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What entices the Stork? Fertility, Education and Family Payments

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Abstract

Developed economies, experiencing concomitant declining fertility and rising educational attainment, have introduced policies to boost fertility. We model substitution of bought in services for parental time in the rearing and education of children in an economy where technological progress leads households to choose fewer, but better educated, children. We analyse the effects on fertility and education of a baby bonus, paid maternity leave and child care subsidies. We establish conditions under which either maternity or child care benefits are more efficacious in raising fertility, and we establish that a lump sum baby bonus will increase fertility only if the bonus increases faster than income per capita. Policies that stimulate fertility also raise parental investment in education.

Key words: Fertility, education, child care, baby bonus, maternity leave

JEL Classifications: H31, J13, O40

1   Introduction

Birth rates and fertility have become part of the Australian economic policy debate since the release of the Treasury's first Intergenerational Report in 2003. This report drew attention to the fiscal implications of a population where the average age is expected to increase over the next few decades due to both increasing longevity and decreasing fertility. The focus of the report and initial policy

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discussion concentrated on the economic implications of demographic change, but more recently interest has moved to the question of whether government policy can actually reverse the long-run decline in fertility. Such interest has been spurred by the observation that such a reversal is actually occurring.

The recent upturn in Australian fertility has caused some commentators to suggest that it is due to the introduction in 2004 of the 'Baby Bonus'[1] and the then Federal Treasurer's exhortation to Australian women.

> The former treasurer's call for Australian couples to have another child "for the country" appears to have paid off, with a spike in the birth rate. The number of births in Australia every year has been knocked out of a rut which had continued since the mid 1990s, when between 255,000 and 260,000 babies were being born per year. In 2005, the year after then treasurer Peter Costello introduced the $3,000 baby bonus along with a parental call to arms, the birth rate climbed to 272,000. Rose (2009)

There has indeed been an increase in births, but it makes more sense to measure the birth rate relative to the female population of child-bearing age rather than just the number of births. The time path of the total fertility rate (births per woman) since 1925 is shown in Figure 1[2] where it can be seen that the increase in fertility started in 2002, well before the introduction of the Baby Bonus.

A more careful assessment of the impact of the Baby Bonus on the birth rate comes from Drago, Sawyer, Sheffler & Wooden (2009) using an econometric model which produces estimates of a bonus-inspired rise between 0.7% and 3.2%. Starting with the 2003 fertility rate of 1.75 births per woman, these predicted rises attributable to the Baby Bonus would have taken the fertility rate to 1.76 or 1.81 respectively, explaining less than one quarter of the observed rise to over 1.99 births per woman in 2008. Guest (2007) argues that the Baby Bonus could have been more effectively targeted if the Australian government had followed the example of Singapore by restricting the payment to second and third children.

Financial child support may not be the only cause of rising fertility. For example, empirical estimates from Feyrer, Sacerdote & Stern (2008) associate a one standard deviation increase in the fraction of housework done by men with an increase in fertility of 0.12 children. An alternative explanation for the recent 'baby bounce-back' comes from Day (2004) who shows that a fertility decline which has resulted from a trend increase in the relative earnings of women, hence the opportunity cost of children, may be reversed eventually if there is sufficient substitutability in raising children between maternal time and child-care services.

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[1] Formally, the Maternity Payment – a lump sum payment to families on the birth or adoption of a child.

[2] Sources: 1925-2006: ABS 3105.0 'Australian Historical Population Statistics, 2008'; 1997-2008: ABS 3301.0 'Births, Australia, 2008'; ABS 3101.0 'Australian Demographic Statistics, Sep 2009'.
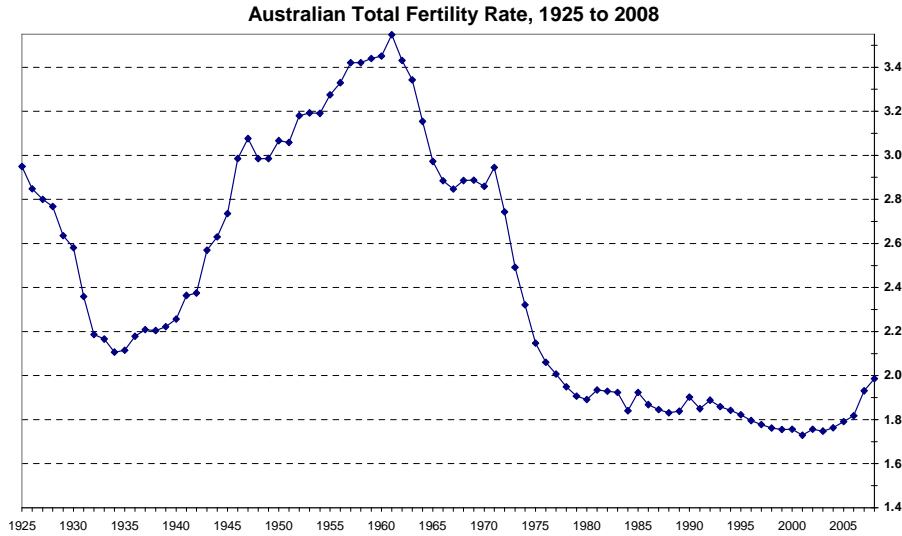
Figure 1: Total Fertility Rate

A review of trends in Australian fertility has been conducted by Lattimore & Pobke (2008). They conclude that some of the recent rise in fertility is due to the 'tempo' effect of births to women who had previously delayed child-bearing. They report that if Australian fertility were to have the same sensitivity to family allowances as OECD countries on average, then the change in allowances (including the Baby Bonus) since 1999 would have caused the total fertility rate to rise by 0.07 babies per woman. Although Lattimore & Pobke (2008) (p. XVII) argue that this result is likely to overestimate the impact of family policy in Australia, the magnitude of their predicted impact is strikingly similar to Drago et al.'s (2009) top-end estimate of a Baby Bonus impact of 0.06 births per woman.

