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WELFARE ENHANCING MERGERS UNDER PRODUCT DIFFERENTIATION

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Welfare Enhancing Mergers under Product Differentiation*

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Abstract

This paper considers a model of duopoly with differentiated products to examine the welfare effects of a merger between two asymmetric firms. We find that for quantity competition, the parameter range for welfare enhancing merger widens if the products are closer substitutes. On the other hand, mergers are never welfare enhancing in this setting when firms compete in prices.

JEL Classification: L11, L12

Keywords: merger, differentiated products, cost asymmetry

1 Introduction

The aim of this paper is to explore the welfare consequences of mergers when products are differentiated and firms compete in prices or quantities. Although the possibility of welfare enhancing mergers is not new, by and large existing results focus on homogenous goods. For example, it is known that for homogenous goods, when the cost difference is substantial, a merger can increase social welfare by improving allocative efficiency in the Cournot equilibrium. In a homogenous product set-up, Perry and Porter (1985)

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and Farrell and Shapiro (1990) provide sufficient conditions for profitable mergers to raise welfare.[1] Our analysis complements the existing literature and extends these results to differentiated goods.

Notwithstanding the research focus on homogenous goods, most mergers involve differentiated products. For example, an examination of the public register of merger decisions for the Australian regulator[2] suggests that only a small fraction of the nearly 500 merger decisions since 2004 have involved markets for homogenous goods. Moreover, mergers continue to be an important part of firms' growth strategies and merger activity does not seem to have significantly slowed down.[3]

This paper follows the set up in Singh and Vives (1984) and Zanchettin (2006) to derive the analytical condition for welfare enhancing mergers with differentiated products when firms compete in quantities. We also show that if firms compete in prices, mergers always reduce total welfare. The positive welfare effect of a merger comes from improved efficiency by allocating more output to the more efficient firm. Since the efficient firm always produces more under price competition than under quantity competition, it follows then that the efficiency gains from a merger are lower under price competition than under quantity competition. This gives rise to the first policy implication: the intensity of product market competition is an important factor in determining the welfare consequences of horizontal mergers. In the presence of cost asymmetry, more intense competition (in this case, Bertrand rather than Cournot) and horizontal mergers can be viewed as alternative means to achieve productive efficiency. This suggests that the antitrust authority should view more favourably horizontal mergers in industries where the product market competition is not intense.[4]

For quantity competition, we show that the parameter range for the merger to be welfare enhancing widens if the products are closer substitutes. Traditionally mergers between firms who offer products which are not close

[1] Hennessy (2000) demonstrates that for a family of well-behaved demand functions the class of profitable mergers – absent cost efficiencies – is larger than the standard analysis of mergers would suggest.

[2] Available at www.accc.gov.au.

[3] See the retrospective on mergers by Sherer (2006).

[4] We thank a referee for suggesting this point.

substitutes are viewed more favorably by the competition authorities. For example, the US merger guideline states that "The price rise [following a merger] will be greater the closer substitutes are the products of the merging firms". This is based on the premise that the merged entity would have more incentive to restrict outputs when the products are closer substitutes. Our results suggest that, when the cost asymmetry is high, it is possible for mergers between firms offering quite different goods to be more harmful to total welfare. As the merged entity shuts down the production in one market, there is greater loss in consumer surplus.

As we focus on a merger to monopoly, consumers' surplus necessarily (weakly) falls regardless of the nature of competition. Thus, our results establish conditions under which efficiency gains are translated into a sufficient increase in profits so that the total surplus increases with the merger. There are three important reasons why such focus on a merger to monopoly and on total surplus is justified. First, many academic economists support the application of an overall social welfare standard and indeed there are antitrust enforcers that use such standard (e.g., Australia's ACCC).[5] Second, our results ought to carry through beyond the case of a merger to monopoly. To the extent that price competition yields lower prices in general, a merger that generates a certain efficiency gain would more likely be welfare-enhancing under quantity competition than under price competition as the starting prices would be lower under the latter.[6] Third, there are no general results in the literature about welfare effects of mergers in the presence of asymmetries (either in costs or demand or both).[7] Our contribution then needs to be placed in this context.

