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ENDOGENOUS MERGERS UNDER MULTI-MARKET COMPETITION

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Abstract

This paper examines a simple model of strategic interactions among firms that face at least some of the same rivals in two related markets (for goods 1 and 2). It shows that when firms compete in quantity, market prices increase as the degree of multi-market contact increases. However, the welfare consequences of multi-market contact are more complex and depend on how two fundamental forces play out. The first is the selection effect, which acts to increase welfare, as shutting down the relatively more inefficient firm is beneficial. The second opposing effect is the internalisation of the Cournot externality effect; reducing the production of good 2 allows firms to sustain a higher price for good 1. This works to increase prices and, therefore, decrease consumer surplus (but increasing producer surplus). These two effects are influenced by the degree of asymmetry between markets 1 and 2 and the degree of substitutability between goods 1 and 2.

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1 Introduction

Multi-market competition refers to the situation in which a firm faces at least some of the same rivals across multiple markets.[1] More specifically, under multi-market competition a firm that is active across multiple markets might find itself competing with firms that are also present in multiple markets as well as firms that are only present in a given market. This is a widespread phenomenon in modern economies and the subject of this paper. In particular, we address the case where the demands for the two products are inter-dependent.

Examples of multi-market competition can be found in the telecommunications, banking, and air transportation industries. Telecommunication carriers compete in mobile and fixed telephony, voice and data services with companies that provide all these services and companies that only provide a subset of the services (e.g., data services). Commercial banks offering a full portfolio of financial products such as insurance, home loans, personal loans, and credit cards compete not only with other full service banks but also with providers that offer only home loans or personal loans or insurance. Full service airlines (and their discount airline subsidiaries) compete with other full service airlines as well as with discount airlines.

In this paper, we propose a new framework to study markets for differentiated products. We employ the standard utility function with two goods and derive the demand curves for the differentiated products. However, we allow for multiple firms in each product market. In our benchmark case, we have two firms in each product market.

Therefore, each firm in our market structure faces competition from another firm offering the same product as well as competition from firms offering the differentiated product. This set-up captures a situation where firms compete with other firms offering products characterised by different degrees of substitutability. Note that the notion of products with different degrees of substitutability is usually examined by address models. However, address models typically feature localised competition where the firm only competes with its neighbours. In our model, all firms compete against all

[1] See Chen and Ross (2007).

rivals in the larger market.

Our market structure also allouws us to examine multi-product competition. Conventional wisdom once suggested that when firms compete against the same rivals in multiple markets, the intensity of competition may suffer.[2] The mechanism(s) through which competition would be softened were not, however, well understood. Bernheim and Whinston's (1990) seminal paper suggested concerted or coordinated effects as a mechanism through which competition would suffer with multi-market contact. By considering a supergame model where firms repeatedly compete with one another, the authors show that when firms interact in multiple markets, the opportunities for punishing deviations from collusive outcomes are enhanced. As punishing deviators becomes easier under multi-market competition, it is easier to sustain cooperative outcomes.[3] We should stress that this theoretical work is by no means conclusive. The collusive equilibrium identified by Bernheim and Whinston is one of the infinitely many equilibria that result from the application of the folk theorem to infinitely-repeated games.

Our emphasis is, however, on unilateral effects. We are interested in the short-run strategic interactions that arise when firms compete across different markets. We provide a direct mechanism through which increased multi-market contact leads to higher prices by introducing a model with both inter- and intra-market competition. Firms may compete with some rival that offers a homogenous good and/or with a rival offering a differentiated good. Furthermore, these firms may offer a single product or they may be multi-product providers.

We explore two related questions in this paper. The first question is normative in nature. We examine different market structures – from no multi-market contact to full multi-market contact – and investigate market

[2] See, for example, Edwards (1955).

[3] Scott (1982, 1983) use cross-industry data and find a positive link between multi-market contact and profits. Phillips and Mason (1992) provide some evidence with an experimental study. Additional support for the hypothesis that multimarket contact leads to higher prices is also found in several single-indusry studies. Examples include Parker and Roller (1997) in telecommunications and Pilloff (1999) in banking. Evans and Kessides (1994) find some evidence of price increases post-merger in the U.S. airline industry in the 1980s largely due to multi-market contacts.

outcomes (e.g., prices and quantities) and welfare. We rank our market structures in terms of social welfare and show how the ranking depends on the degrees of asymmetry and substitution between the products. The second question is positive in nature. We ask the following question: if we allow firms to merge, which mergers would be profitable and what would be the likely resulting market structure? To answer this question, we examine the profitability of mergers in our market structure. We identify how the traditional results of merger profitability need to be modified with the introduction of product differentiation, the presence of inter and intra-market competition, and multi-market competition.