Lattimore & Pobke (2008) suggest that the current moderate level of Australian fertility is likely to be sustained into the future; the fact that it is slightly below the demographic replacement rate, which is the fertility rate required to keep population stable in the absence of migration, should not be regarded as a matter of concern. They suggest (pp. 101-102) that current Australian public policies that lower the costs of children and that reduce the trade-offs between careers and bearing children are important in reducing the risk of a long-run shift to the much lower fertility levels that currently exist in some European countries.

There is further support for the proposition that child assistance is effective in raising fertility. For OECD economies, Feyrer et al. (2008) estimate that doubling government spending on family assistance raises fertility by 0.13 children.

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However, caution needs to be exercised when interpreting coefficient estimates. For instance, a negative coefficient on federally mandated paid maternity leave, estimated by Feyrer et al. (2008) for OECD countries, may reflect the endogeneity of pro-natalist policies. That is, countries with low fertility may respond by increasing paid maternity leave.

McDonald (2006) reviews the demographic evidence on the efficacy of public policy and concludes that fertility does respond positively not only to cash payments but also to resources that support women's ability to combine work with raising more than one child. Apps & Rees (2004) argue that subsidies to child care can be more effective than cash grants in increasing fertility (and women's supply of labour). This is a theme that we develop in this paper.

Following the lead of both Apps & Rees (2004) and McDonald (2006), we propose to model the effects of the two types of family assistance that currently operate in Australia – cash payments and child-care subsidies – and we extend the modelling by including a third type of assistance, soon to be introduced, maternity pay for working mothers. We also introduce the effects of family policy on education of children following the lead of Becker & Lewis (1973) who explore the notion that families choose between the quantity and quality of children, where education is regarded, in somewhat crude economistic terms, as investment in quality.

In the next section we review the literature that has analysed multi-faceted family assistance. We then set out our model which allows us to analyse three types of family assistance in an overlapping generations model of endogenous growth which allows parents to choose both the quantity of children and their quality or level of education.

2   Review of related models

A central theme to the existing theoretical literature on the relationship between public finance and family size concerns the socially optimal level of taxation and child support, given interrelations between child rearing, labor supply and the degree of inequality aversion exhibited by the social welfare function (Balestrino, Cigno & Pettini (2002) and Cigno & Pettini (2001)). The more restrictively positivist approach of this paper is similar to that of Apps & Rees (2004) in that we assume that the government would like to increase fertility and we want to compare and contrast the effectiveness of different instruments of child support for doing so.

Our analysis differs from Apps & Rees (2004) in three ways. First, we consider a wider set of instruments that includes paid maternity leave. Second, we allow for parents to care about quality, as well as the number of, children. Third, we model household decision making within an endogenously growing economy where technological progress induces households to trade child quantity for quality. The first two aspects allow us to explore the difference in effectiveness of maternity benefits and child care benefits. The latter aspect allows us to distinguish the effects of a lump sum baby bonus from the other two forms of child

support. Moreover, by modelling simultaneous decisions over number of children and education, we can also predict the effects of child support on education.

Apps & Rees (2004) find that, all else equal, raising either child care subsidies or a lump sum baby bonus unambiguously increase fertility. Given the similarity of our modelling assumptions, our results do not fundamentally contradict those of Apps & Rees (2004). It is not surprising, however, to find that our extension to an extra choice variable (education) and extra policy instrument (maternity leave) does lead to some interesting differences. We find, for instance, that raising a lump sum baby bonus has an ambiguous effect on fertility. Moreover, the relative effectiveness of raising the rate of maternity benefit and child care benefit depends both on the relative efficiency of child care and on prevailing input prices.

We embed our analysis of household choice in an extended version of the model of 'Modern Economic Growth' developed by Galor & Weil (2000) who analyse the interplay between population, technology and human capital. They present a growth model that endogenises the level of education and partially endogenises the rate of technological progress. In brief, rapid technological change raises the rate of return to human capital, inducing a rise in education levels. Higher education levels in turn increase the speed of technological progress. The joint evolution of technological change and education drives economic growth. Rapid technological change induces parents to choose quality (education) over quantity of children. Thus, the population growth for the current generation is inversely related to the education level of the next generation. We extend the Galor & Weil (2000) model by introducing bought in services as an input in both the rearing and education of children and by introducing family policy.

3   The Model

Consider an overlapping generations model in which people live for two periods: childhood $(t-1)$ and adulthood (t). In childhood, individuals consume both time and goods and services provided by their parents. Parents allocate inputs across number and quality (education) of children. In the second period of life, each individual is endowed with one unit of labour.

3.1   Production of Final Output

Following Galor & Weil (1998), the economy produces a single homogeneous good at time t according to the production function:

$$Y_t = Z_t K_t^{\beta} H_t^{\alpha(1-\beta)} X^{(1-\alpha)(1-\beta)}; \quad \alpha \in (0,1) \quad \beta \in (0,1) \tag{1}$$

where Z_t, K_t, H_t and X denote technology, physical capital, human capital and fixed natural resources, respectively. In a small open economy with perfect capital mobility, capital accumulation has no role in the mechanism for growth.[3]

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[3]Given an interest rate, r, in a competitive world capital market, capital is employed up to the point where $MP_K = r$. From this condition, the optimal capital stock is a function of

The production function in per worker terms is

$$y_t = h_t{}^\alpha x_t^{1-\alpha} \tag{2}$$

where $h_t = H_t/L_t$ is human capital per worker and $x_t = J_t X/L_t$ is the amount of effective resources per worker at time t.