We present the model set up in the next section and solve first for the optimisation problem the merged entity faces. Sections 3 and 4 analyse the quantity competition and price competition games in turn and derive the

[5] See, for example, Coate (2005) for an overview of papers addressing the two standards.

[6] For the simple set-up of a two goods model, the results would carry through to the case of more than 2 firms in the per-merger market. For Cournot competition, we face the same trade-off between price and efficiency, and for some parameter ranges, we would have welfare enhancing mergers. We briefly discuss the case for Bertrand competition in Section 4.1.

[7] See, for example, Motta (2004) for a review of models of horizontal mergers.

welfare results of mergers. The final section presents our conclusions.

2 The Model

Let the representative consumer's utility be a quadratic function of two differentiated products,

$$U = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + m, \tag{1}$$

where q_1 and q_2 are the quantities of the two differentiated goods and m is a numeraire good. The parameter γ measures the degree of product differentiation. If $\gamma = 0$, the demand for the two goods are independent. We assume that the two goods are substitutes so that $0 \leq \gamma \leq 1$.[8]

We consider the set up with one monopoly firm in each sector. The inverse demand curves for the two goods are

$$p_1 = \alpha_1 - (q_1 + \gamma q_2) \tag{2}$$

and

$$p_2 = \alpha_2 - (\gamma q_1 + q_2). \tag{3}$$

We assume that the marginal costs of production in markets 1 and 2 are equal to c_1 and c_2, respectively, and that there are no fixed costs.

Following Zanchettin (2006), we define an index a to measure the asymmetry between the two firms.

Definition 1 *Let $a \equiv (\alpha_1 - c_1) - (\alpha_2 - c_2)$ and $\alpha_1 - c_1 = 1$. Without loss of generality, assume $a \geq 0$.*

For $a = 0$, two firms are symmetric. For $a \geq 1 - \frac{\gamma}{2}$, the asymmetry between the firms is so large that in equilibrium firm 1 sets its quantity at the monopoly level, $q_1 = q_1^M$, and at that quantity firm 2 is priced out of the market (i.e., $q_2 = 0$). We focus on the case where $a \leq 1 - \frac{\gamma}{2}$.

From the utility function (1), the total surplus is

$$TS = q_1 + (1 - a)\, q_2 - \frac{1}{2}(q_1 + q_2)^2 + (1 - \gamma)\, q_1 q_2. \tag{4}$$

[8] Mergers of firms offering complementary goods are always welfare enhancing in this set-up.

Consumer surplus is defined as

$$CS = TS - \pi_1 - \pi_2, \tag{5}$$

where π_1 and π_2 are the two firms' profits.

We denote by Q_i (q_i) the quantity choice of the merged entity (differentiated duopolists) in market i, $i \in \{1, 2\}$. For the merged entity, who is a monopolist over markets 1 and 2, setting quantity is equivalent to setting price. The merged entity's optimisation problem is:

$$\max_{\{Q_1, Q_2\}} (p_1 - c_1) Q_1 + (p_2 - c_2) Q_2. \tag{6}$$

The first order conditions yield

$$Q_1 = \frac{1 - 2\gamma Q_2}{2} \text{ and } Q_2 = \frac{(1-a) - 2\gamma Q_1}{2}. \tag{7}$$

For $a < 1 - \gamma$, the solution is interior:

$$Q_1^* = \frac{1 - \gamma (1-a)}{2(1-\gamma^2)} \quad \text{and} \quad Q_2^* = \frac{(1-a) - \gamma}{2(1-\gamma^2)}. \tag{8}$$

This gives the merged entity's profit equal to $\Pi = \frac{1-2\gamma(1-a)+(1-a)^2}{4(1-\gamma)(1+\gamma)}$, and the resulting consumer surplus is $CS = \frac{(1-a)^2-2\gamma(1-a)+1}{8(1+\gamma)(1-\gamma)}$.