For the notion of multi-market competition, our paper is closely related to Chen and Ross (2007). They focused on the effects of multi-market competition on prices and welfare when firms serve two different markets with a single production facility and an increasing marginal cost technology. Although the demand functions are independent in their model, the link between the markets arises as the larger the output is in one market, the higher the marginal cost is in the other market. These authors then use this framework to explain phenomena that are not fully understood in competition analysis: the issues of recoupment (lower prices in one market are compensated by higher prices in other markets) and retaliatory entry. In contrast, our model considers inter-dependent demands and focuses on the impacts of mergers on prices, welfare and market structure with constant marginal costs. With independent demand across product markets, the presence of multi-market contact has no effect on the equilibrium when markets clear simultaneously in Chen and Ross. In contrast, we consider products that are substitutes and show that the impacts of multi-market competition on market outcomes depend on the degree of substitutability.

Our framework also allows us to analyse the profitability of mergers in the presence of multi-market contact. The standard literature on mergers do not consider product differentiation.[4] With the inclusion of product

[4] One exception is Bárcena-Ruiz and Garzón (2003). They consider a merger to monopoly with differentiated products for mixed duopoly. Mixed duopoly refers to the situation when one of the merging firms is a public firm whose objective is to maximise social welfare.

differentiation and the presence of both inter and intra-market competition, we show how the standard profitability results in Salant *et al.* (1983) change. As in Salant *et al.*, we use linear demands and costs for comparison. We show that the threshold combined pre-merger output required for profitable mergers is a lot lower than in Salant *et al.* This is a starting point of including differentiated products and multi-product firms in the analysis. We are currently working on the merger profitability condition for general demand functions.[5]

In our model, mergers do not create synergy. However with asymmetric markets, there are efficiency gains for firms operating in multi-market. Since the products are not perfect substitutes, both the efficiency gain and welfare effects depend on the degree of substitutability between the products. In this paper, as a starting point, we take the standard approach and compare the profitability of mergers for different given groups of firms. We do not consider equilibrium coalition formation as in Horn and Persson (2001) and Possajennikov (2001).

We present the basic setup in the next section, and also discuss price and welfare comparisons for given market structures. Section 3 gives the results on merger profitability. Section 4 concludes. Most proofs are collected in the appendix.

2 The Basic Setup

In this paper, we follow Singh and Vives (1984) and consider preferences for goods 1 and 2 represented by the following social welfare function:[6]

$$U\left(Q_1, Q_2, m\right) = \alpha_1 Q_1 + \alpha_2 Q_2 - \frac{1}{2}\left(Q_1^2 + 2\gamma Q_1 Q_2 + Q_2^2\right) + m, \qquad (1)$$

[5] Papers on merger profitability with general demand functions typically deal with homogenous products. See Farrell and Shapiro (1990), Cheung (1992), Faulí-Oller (1997, 2002).

[6] This can be extended to N varieties with one firm producing one good each as in Häckner (2000).

where m represents all other goods in the economy. The FOC determining the optimal consumption of good i is

$$\frac{\partial U}{\partial Q_i} = \alpha_i - Q_i - \gamma Q_j - P_i = 0, \tag{2}$$

where $i \in \{1, 2\}$ and $i \neq j$.

The parameter γ measures the degree of product differentiation, $\gamma \in [-1, 1]$. If $\gamma = 0$, the demand for the two goods are independent. If $\gamma > 0$ the two goods are substitutes with $\gamma = 1$ representing the case of perfect substitutes. The two goods are complements if $\gamma < 0$. The analysis in this paper focuses on the case of substitutes. We assume that the marginal costs of production in markets 1 and 2 are equal to c_1 and c_2, respectively, and there are no fixed costs. Under this framework, the total surplus (denoted by TS) is derived from the utility function given in Equation 1,

$$TS = \alpha_1 Q_1 + \alpha_2 Q_2 - \frac{1}{2}\left(Q_1^2 + 2\gamma Q_1 Q_2 + Q_2^2\right) - c_1 Q_1 - c_2 Q_2. \tag{3}$$

Consumer surplus (CS) is defined as $TS - \Pi$, where Π is the sum of firms' profits.

