. Then, over the feasible set of allocations, the private good is everywhere truly a good⁵.

⁴Had the locus of satiation points been everywhere downward-sloping, the possibility of violation of normality would have been present for augmenting the public characteristic.

⁵To make the example easy to work with, we assume a functional form for preferences that generates linear income expansion paths. This violates our assumption that the individual will always choose a strictly positive quantity of the private good. However, this does not matter for the points that we wish to make. A more general example would involve rather more intricate workings to identify all potential equilibria, but would produce the same qualitative conclusions.

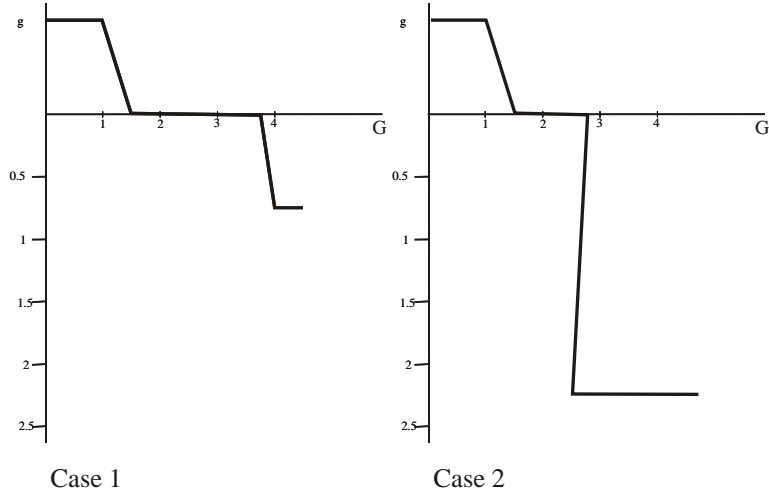


Figure 4: The problem with nonnormality

6.2.1 Case 1

Suppose, initially, that

$$(c^a, c^b) = \left(\frac{4}{3}, \frac{4}{3} \right).$$

Simple algebra confirms that, for values of G consistent with the player choosing to augment provision $[\frac{5}{6} \leq G \leq \frac{3}{2}]$, her replacement function takes the form

$$\hat{g} = r(\cdot) = \frac{9}{4} - \frac{3}{2}G.$$

Further, for values of G consistent with her choosing to reduce prevailing provision $[\frac{15}{4} \leq G \leq 4]$, the relationship is

$$\hat{g} = r(\cdot) = \frac{45}{4} - 3G.$$

Stitching the graphs together, the player's replacement graph is as shown in the left hand panel of Figure 4:

(The horizontal segments away from the axis reflect the fact that the example allows the player to choose to consume nothing of the private good – a feature we did not allow in our analysis above).

A community of such players as the one described would have a single Nash equilibrium.

6.2.2 Case 2

Now suppose, instead, that

$$(c_i^a, c_i^b) = \left(\frac{4}{3}, \frac{4}{9} \right).$$

All other assumptions about the player are the same as in Case 1. The unit cost associated with depletion of the public characteristic is now quite small. The relevant stretch of the graph for a player who chooses to deplete the level is now described by the equation

$$\hat{g} = r(\cdot) = 18G - \frac{189}{4}.$$

Stitching the two graphs together, and filling in the remaining stretches, the graph of the replacement relationship is as shown in the right hand panel of Figure 4.

This no longer graphs a single-valued function. For example, there are not one, but three, allocations that are consistent with $G = 2.6$. These are $(G_{-i}, \hat{g}) = (2.6, 0)$, $(G_{-i}, \hat{g}) = (3.5, -0.9)$, and $(G_{-i}, \hat{g}) = (4.85, -2.25)$.