For $a \geq 1 - \gamma$, the first order conditions yield $Q_2^* = 0$ and $Q_1^* = \frac{1}{2}$. The merged firm profit is equal to $\frac{1}{4}$, and the consumer surplus is equal to $\frac{1}{8}$.

3 Quantity competition

For a differentiated duopoly competing by setting quantities, each firm i solves $\max_{q_i} (p_i - c_i) q_i$. This yields the best response function:

$$q_i = \frac{\alpha_i - \gamma q_j - c_i}{2}, \; i, j \in \{1, 2\} \text{ and } i \neq j. \tag{9}$$

For $a \leq 1 - \frac{\gamma}{2}$, both firms produce positive outputs:

$$q_1^C = \frac{2 - \gamma(1-a)}{4 - \gamma^2} \text{ and } q_2^C = \frac{2(1-a) - \gamma}{4 - \gamma^2}. \tag{10}$$

The resulting profits are

$$\pi_1 = \left(\frac{2 - \gamma(1-a)}{4 - \gamma^2}\right)^2 \text{ and } \pi_2 = \left(\frac{2(1-a) - \gamma}{4 - \gamma^2}\right)^2. \tag{11}$$

This gives consumer surplus

$$CS = \frac{\left(4 - 3\gamma^2\right)(1-a)^2 + 2(1-a)\gamma^3 + 4 - 3\gamma^2}{2(\gamma+2)^2(2-\gamma)^2}. \tag{12}$$

Combining this with the output decision of the merged entity analysed in Section 2, we plot the different cases, depending on whether or not there is a corner solution, in Figure 1. Our analysis focuses on cases 1 and 2.

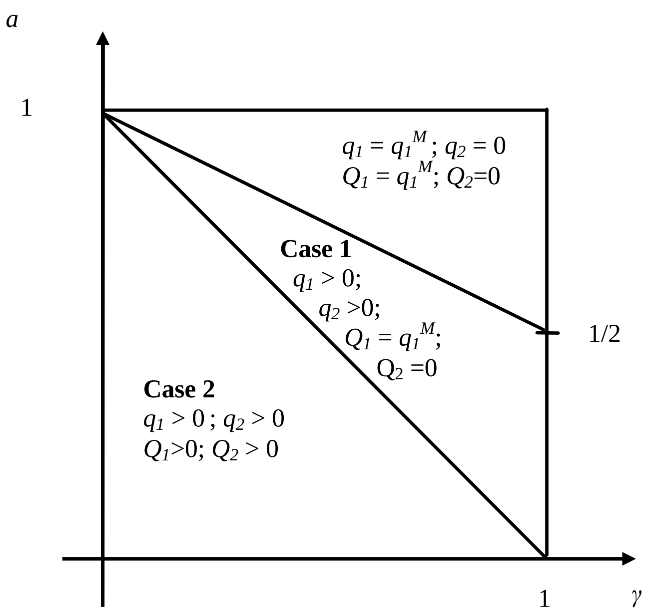

Figure 1: Output equilibirum in (γ, a) space under quantity competition.

3.1 Welfare Results

For the entire parameter range, industry profit increases and consumer surplus decreases after the merger. In any two-to-one merger, the merged entity can always mimic the pre-merger behaviour of the firms and, therefore, profits have to be (weakly) higher. For substitute goods, this means (weakly) higher price and, therefore, lower consumer surplus. However, we show below that under quantity competition, for a given parameter range, two-to-one mergers can be welfare improving; the increase in profits dominate the fall in consumer surplus.