If there are no property rights over land, factor payments to land are zero. By this simplifying assumption and Euler's Theorem, each worker receives a wage[4] per efficiency unit of labor:

$$w_t = (x_t/h_t)^{1-\alpha} \tag{3}$$

3.2 Production of Human Capital

The individual level of human capital is determined by the individual's quality (education) as well as by the technological environment. Galor and Weil (2000) assume human capital is depreciated by (decreasing and convex in) the *rate* of technological change, g_{t+1}. The level of human capital of children of members of generation t, h_{t+1}, is an increasing function of their education, e_{t+1}, and is depreciated by the rate of progress in the state of technology from period t to period $t+1$, g_{t+1}:

$$h_{t+1} = h(e_{t+1}, g_{t+1}) \tag{4}$$

where $h_e > 0$, $h_{ee} < 0$ and $h_g < 0$, $h_{gg} < 0$. Education lessens the adverse effect of technological change: $h_{eg} > 0$.

3.3 Household Optimisation

Motivated by intergenerational altruism or transfers from children in old age, households derive direct utility from the aggregate full income of their offspring.[5] The preferences of members of generation t are represented by the utility function:

$$u^t = \gamma \ln(w_{t+1} h_{t+1} n_t) + (1-\gamma) \ln c_t \tag{5}$$

where n_t is the number of children.[6]

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r, α, β and the other inputs. The production function can be rewritten as:

$$Y_t = \left[(\beta/r)^{\beta/1-\beta} Z_t^{1/1-\beta} \right] H_t^\alpha X^{(1-\alpha)} = H_t^\alpha (J_t X)^{(1-\alpha)}$$

where the proportionate change in J_t is a fixed multiple of the proportionate change in the original technological parameter, Z_t.

[4] The price of the homogeneous good is normalized to unity in every period, so that the real wage equals the nominal wage.

[5] See Barro & Becker (1988) and Becker, Murphy & Tamura (1990) on parental altruism; Cigno & Rosati (1996) for a model of intergenerational transfers. If parents derive utility from their children's actual income, we obtain a dynastic utility function as per Barro & Becker (1988).

[6] The model structure up to this point corresponds to Galor and Weil (2000). The analysis from this point on is the original work of the authors.

3.3.1 With bought in child care and education

In this paper, we want to examine the effects of child support paid to the household in the form of a baby bonus, subsidised child care and paid maternity leave. We need to introduce external services which can be purchased to substitute for parental time in raising and educating children.

In the second period of life, each individual faces the budget constraint:

$$w_t h_t [(\hat{z}^q + \hat{x}^q) + (\hat{z}^e + \hat{x}^e) e_{t+1}] n_t \leq w_t h_t \tag{6}$$

where $\hat{z}_q$ and $\hat{z}_e$ denote the fraction of the individual's unit time endowment required to raise a child regardless of quality and for each unit of education per child, respectively; $\hat{x}_q$ and $\hat{x}_e$ denote the services required to raise a child regardless of quality and for each unit of education per child, respectively.

Note that we price human capital used in bought in services and parenting at the same rate. Both bought in services and parental time use human capital. However, if bought in services and parental time are priced at the same rate, then the household should be indifferent between bought in and parental time. Thus, Galor & Weil (2000) state:

> Since all members of a generation are identical in their endowments, the budget constraint is not affected if child quality is produced by professional educators rather than by parents

We suggest that whilst members are identical in their human capital endowments, professional child carers and educators differ from parents in the productivity of their human capital. Thus, the prices of bought in services and parental time differ due to heterogeneous efficiency, incorporated in the general production functions:

$$n_t = f(Bz^q, Ax^q) \tag{7a}$$

$$e_{t+1} = g(Dz^e, Ex^e) \tag{7b}$$

which are linear homogeneous, continuously differentiable and strictly quasi-concave and where $A(E)$ and $B(D)$ denote the efficiency of *total* bought in services and *total* parental time, z^i and x^i ($i = q, e$), respectively, in child rearing (education).

The household's optimal choice of child quantity and quality, derived by maximising (5) subject to (4) and (6) is

$$n_t = \frac{\gamma}{(\hat{z}^q + \hat{x}^q) + (\hat{x}^e + \hat{z}^e) e_{t+1}} \tag{8}$$

$$e_{t+1} = \frac{h_{t+1}}{h_e} - \frac{(\hat{z}^q + \hat{x}^q)}{(\hat{x}^e + \hat{z}^e)} \tag{9}$$

where, for the moment, we note that $\hat{z}^i(B, D)$ and $\hat{x}^i(A, E)$. We explicitly solve for the cost minimizing inputs under a system of child support in the following section.

Equation (8) implies the well known child 'quality-quantity trade-off', a phrase coined by Becker & Lewis (1973).

Proposition 1 *If child rearing is relatively intensive in parental time, then the elasticity of child quantity with respect to child quality increases with the introduction of bought in child care and education*

Proof. The elasticity of child quantity with respect to child quality is

$$\frac{d \ln n_t}{d \ln e_{t+1}} = -\frac{(\hat{x}^e + \hat{z}^e) \, e_{t+1}}{(\hat{z}^q + \hat{x}^q) + (\hat{x}^e + \hat{z}^e) \, e_{t+1}}$$

where

$$\frac{d \ln n_t}{d \ln e_{t+1}} > \frac{d \ln n_t}{d \ln e_{t+1}} \mid_{\hat{x}^e = \hat{x}^q = 0} \Leftrightarrow \frac{\hat{z}^q}{\hat{x}^q} > \frac{\hat{z}^e}{\hat{x}^e}$$

∎

Note, in their discussion of the quality-quantity trade-off, that Galor & Weil (2000) (footnote 9, p. 812) state

> If both time and goods are required to produce child quality, the process we describe would be intensified.