6.2.3 Implications for Equilibrium

Imagine three players, each of whom has the parameter values described in Case 2. Suppose that nature's endowment of G is $G^0 = 4.5$. An equilibrium must satisfy the requirement that

$$G = G^0 - \sum_{j=1}^3 g_j.$$

Equilibrium may involve one interior player and two choosing $b [= g] = 0$. Then the equilibrium value of G satisfies

$$G = 4.5 + \left(18G - \frac{189}{4} \right) \implies G = 2.5147.$$

Alternatively, maybe two players are at an interior point and the third contributes zero, in which case

$$G = 4.5 + 2 \left(18G - \frac{189}{4} \right) \implies G = 2.5714.$$

Finally, all three may be at an interior point:

$$G = 4.5 + 3 \left(18G - \frac{189}{4} \right) \implies G = 2.5896.$$

These three equilibria are illustrated in Figure 5, which shows the relevant part of the graph of the replacement correspondence.

They are where the dashed line intersects branches of the 'aggregate replacement graph'. We have stretched out the horizontal axis and compressed

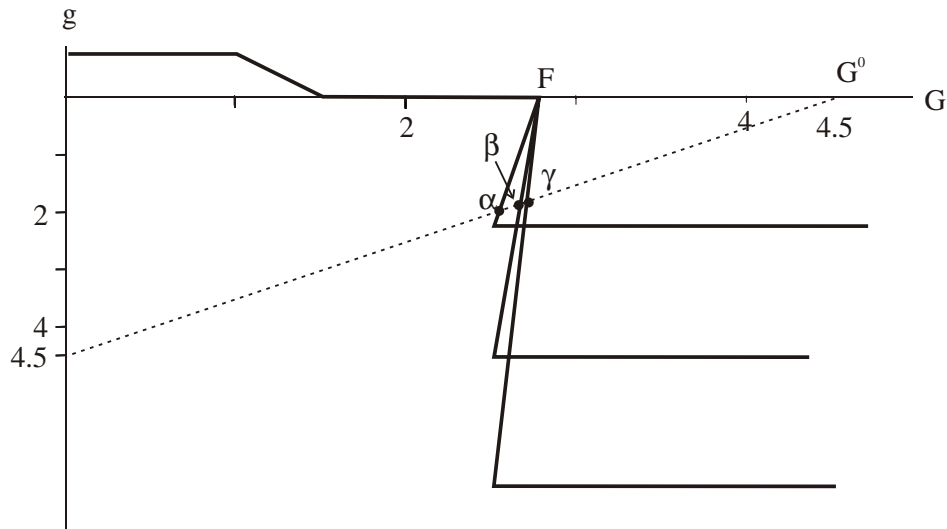


Figure 5: Normality violation and multiple equilibria

the vertical axis in this figure to try to show more clearly the relevant features of this graph.

We should now explain the graph. At the point F , all three players are choosing zero contributions to G . The positively sloped ray emanating from F , passing through the point α , is identical with the individual replacement graph. The point α is an equilibrium at which one player is on the interior branch of her replacement graph, the other two are choosing $g = 0$, which is on their zero branch, and their choices are consistent with the amount by which the realised level of G falls short of its initial value G^0 . At the point β , two players are on their interior branches and one is contributing zero. Finally, at γ all three players are choosing interior points.

If nature's endowment were low — indeed, for any value of G^0 below 2.625 — there would be a unique equilibrium. For example, if $G^0 = 2$, the unique equilibrium involves zero contributions to changing G , and $G^{eq} = G^0$. Sufficiently high natural endowments of G will again lead to a unique equilibrium outcome in the present example.

7 Comparative Statics and Other Properties of Equilibrium

Let us return to the model in which the normality assumption is imposed. We are now able to make statements about the normative and comparative static properties of the Nash noncooperative equilibrium.

7.1 Normative Properties

Since some individuals may like, and others dislike, the public characteristic, there is no general presumption that it will be underprovided at equilibrium. The equilibrium level may either exceed or fall short of a level consistent with Pareto optimality.