Proposition 1 *For the parameter range,* $\frac{(2-\gamma)\left(12-4\gamma-3\gamma^2\right)}{2(12-\gamma^2)} \leq a \leq 1-\frac{\gamma}{2}$*, the total surplus increases post merger under quantity competition.*

Proof. For $1-\gamma \leq a \leq 1-\frac{\gamma}{2}$, for the differentiated duopolists, $q_1 = q_1^C$ and $q_2 = q_2^C$. This gives $\pi_1 = \frac{(2-\gamma(1-a))^2}{(4-\gamma^2)^2}$ and $\pi_2 = \frac{(2(1-a)-\gamma)^2}{(4-\gamma^2)^2}$. The resulting consumer surplus is $CS = \frac{\left(4-3\gamma^2\right)(1-a)^2+2(1-a)\gamma^3+4-3\gamma^2}{2(\gamma+2)^2(2-\gamma)^2}$.

For the merged entity, $Q_1 = \frac{1}{2}$ and $Q_2 = 0$. The merged firm's profit is equal to $\frac{1}{4}$ and the consumer surplus is equal to $\frac{1}{8}$.

The total surplus post merger increases if

$$\begin{aligned} \frac{3}{8} \quad \geq \quad & \frac{(2-\gamma(1-a))^2}{(4-\gamma^2)^2} + \frac{(2(1-a)-\gamma)^2}{(4-\gamma^2)^2} \\ & + \frac{\left(4-3\gamma^2\right)(1-a)^2 + 2(1-a)\gamma^3 + 4 - 3\gamma^2}{2(\gamma+2)^2(2-\gamma)^2}. \end{aligned} \tag{13}$$

This holds for $a \geq \frac{(2-\gamma)\left(12-4\gamma-3\gamma^2\right)}{(24-2\gamma^2)}$. Finally, note that $1-\gamma \leq \frac{(2-\gamma)\left(12-4\gamma-3\gamma^2\right)}{(24-2\gamma^2)} \leq 1-\frac{\gamma}{2}$. ■

The possibility of welfare gain comes from the efficiency gain of shutting down production of the high cost product. It follows that there is welfare gain only if the asymmetry between firms is sufficiently large. Furthermore, the lower bound of the cost asymmetry required for welfare enhancing merger depends on the degree of product substitutability. The band for a welfare increasing merger is equal to

$$\Delta = 1 - \frac{\gamma}{2} - \frac{(2-\gamma)\left(12-4\gamma-3\gamma^2\right)}{(24-2\gamma^2)}. \tag{14}$$

As the products become closer substitutes, the parameter range for welfare enhancing merger widens:

$$\frac{\partial\Delta}{\partial\gamma} = \frac{\left(\gamma^4 - 32\gamma^2 + 48\right)}{(12-\gamma^2)^2} \geq 0. \tag{15}$$

Our result is in contrast to the traditional view that mergers between firms who offer products which are not close substitutes should be viewed more favorably. Once we take into consideration corner solutions, a merger between firms offering quite different goods may be more harmful to total welfare since there is greater consumer surplus loss if the merged entity ceases production in one of the markets.

4 Price competition

From the inverse demand curves given in Equations (2) and (3), we obtain the demand curves $q_1 = \frac{(\alpha_1 - p_1) - \gamma(\alpha_2 - p_2)}{(1-\gamma^2)}$ and $q_2 = \frac{(\alpha_2 - p_2) - \gamma(\alpha_1 - p_1)}{(1-\gamma^2)}$. Firm i's optimisation problem can be written as $\max_{p_i} (p_i - c_i) q_i$. For $p_1 > c_1$ and $p_2 > c_2$, this yields the following best response functions:

$$p_1 = \frac{\alpha_1 + c_1 - \gamma(\alpha_2 - p_2)}{2} \text{ and } p_2 = \frac{\alpha_2 + c_2 - \gamma(\alpha_1 - p_1)}{2}. \tag{16}$$

This gives the interior solutions

$$p_1^B = \frac{2\alpha_1 + 2c_1 - \gamma^2\alpha_1 - \gamma(1-a)}{(2+\gamma)(2-\gamma)} \text{ and } p_2^B = \frac{2\alpha_2 + 2c_2 - \gamma^2\alpha_2 - \gamma}{(2+\gamma)(2-\gamma)}. \tag{17}$$

Note that the assumption $a \geq 0$ implies that $p_1 - c_1 \geq p_2 - c_2$.