The process they describe is one where technological progress expected to occur during a child's lifetime induces a decline in parents' chosen quantity of children and a rise in their quality. Referring to the proof of Proposition 1, the elasticity of child quantity with respect to child quantity intensifies if both time and goods are required to produce child quality, but only time is required to produce child quality ($\hat{x}^q = 0$). This is a special case of Proposition 1. In general, the process of declining fertility and rising education intensifies or attenuates depending relative factor intensities, which in turn depend on efficiency parameters and child support benefits, which we now analyse.

3.3.2 With a system of child support

The production function for child rearing is a David & van de Klundert (1965) variant of CES form[7]

$$n_t = [(Bz^q)^\rho + (Ax^q)^\rho]^{1/\rho} \, ; \rho \neq 0 \tag{10}$$

where z and x denote *total* parental time and *total* bought in child care, respectively, and the positive coefficients A and B represent the levels of efficiency of parental time and bought in child care, respectively. The ratio A/B can be interpreted as returns to specialisation by professional child carers. Alternatively, we could define $B = \delta^{1/\rho}$ and $A = (1 - \delta)^{1/\rho}$, where δ is a distribution parameter that could be interpreted as measuring the relative factor shares in production. The constant elasticity of substitution between parental care and bought in care is $\varepsilon = 1/(1 - \rho)$.

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[7]Klump & Preissler (2000) show that David & van de Klundert (1965) is a CES variant with input augmenting efficiency parameters that follows directly from the definition of the elasticity of substitution.

Similarly, the production function for child education is

$$e_{t+1} = [(Dz^e)^a + (Ex^e)^a]^{1/a}\,; a \neq 0 \qquad (11)$$

where z and x denote *total* parental time and *total* bought in education (schooling), respectively, and the positive coefficients D and E represent the levels of efficiency of parental time and bought in education, respectively. The ratio E/D can be interpreted as returns to specialisation by professional teachers.

Because the production functions for child-rearing and education are homogeneous of degree one, the household optimization problem can be solved in two stages. The household first chooses, for a given n_t and e_{t+1}, the cost minimizing input mix and then chooses n_t and e_{t+1}, given the efficient input mix, so as to maximize utility subject to a budget constraint.

Cost minimization

The total cost of rearing children is

$$C_t = w_t h_t [(1-m)\hat{z}^q + (1-\sigma)\hat{x}^q + (\hat{x}^e + (1-m)\hat{z}^e)\,e_{t+1}]n_t \qquad (12)$$

where m is the rate of maternity benefit, σ is the rate at which child care is subsidized.

The household first chooses the input mix, for a given n_t and a given e_{t+1}, so as to minimize (12) subject to (10) and (11). Substituting from the input demands in the appendix, the cost of raising a child with no educational attainment as a share of household income is

$$
\begin{aligned}
p^n\,(m, \sigma, B, A) &= (1-m)\hat{z}^q + (1-\sigma)\hat{x}^q \\
&= \left[\left(\frac{(1-m)}{B}\right)^{\frac{\rho}{\rho-1}} + \left(\frac{(1-\sigma)}{A}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}} \qquad (13)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
p^e\,(m, D, E) &= \hat{x}^e + (1-m)\hat{z}^e \\
&= \left[\left(\frac{(1-m)}{D}\right)^{\frac{a}{a-1}} + \left(\frac{1}{E}\right)^{\frac{a}{a-1}}\right]^{\frac{a-1}{a}} \qquad (14)
\end{aligned}
$$

Utility maximisation

In the second period of life, each individual faces the budget constraint:

$$w_t h_t [p^n\,(m, \sigma, B, A) + p^e\,(m, D, E)\,e_{t+1}]n_t - bn_t + (1+\tau)\,c_t \leq w_t h_t \qquad (15)$$

where τ is the rate at which consumption is taxed and b is a lump sum "baby bonus" payment per child.

The government raises revenue from taxation on consumption at the rate, τ.[8] The government budget constraint in per household terms is

$$\tau c_t - \sigma x^q n_t - m z^q n_t - b n_t = 0 \qquad (16)$$

Each household chooses the number and quality of children, to maximise (5) subject to (15) and (16), yielding

$$n_t = \frac{\gamma}{p^n(m, \sigma, B, A) + p^e(m, D, E) e_{t+1} - b/y_t} \qquad (17a)$$

$$e_{t+1} = \frac{h_{t+1}}{h_e} - \left[\frac{p^n(m, \sigma, B, A) - b/y_t}{p^e(m, D, E)} \right] \qquad (17b)$$

where $y_t = w_t h_t$ and $y_t [p^n(m, \sigma, B, A) + p^e(m, D, E) e_{t+1}]$ is the per unit cost of raising children with education, e_{t+1}. We assume that b is sufficiently small: $y_t [p^n(m, \sigma, B, A) + p^e(m, D, E) e_{t+1}] > b$.

By (17a), household fertility is inversely related to the level of education each child receives. Child support, in the form of a maternity benefit, child care benefit and a baby bonus, affect fertility. We analyse the difference in effects in Section 5. By (17b), de_{t+1}/h_{t+1} is unaffected by child support.

By (17a) and (17b), household fertility and the level of education each child receives is unaffected by household income in the absence of a lump sum baby bonus ($b = 0$). Intuitively, because household preferences are log utility, the household will allocate γ proportion of their income to potential income of their children and $(1 - \gamma)$ proportion of their income to consumption. To generate potential income of their children, the household switches between quantity and quality of children, depending on the relative return, which is determined by the rate of technological progress expected over the lifetime of the child. We describe this process in the following section. Household income does not influence the household's choice between quantity and quality of children.