Furthermore, even if we regard the equilibrium level of its provision as optimal, resource allocation may still be inefficient, for the reason already present in existing models of public good provision. This aspect of the model, however, is more evident in the present formulation. In the models of Buchholz and Konrad (1995) and Ihori (1996) of public good provision with productivity differentials, it is possible for inefficiency to arise because the ‘wrong’ players are contributing to the public good. Suppose, in that context, that there is a positive contributor whose unit cost exceeds that of the most efficient generator of the public good. Then, by reducing that individual’s contribution, and increasing that of the lower-cost player, extra resources can be devoted to private good consumption at an unchanged level of the public good. If redistribution is permitted, then a Pareto superior allocation can be achieved.

In the present context, this remains true. But it is also true, as we showed earlier, that Pareto efficiency is inconsistent with there being both positive augmenters and positive depleters at equilibrium. For example, the equilibrium depicted in Figure 3 must be Pareto inefficient.

7.2 Comparative Static Properties

The comparative static properties of equilibrium are, for the most part, straightforward extensions of those possessed by the public good model. For example, the standard neutrality property holds for redistributions amongst the current set of positive augmenters, and also for redistributions amongst the current set of positive depleters. Since the properties of the present model generally involve rather minor modifications of known properties of the basic model, we need not rehearse them all here. But it is worth stating one or two specific corollaries of these properties that are of interest in their own right.

7.2.1 A ‘Neutral Growth’ Property

A natural and interesting extension of the neutrality result associated with income redistribution in the basic public good model is the following neutrality property associated with an increase in players’ resource endowments:

Proposition 3 *Let players j and k be, respectively, an active augmentser and an active depleter at equilibrium. Now suppose that each receives an equal extra increment of income. Then*

- if $c_j^a < c_k^b$, the equilibrium level of the public characteristic increases,
- if $c_j^a = c_k^b$, the equilibrium level of the public characteristic remains unchanged,

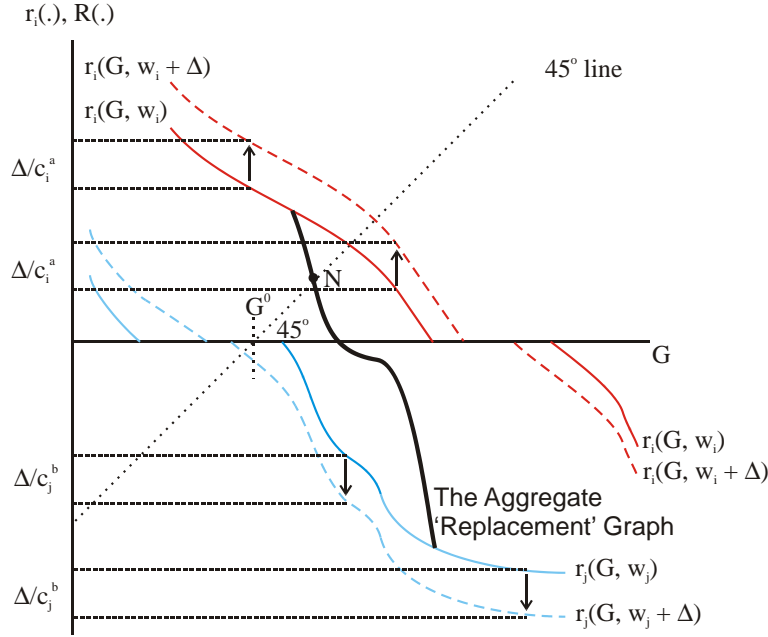


Figure 6: Resource growth and neutrality

- if $c_j^a > c_k^b$, the equilibrium level of the public characteristic falls.

The second of these possibilities extends existing neutrality propositions. If the two players have the same unit costs at the equilibrium allocation, then a net increase in the resources available to the economy may have no real consequences. It is all wasted in additional conflict over the level of the public characteristic, the players' private consumption levels being unchanged. The reasoning precisely follows that used in Cornes and Hartley [11]: in the neighbourhood of the equilibrium, the graph of player j 's replacement function shifts upward by some amount. That of player k moves downward by precisely the same amount. Therefore the aggregate replacement graph remains unchanged in the neighbourhood of the initial equilibrium. Figure 6 depicts this neutrality proposition.