In contrast to the quantity setting game, under differentiated Bertrand, the efficient firm may be able to charge a low enough price to drive the inefficient firm out of the market even if $a < 1 - \frac{\gamma}{2}$. This will occur when $q_2 \leq 0$ or $\frac{(\alpha_2 - p_2) - \gamma(\alpha_1 - p_1)}{(1-\gamma^2)} \leq 0$. This holds for $p_2 \geq \alpha_2 - \gamma(\alpha_1 - p_1)$. To enforce this price below c_2, firm 1 needs to choose a price such that: $p_1 \leq \alpha_1 - \frac{1-a}{\gamma}$. In this case, firm 1 charges a price just low enough to drive firm 2 out of the market. Zanchettin (2006) terms the pricing behaviour in this parameter range the limit-pricing equilibrium. For $1 - \frac{\gamma}{2-\gamma^2} \leq a \leq 1 - \frac{\gamma}{2}$, the equilibrium is

$$p_1 - c_1 = \frac{\gamma - (1-a)}{\gamma},\ q_1 = \frac{1-a}{\gamma}, \text{ and } p_2 - c_2 = q_2 = 0. \tag{18}$$

Note that in this parameter range, for quantity competition, both firms produce positive output. The ability of firm 1 to exercise limit pricing is the key for Zanchettin's result that the efficient firm prefers price competition.

For $a < 1 - \frac{\gamma}{2-\gamma^2}$, we have the usual interior solution for differentiated Bertrand with the equilibrium $p_1 = p_1^B$ and $p_2 = p_2^B$. We plot the price competition equilibrium against the merged entity's optimal choices in the following diagram. Focusing on the parameter range $a < 1 - \frac{\gamma}{2}$, there are three cases according to the nature of equilibrium outcome[9]: (I) limit pricing behaviour: $q_1 > q_1^M$, $q_2 = 0$, $Q_1 = q_1^M$, and $Q_2 = 0$; (II) $q_1 > 0$, $q_2 > 0$,

[9] As the previous notations, q_i, $i \in \{1, 2\}$, denotes the output of the duopolist i while Q_i, $i \in \{1, 2\}$, denotes the merged entity's output choice in market i.

$Q_1 = q_1^M$, and $Q_2 = 0$; (III) interior solution: $q_1 > 0$, $q_2 > 0$, $Q_1 > 0$, and $Q_2 > 0$.

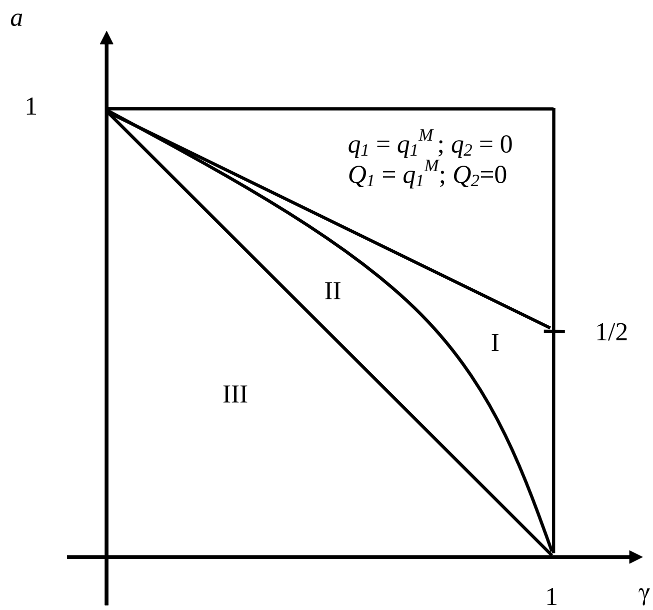

Figure 2: Output equilibirum in (γ, a) space under price competition.

4.1 Welfare Results

While two-to-one mergers can be welfare enhancing under quantity competition, the next result shows that this is not the case for price competition.