As a portion of household income, the opportunity cost of raising a child is constant. Thus, household income does not effect the individual's choice of total time spent on children and consumption. The income expansion path is horizontal at γ in terms of time devoted to child rearing.

With the introduction of a lump sum baby bonus, the opportunity cost of raising a child, as a portion of income, rises with income. Whether this has a positive (or negative effect) on household fertility choice depends on whether the baby bonus increases proportionately more (or less) than household income.

4   Dynamic System

From (2), the evolution of the economy is described by the evolution of human capital and effective resources, h_t and x_t, respectively. For the following analysis,

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[8]This form of taxation does not affect the household's fertility and education choice.

we note that the size of the working age population at time $t+1$ is

$$L_{t+1} = n_t L_t \tag{18}$$

where $n_t - 1$ is the rate of population growth.

We assume that initially, $b = 0$, which reduces the dimensionality of the system to be studied.

4.1 The evolution of education

Equation (17b) can be rewritten as

$$
\begin{aligned}
& G\left(e_{t+1}, g_{t+1}\right) \\
= \; & \left[p^n\left(.\right) + p^e\left(.\right) e_{t+1}\right] h_e(e_{t+1}, g_{t+1}) - p^e\left(.\right) h(e_{t+1}, g_{t+1}) \\
= \; & 0 \tag{19}
\end{aligned}
$$

where $p^n\left(.\right) = p^n\left(m, \sigma, B, A\right)$ and $p^e\left(.\right) = p^e\left(m, D, E\right)$. Referring to the appendix, total differentiation of (19) yields an implicit functional relationship between a child's education level (e_{t+1}) and technological progress expected to take place between the first and second period of a child's lifetime (g_{t+1}):

$$e_{t+1} = e(g_{t+1}) \tag{20}$$

where $e^{'}\left(g_{t+1}\right) > 0$. We may reasonably assume from diminishing returns to education that this implicit function is also concave: $e^{''}\left(g_{t+1}\right) > 0$.

The reduced form rate of technological progress is

$$g_{t+1} \equiv \frac{J_{t+1} - J_t}{J_t} = g\left(e_t, L_t\right) \tag{21}$$

where $g_i\left(e_t, L_t\right) > 0$ and $g_{ii}\left(e_t, L_t\right) < 0, i = e_t, L_t$.[9]

Thus, the evolution of education is described by

$$e_{t+1} = e(g\left(e_t, L_t\right)) \tag{22}$$

4.2 The evolution of effective resources

The evolution of effective resources per worker is

$$x_{t+1} = \frac{1 + g_{t+1}}{n_t} x_t$$

Substituting from (21), (17a) and (22) yields

$$
\begin{aligned}
x_{t+1} & = \left[1 + g\left(e_t, L_t\right)\right]\left[p^n\left(.\right) + p^e\left(.\right) e(g\left(e_t, L_t\right))\right] x_t/\gamma \\
& = \phi\left(e_t, L_t\right) x_t \tag{23}
\end{aligned}
$$

where, under a system of child support paid in the form of maternity and child care benefits, the evolution of effective resources per worker is driven by education and working age population size.

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[9] (21) is a reduced form implied by the R&D equations of Romer (1990) and Aghion & Howitt (1992).
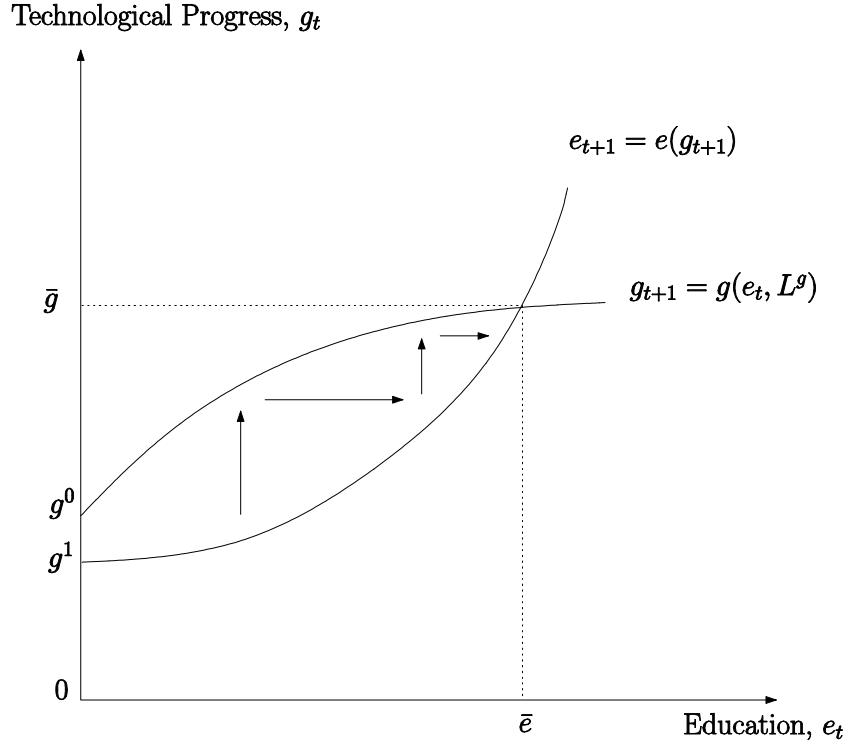
Technological Progress, g_t

Figure 2: Evolution of Technology and Education for a given population

4.3 The dynamic system

Thus, the evolution of the economy is governed by a two-dimensional system

$$\begin{cases} x_{t+1} = \phi\left(e_t, L_t\right) x_t \\ e_{t+1} = e(g\left(e_t, L_t\right)) \end{cases} \tag{24}$$

To simplify the exposition, we assume, as do Galor & Weil (2000), that an increase in population size has no effect on technological progress. That is, $g_L\left(e_t, L_t\right) = 0$. Thus, we analyse the evolution of x_t and e_t for a given population size, L.