Denoting the sets of augmenters and depleters at the initial equilibrium respectively by I_a and I_b , we have the following straightforward corollary, which contains as special cases the standard neutrality proposition associated with the basic public good model, and provides a general statement of the impact of income changes in the general public characteristic model.

Proposition 4 *Consider a set of changes in the incomes of augmenters and depleters. If the sets of augmenters and depleters remain unchanged at the new equilibrium, then the aggregate level of the public characteristic rises, remains*

unchanged or falls according to whether

$$\sum_{j \in I_a} \frac{\Delta w_j}{c_j^a} >, =, \text{ or } < \sum_{k \in I_b} \frac{\Delta w_k}{c_k^b}.$$

7.2.2 Pareto-improving Taxation

We have emphasized that, in the present model, inefficiency is generated not only by the wrong level of provision of the public characteristic but also by the presence of resources devoted both to augmenting [lobbying in support of] and depleting [lobbying against] the provision of the public characteristic. This suggests that policies that prohibit or discourage such mutually opposed uses of resources, while leaving the level of public characteristic unchanged, may be Pareto-improving. In particular, the simultaneous imposition of taxes on both the augmenting and the depleting activities may be Pareto-improving. Certainly, if there is a net redistribution of tax revenue to noncontributors which leaves the sets of augmenters, noncontributors and depleters unchanged, it is easy to see how such taxation can reduce the level of resources wasted in the contest over the level of G . One should, of course, be wary of the possibility that, in the absence of such a redistribution, the imposition of such a tax may be undone in the manner suggested by the literature on neutrality. However, even without such redistribution, Andreoni and Bergstrom [1] show that it is possible in the basic public good model with equal marginal costs of contribution across contributors for an appropriately designed tax intervention to overcome the neutrality property and have real effects, by genuinely changing the opportunity cost facing potential contributors. Their analysis carries over to the present context. It is possible to tax both augmenters and depleters, leave the current equilibrium level of G unaffected, and reduce the aggregate equilibrium levels of both augmenting and depleting activities, thereby increasing the resources devoted to private consumption and creating the possibility of a Pareto improvement.

8 Concluding Remarks

By fully exploiting the aggregative structure inherent in the public good model, its extension to encompass characteristics that some regard as bads, and that they can take actions to reduce, becomes relatively straightforward. Normality is not quite such an innocuous assumption in this setting, but if normality as we have defined it holds then the resulting model possesses a unique equilibrium. In any event, equilibrium properties are easily analyzed by exploiting fully the aggregative structure of the public characteristic model. There are no great surprises amongst its properties. However, this formulation highlights the fact that not only does the equilibrium level of the public characteristic matter for normative evaluation, but that we should also be worried about the possible waste of resources implied in the process by which that equilibrium is brought

about. Also, the analysis shows an interesting twist on the neutrality proposition, in which resource growth may be entirely dissipated by conflict over the public characteristic.

It is, of course, easy to imagine extensions of the present simple model. The idea of viewing the public characteristic level as a consequence of a production process suggests that it is worthwhile to consider more general additively separable social composition functions that describe the way in which individual contributions influence the total level of the public characteristic. Furthermore, our current understanding of the global warming debate makes it clear that activities that augment or deplete a public characteristic may well involve joint production technologies. The level of carbon dioxide emitted into the atmosphere is, after all, typically a byproduct of the production and consumption of private goods. On the other side of the ledger, its reduction can be a significant byproduct of various agricultural practices, and of afforestation, activities often motivated by their role in generating other outputs such as the mitigation of soil erosion, desalination and so on. The tractability of such models of contentious ‘impure’ public characteristics has yet to be tested.

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