Proposition 2 *When goods are substitutes, a merger from duopoly to monopoly always reduces total welfare if firms compete in prices.*

Proof. See the appendix. ■

The intuition for this result relies on Zanchettin's (2006) observation that the efficient firm produces more under price competition than under quantity competition over the entire parameter space. This is most apparent in the limit pricing range where the inefficient firm is driven out of the market under price competition while it remains active under quantity competition.

Therefore, the efficiency gain from a merger is lower under price competition and not sufficient to outweigh the decline in consumer surplus post merger.

In this paper, we discuss the simple case of duopoly competition. The same intuition would carry through to more than 2 firm analysis. If as we specify in this paper, the asymmetry is between market 1 and 2 with homogenous firms within each market, Bertrand competition within each market would have the pre-merger market prices equal to the respective marginal costs. Mergers in this set-up would not generate any efficiency gains. If there is cost asymmetry between firms within each market with more than 1 firm offering each product, Bertrand competition within each market would again ensure that the low-cost firm would sell to all the consumers[10]. In this case, mergers again would not generate any efficiency gain. This suggests that with price competition, it is unlikely that mergers would increase total welfare. If we consider N differentiated products with only 1 firm offering each product, although the analysis would be slightly different, we should still expect that welfare improving mergers would be less likely to occur compared with the case when firms compete in quantities.[11]

5 Conclusion and Discussion

In this paper we develop the analytical condition for a merger of differentiated goods duopolists to be welfare enhancing. If firms compete in prices, a merger between duopolists always reduces total welfare. For quantity setting firms, a merger between firms offering substitute goods increases social welfare if the cost difference is substantial. Furthermore, the parameter range for the merger to be welfare enhancing widens if the products are closer substitutes. Traditionally mergers between firms who offer products which are not close substitutes are viewed more favorably by the competition authorities. This is based on the premise that the merged entity would have more incentive to restrict outputs when the products are closer substitutes. Our results suggest that, in contrast, when the cost asymmetry is high, mergers

[10] With the premerger price slightly lower than the high-cost firm's marginal cost.

[11] Häckner (2000) compares quantity competition and price competition with N differentiated firms. The welfare analysis is absent in the paper.

between firms offering quite different goods may be more harmful to welfare since as the merged entity shuts down the production in one market, there is greater loss in consumer surplus.

We use the framework of a representative consumer to study the effects of mergers with product differentiation. Given the focus of the comparison between price and quantity competition, we argue that for available frameworks on product differentiation, this is the suitable one to use. For example, another popular model for analysing product differentiation is the Hotelling address model. However, in the Hotelling model, it is not clear on what happens if firms choose quantities as the strategic variable. When the two firms sells to the same consumer, with inelastic demand and without any tie-breaking rules, it is unclear what the equilibrium would be in the quantity setting game. As an extension to the current framework, we are now working on a model with both inter- and intra-market competition.

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6 Appendix

Proof of Proposition 2:. **Case 1:** $1-\frac{\gamma}{2-\gamma^2} \leq a \leq 1-\frac{\gamma}{2}$: For the duopolists, $p_1 = \alpha_1 - \frac{1-a}{\gamma}$, $\pi_1 = \frac{(\gamma+a-1)(1-a)}{\gamma^2}$, and $p_2 - c_2 = q_2 = 0$. Consumer surplus is $CS = \frac{(1-a)^2}{2\gamma^2}$. For the the merged entity, the equilibrium is $Q_1 = \frac{1}{2}$ and $Q_2 = 0$. Total surplus goes down post merger if

$$\frac{3}{8} \leq \frac{(\gamma+a-1)(1-a)}{\gamma^2} + \frac{(1-a)^2}{2\gamma^2}. \tag{19}$$

This holds since in this case since $(2a+3\gamma-2)(2a+\gamma-2) \leq 0$.