Analysis of the dynamic system is simplified by the fact that the joint evolution of e_t and g_t is determined independently of x_t. Technology and education evolve over time so as to satisfy in every period t

$$g_{t+1} \quad = \quad g\left(e_t, L\right) \tag{25a}$$
$$e_{t+1} \quad = \quad e(g_{t+1}) \tag{25b}$$

where both functions are increasing and strictly concave.

Proposition 2 *For a sufficiently large population, the economy converges to a steady state equilibrium with a constant rate of technological progress and constant levels of education (and human capital per worker), $\bar{g}$, $\bar{e}$, and $\bar{h}$, respectively.*

Proof. *Existence* Referring to Figure 2, a plot of both functions in (g_t, e_t) space reveals that increasing and strict concavity of both functions is not sufficient for existence of a steady state equilibrium. Let g^0 and g^1 denote $g_{t+1} = g(0, L)$ and $0 = e(g_{t+1})$, respectively. For $g_{t+1} = g(e_t, L)$ to intersect $e_{t+1} = e(g_{t+1})$ at positive levels of $(\bar{e}, \bar{g})$, we require $g^0 > g^1$. Recognising that $g_{t+1} = g(e_t, L)$ shifts up in (g_t, e_t) space with an increase in L, this condition is satisfied for a sufficiently large population, L^g.

Convergence Referring to Figure 2, a plot of both functions in (g_t, e_t) space reveals that increasing and strict concavity of both functions is sufficient for convergence to a steady state equilibrium. ∎

Corollary 1 (to Proposition 2) *In steady state equilibrium, income per capita grows at the rate of technological progress and fertility is constant.*

Proof. From the production function, $\bar{g}$, $\bar{e}$ and $\bar{h} \Rightarrow (y_{t+1} - y_t)/y_t = \bar{g}$. From (17a), $\bar{e} \Rightarrow \bar{n}$. ∎

5   Types of Child Support

Because this paper focuses on comparing and contrasting the effectiveness of different forms of child support, we assume initially that any increase in child support payment is financed by an increase in the consumption tax rate.

5.1   Fertility Effects

Maternity benefit and Child Care Benefits only

The relevant partial derivatives are, in the absence of a baby bonus,

$$\frac{\partial \bar{n}}{\partial m} = -\frac{\partial \bar{n}}{\partial(1-m)} = \frac{(\hat{z}^{q*} + \hat{z}^{e*}e_{t+1})\bar{n}}{p^n(m,\sigma,B,A) + p^e(m,D,E)\bar{e}} > 0 \qquad (26\text{a})$$

$$\frac{\partial \bar{n}}{\partial \sigma} = -\frac{\partial \bar{n}}{\partial(1-\sigma)} = \frac{\hat{x}^{q*}\bar{n}}{p^n(m,\sigma,B,A) + p^e(m,D,E)\bar{e}} > 0 \qquad (26\text{b})$$

Dividing (26a) by (26b) and comparing proportionate fertility effects,

$$\frac{\partial \bar{n}}{\partial m}\frac{m}{\bar{n}} > \frac{\partial \bar{n}}{\partial \sigma}\frac{\sigma}{\bar{n}} \Leftrightarrow m(\hat{z}^{q*} + \hat{z}^{e*}\bar{e}) > \sigma\hat{x}^{q*}$$

which yields the following

Proposition 3 (Maternity benefit versus Child-care benefit) *An increase in m boosts fertility proportionately more than does an equivalent proportionate increase in σ if and only if $m(\hat{z}^{q*} + \hat{z}^{e*}\bar{e}) > \sigma\hat{x}^{q*}$*

That is, fertility responds more to an increase in a maternity benefit than to a proportionate increase in a child care benefit when the subsidized time input of raising a child with education, $\bar{e}$, exceeds the subsidised amount of bought in child care per child.

Thus, by recognising that a paid maternity benefit subsidises parental time, used in not only child quantity but also quality, we conclude that a maternity benefit is a more effective lever in boosting fertility than a child care benefit.

To both identify critical values satisfying the condition in Proposition 3 and test the robustness of our conclusion, we explore the possibility that parental time devoted to education is relatively small. Consider the case where $\hat{z}^{e*} \to 0$. To simplify the exposition, we consider the elasticity of fertility with respect to $(1-m)$ and $(1-\sigma)$, respectively:

$$-\frac{\partial \bar{n}.(1-m)}{\partial(1-m).\bar{n}} > -\frac{\partial \bar{n}.(1-\sigma)}{\partial(1-\sigma).\bar{n}} \Leftrightarrow (1-m)\hat{z}^{q*} > (1-\sigma)\hat{x}^{q*}$$

which, together with the input demands (see appendix), implies that m has greater efficacy than σ in raising fertility if and only if $(1-m)\hat{x}^{q*}/(1-\sigma)\hat{z}^{q*} = \left[\frac{A(1-m)}{B(1-\sigma)}\right]^{\rho/(1-\rho)} < 1$.

As $\hat{z}^{e*} \to 0$, we can identify a critical value of maternity benefit, m, for given values of A, B and σ, or a critical ratio of $(1-\sigma)/(1-m)$, for a given relative efficiency of bought in child care, (A/B), which imply

$$\hat{x}^{q*}/\hat{z}^{q*} = \left[\frac{A(1-m)}{B(1-\sigma)}\right]^{\rho/(1-\rho)} = 1$$

Households substitute bought in child care for parental time in child rearing, suggesting $0 < \rho < 1$.

Corollary 2 (to Proposition 3) *Assuming $\varepsilon > 1\,(0 < \rho < 1)$, $\hat{z}^{q*} > \hat{x}^{q*}$ if and only if $(1-\sigma)/(1-m) > (A/B)$ or, equivalently, $m > m^*$ where $m^* = 1 - (1-\sigma)\frac{B}{A}$*

This corollary suggests that the higher the rate of child care benefit (σ) and the higher the relative efficiency of bought in child care $(A > B)$, the higher the rate of maternity benefit required to induce a proportionately larger increase in fertility than that induced by a proportionately equivalent increase in the rate of child care benefit. The existence of a child care industry suggests $A > B$. By implication, the higher m relative to σ, the more likely it is that m is the more effective lever to raise fertility.

To indicate whether the existing maternity benefit is sufficiently high to render it the more effective lever, we provide an illustrative example, which is representative of the Australian economy.

Example 1 *Consider an average annual income of $60,000 and a minimum annual income of $30,000. Eighteen weeks paid maternity leave at minimum wages translates to $m = \$10,000/\$60,000 = 1/6$. An annual child care cost of $15,000 with a government rebate of $7,500 translates to $\sigma = 1/8$.*

To the extent that these payments are used to raise fertility, the corollary to Proposition 3 indicates which is the most effective lever. Bought in child care services would have to be less than 5% more efficient than parental time $(A < 1.05B)$, for m to be the more effective lever at the existing rate. To the extent that the relative efficiency of bought in child care exceeds 5%, we can infer that a higher rate of maternity benefit will render it more effective.

Before drawing too many inferences from this Proposition and its corollary, we should note some critical assumptions. For instance, we model market clearing demand for parental time and bought in child care in child rearing. Importantly, we assume that the supply of bought in child care is elastic. In Australia, as in many European countries, child care markets work differently than in the United States: they are characterized by heavy subsidies, which also means excess demand can lead to a rationing problem. When analysing the responsiveness of $\hat{x}^{q*}/\hat{z}^{q*}$ to changes in benefits and efficiency, we could incorporate a supply constraint in bought in child care (i.e. $\hat{x}^{q*} \leq \hat{x}^c$). Under this scenario, m^* would be lower and we could contrast the effects of reduced rationing and increased rate of child care benefit on $\bar{n}$ and $\hat{z}^{q*}$.

Lump Sum Baby Bonus

With the introduction of a baby bonus,

$$\frac{d\bar{n}}{db} = \frac{\bar{n}}{p\left(m, \sigma, B, A\right) + p^e\left(m, D, E\right)\bar{e} - b/y\left(\bar{g}\right)} \left[d\left(\frac{b}{y}\right)\right] \qquad (27)$$

where

$$\frac{d\bar{n}}{db} \gtreqless 0 \Leftrightarrow \frac{db}{b} \gtreqless \frac{dy}{y}$$

which we summarize with the following

Proposition 4 (Lump Sum Baby Bonus) *The introduction of a baby bonus (b) is predicted to increase fertility. Any further increase in fertility will require b to be rising faster than y.*

In contrast to a maternity benefit and a child care benefit, an increase in the lump sum baby bonus has an ambiguous effect on fertility when income per capita is rising. The intuition for this result lies in the fact that a lump sum payment implies the net cost of raising children as a proportion of income rises as household income rises (refer to discussion accompanying (17a) and (17b)). In a steady state equilibrium, household income grows at the rate of technological progress. The implication is that the lump sum nature of the baby bonus, instead of an income related program, may explain why we may expect to see a limited rise in fertility due to the baby bonus payment.

As noted in the previous section, the introduction of a lump sum baby bonus increases the dimensionality of the dynamic system. The dynamic system is outlined in the appendix.

5.2   Education Effects

By (17b), the steady state level of education, $\bar{e}$, will be decreasing in p^n (.) and increasing in p^e (.):

$$\frac{\partial \bar{e}}{\partial p^n (m, \sigma, B, A)} \quad < \quad 0 \qquad (28a)$$

$$\frac{\partial \bar{e}}{\partial p^e (m, D, E)} \quad > \quad 0 \qquad (28b)$$

It is straightforward to show that $\partial p^n (.) / \partial \sigma < 0$ and $\partial p^i (.) / \partial m < 0$ $(i = n, e).$, since any rise in input demand due to a reduction in price (for instance, a rise in $\hat{x}^{q*}$ due to a reduction in $(1 - \sigma)$) will be less than the reduction price. We summarise this discussion with the following

Proposition 5 *An increase in σ raises the level of education*

The effect on $\bar{e}$ of an increase in m depends on the relative decrease in $p^n (.) - b/y_t$ and $p^e (.)$. Since parental education and bought in education are more likely complementary, whereas parental child care and bought in child care are substitutable, we may reasonably intuit that $p^n (.)$ will decrease relative to $p^e (.)$ when we subsidise parental time:

$$p^n (.) \quad = \quad (1 - m) \downarrow (\hat{z}^{q*}) \uparrow + (1 - \sigma) (\hat{x}^{q*}) \downarrow \qquad (29a)$$

$$p^e (.) \quad = \quad (1 - m) \downarrow (\hat{z}^{e*}) \uparrow + (\hat{x}^{e*}) \uparrow \qquad (29b)$$

By (17b), if $p^n (.)$ and $p^e (.)$ fall by the same proportion, then an increase in m has a positive effect on $\bar{e}$. Thus, a disproportionately larger fall in $p^n (.)$ is sufficient but not necessary for $\partial \bar{e} / \partial m > 0$. We summarise this discussion with the following

Proposition 6 *If $0 < \rho < 1$ and $-\infty < a < 0$, then an increase in m raises the level of education*

Thus, we find that increasing child support in various forms also raises the steady state level of education. Specifically, increasing the:

- child care benefit raises both fertility and education;

- maternity benefit raises fertility and raises the level of education if parental time and bought in education are complementary;

- lump sum baby bonus raises both fertility and education if the baby bonus is rising faster than household incomes.