Case 2: $1-\gamma \leq a \leq 1-\frac{\gamma}{2-\gamma^2}$: For the duopolists, $p_1 = p_1^B$ and $p_2 = p_2^B$. This gives

$$\pi_1 = \frac{\left(a\gamma-\gamma-\gamma^2+2\right)^2}{(\gamma+2)^2(2-\gamma)^2(1-\gamma^2)} \text{ and } \pi_2 = \frac{\left(a\gamma^2-\gamma-\gamma^2-2a+2\right)^2}{(\gamma+2)^2(2-\gamma)^2(1-\gamma^2)}. \tag{20}$$

Consumer surplus is

$$CS = \frac{a^2\left(4-3\gamma^2\right) - 2(1-\gamma)(2+\gamma)^2 a + 2(1-\gamma)(2+\gamma)^2}{2(\gamma+2)^2(2-\gamma)^2(1+\gamma)(1-\gamma)}. \tag{21}$$

The merged entity produces $Q_1 = \frac{1}{2}$ and $Q_2 = 0$.

The total surplus goes down post merger if

$$\begin{aligned}\frac{3}{8} \quad \leq \quad & \frac{\left(a\gamma-\gamma-\gamma^2+2\right)^2}{(\gamma+2)^2(2-\gamma)^2(\gamma+1)(1-\gamma)} + \frac{\left(a\gamma^2-\gamma-\gamma^2-2a+2\right)^2}{(\gamma+2)^2(2-\gamma)^2(\gamma+1)(1-\gamma)} \\ & + \frac{a^2\left(4-3\gamma^2\right) - 2(1-\gamma)(2+\gamma)^2 a + 2(1-\gamma)(2+\gamma)^2}{2(\gamma+2)^2(2-\gamma)^2(1+\gamma)(1-\gamma)}.\end{aligned} \tag{22}$$

This holds since

$$
\begin{aligned}
& 4\left(2\gamma^4 - 9\gamma^2 + 12\right)a^2 + 8(1-\gamma)(2\gamma-3)(\gamma+2)^2 a \\
& -(1-\gamma)\left(16\gamma - 9\gamma^2 + 3\gamma^3 - 12\right)(\gamma+2)^2 \\
\geq\ & 0. \qquad (23)
\end{aligned}
$$

Case 3: $a \leq 1-\gamma$: For the duopolists, $p_1 = p_1^B$ and $p_2 = p_2^B$. The merged entity produces $Q_1 = Q_1^*$ and $Q_2 = Q_2^*$ as given in Equation 8.

The total surplus goes down post merger if

$$
\begin{aligned}
& \frac{1-2\gamma(1-a)+(1-a)^2}{4(1-\gamma)(1+\gamma)} + \frac{(1-a)^2 - 2\gamma(1-a)+1}{8(1+\gamma)(1-\gamma)} \\
\leq\ & \frac{\left(a\gamma - \gamma - \gamma^2 + 2\right)^2}{(\gamma+2)^2(2-\gamma)^2(\gamma+1)(1-\gamma)} + \frac{\left(a\gamma^2 - \gamma - \gamma^2 - 2a + 2\right)^2}{(\gamma+2)^2(2-\gamma)^2(\gamma+1)(1-\gamma)} \\
& + \frac{a^2\left(4-3\gamma^2\right) - 2(1-\gamma)(2+\gamma)^2 a + 2(1-\gamma)(2+\gamma)^2}{2(\gamma+2)^2(2-\gamma)^2(1+\gamma)(1-\gamma)}. \qquad (24)
\end{aligned}
$$

This holds for

$$
a \leq \frac{-(1-\gamma)(4-3\gamma)(\gamma+2)^2 + \sqrt{(1-\gamma)(3\gamma+4)(4-3\gamma)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}}{\gamma\left(12-5\gamma^2\right)}. \qquad (25)
$$

Since $\frac{-(1-\gamma)(4-3\gamma)(\gamma+2)^2 + \sqrt{(1-\gamma)(3\gamma+4)(4-3\gamma)(\gamma+1)(\gamma-2)^2(\gamma+2)^2}}{\gamma(12-5\gamma^2)} \geq 1-\gamma$, total welfare always goes down post merger in this case. ■