Our analysis of comparative statics can be extended by relaxing the assumption that any increase in child support is financed by an increase in the consumption tax rate. For example, we can analyse the effect of an exogenous increase in maternity benefit, with parental time input, bought in services and child care subsidies as endogenously determined.

6   Conclusion

This paper analyses the effects of three forms of child support (paid maternity leave, child care subsidies and a baby bonus) in a model of endogenous fertility in which technological progress induces households to choose fewer, but better educated, children. The comparative static analysis predicts that:

1. Whilst the introduction of a lump sum baby bonus increases fertility, any further increase in fertility would require the lump sum baby bonus to rise faster than household incomes.

2. If parental time is used to rear and educate children, paid maternity leave may be more effective than child care subsidies in raising fertility.

3. The relative effectiveness of child care subsidies and paid maternity leave in raising fertility depends on the relative efficiency of child care and the prevailing rates of subsidy and maternity pay.

4. The three forms of child support increase not only the level of fertility but also the level of education.

Existing theoretical models find that, regardless of whether it is the most effective lever, raising the lump sum baby bonus would unambiguously increase fertility (Apps & Rees 2004). In contrast, we find that raising the lump sum baby bonus will have no effect on fertility if the rise is proportionate to the rise in household income. We also find that the relative effectiveness of raising the rate of maternity benefit and child care benefit depends on the relative efficiency of child care and prevailing input prices. Our final prediction that child support also raises the level of education suggests that policies designed to boost fertility may also enhance economic growth. Using our framework to analyse the overall impact of child support policies on economic growth is a challenging, but feasible direction for further research.

A   Appendix

A.1   Input Demands

The input demands for parental time and bought in services used in child care and education, respectively, are

$$z^{q*} = \left[\left(\frac{(1-m)}{B} \right)^{\frac{\rho}{\rho-1}} + \left(\frac{(1-\sigma)}{A} \right)^{\frac{\rho}{\rho-1}} \right]^{-\frac{1}{\rho}} (1-m)^{\frac{1}{\rho-1}} B^{-\frac{\rho}{\rho-1}} n_t$$

$$x^{q*} = \left[\left(\frac{(1-m)}{B} \right)^{\frac{\rho}{\rho-1}} + \left(\frac{(1-\sigma)}{A} \right)^{\frac{\rho}{\rho-1}} \right]^{-\frac{1}{\rho}} (1-\sigma)^{\frac{1}{\rho-1}} A^{-\frac{\rho}{\rho-1}} n_t$$

$$z^{e*} = \left[\left(\frac{(1-m)}{D}\right)^{\frac{a}{a-1}} + \left(\frac{1}{E}\right)^{\frac{a}{a-1}}\right]^{-\frac{1}{a}} (1-m)^{\frac{1}{a-1}} D^{-\frac{a}{a-1}} e_{t+1}$$

$$x^{e*} = \left[\left(\frac{(1-m)}{D}\right)^{\frac{a}{a-1}} + \left(\frac{1}{E}\right)^{\frac{a}{a-1}}\right]^{-\frac{1}{a}} E^{-\frac{a}{a-1}} e_{t+1}$$

A.2 Derivation of equation (20)

The total derivative of (19) is

$$\frac{\partial G}{\partial g_{t+1}} dg_{t+1} + \frac{\partial G}{\partial e_{t+1}} de_{t+1} = 0$$

where

$$\frac{\partial G}{\partial g_{t+1}} = [p^n(.) + p^e(.) e_{t+1}] h_{eg} - p^e(.) h_g > 0$$

$$\frac{\partial G}{\partial e_{t+1}} = [p^n(.) + p^e(.) e_{t+1}] h_{ee} < 0$$

implying

$$\frac{de_{t+1}}{dg_{t+1}} > 0$$

A.3 Dynamic system with baby bonus

$$
\begin{aligned}
& G\left(e_{t+1}, g_{t+1}\right) \\
= & \left[p^n(.) + p^e(.) e_{t+1} - b/y(e_t, g_t, x_t)\right] h_e\left(e_{t+1}, g_{t+1}\right) - p^e(.) h\left(e_{t+1}, g_{t+1}\right) \\
= & 0
\end{aligned}
$$

$$
x_{t+1} = \begin{cases}
[1 + g(e_t, L_t)][p^n(\sigma, m, A, B) + p^e(m, D, E) e(g(e_t, L_t))] x_t/\gamma \\
\qquad = \phi^a(e_t, L_t) x_t \quad \text{if } b = 0 \\
\\
[1 + g(e_t, L_t)][p^n(\sigma, m, A, B) + p^e(m, D, E) e(g(e_t, L_t)) - b/y(e_t, g_t, x_t)] x_t/\gamma \\
\qquad = \phi^b(e_t, g_t, x_t, L_t) x_t \quad \text{if } b > 0
\end{cases}
$$

which implies the evolution of the economy is governed by either a two-dimensional system or a three-dimensional system depending on whether or not the initial system of child support comprises a lump sum baby bonus payment.

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