The following definition is helpful in keeping our notations as simple as possible:

Definition 1 *Let* $a \equiv (\alpha_1 - c_1) - (\alpha_2 - c_2)$ *and* $\alpha_1 - c_1 = 1$. *Without loss of generality, assume* $a \geq 0$.

The index a summarises the asymmetry between the two markets. For $a = 0$, the two markets are symmetric. We refer to market 2 as relatively inefficient vis-a-vis marekt 1. This asymmetry may be created by lower intercept, α, or higher cost, c.

We discuss three market structures in this paper. In our benchmark market structure B (see Figure 1), there are two firms (A and B) that produce good 1 and two firms (C and D) that produce good 2. Firms compete by setting quantities. This simple framework allows us to capture both closer intra-market competition (e.g., between firms A and B) and more distant inter-market competition (e.g., between firms in market 1 and firms in market 2). Moreover, it also allows us to investigate the consequences of changes in the market structure that affect intra- and inter-market competition. In particular, we consider a market structure where one firm can offer both goods while facing different rivals in each market (see Figure 2). We refer to this market structure as partial multi-market contact, market structure P. In the last structure we consider, there are two firms competing with each other in both markets (see Figure 3). We refer to this as full market contact, market structure F.

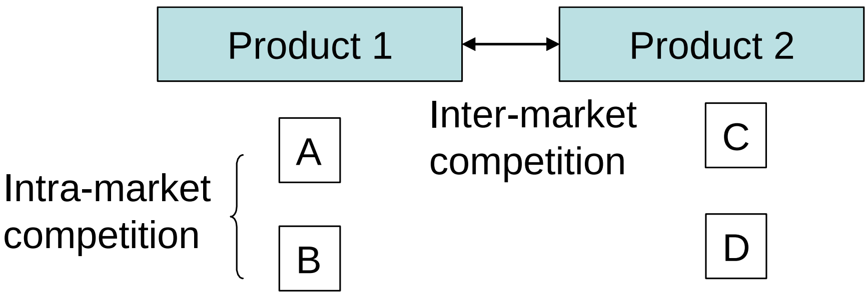

Figure 1: The Benchmark (B).

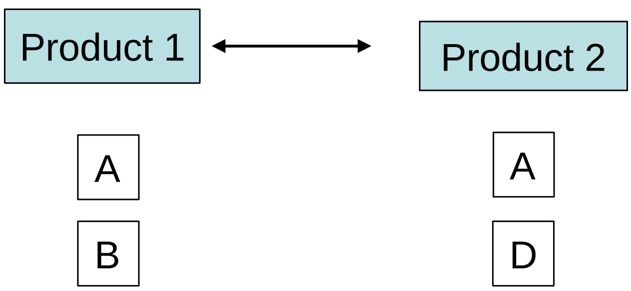

Figure 2: Partial Multi-Market Contact (P).

2.1 Market Equilibrium

In this section we characterise the market equilibria under the various market structures. This is presented in Table 1 below. As the degree of asymmetry, a, increases, firms cease offering product 2. Different market structures have

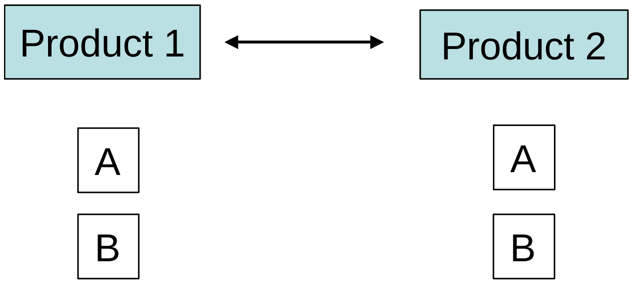

Figure 3: Full Multi-Market Contact (F).

different critical a values for corner solutions to eventuate. For a multi-product firm, since the externality between the markets is internalised, the firm would produce less in market two and the critical a required would be lower than a single-good only producer. Given our three market structures, we have the critical value for market structure B, $a_B = \frac{3-2\gamma}{3}$, being the highest. In market structure P, the critical value required for the multi-product firm firm to exit market 2 is the lowest, $a_P \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.[7] With two symmetric firms serving both markets, the critical a value required for both firms to cease production in market 2 is in between the previous critical values, $a_F \geq 1-\gamma$. Since for $a \geq \frac{3-2\gamma}{3}$, all market structures yield the same market outcome with $Q_2 = 0$, we present the analysis for the case $a < \frac{3-2\gamma}{3}$.[8] Omitted for simplicity, in all cases, the consumer surplus is computed as:

$$CS = TS - \Pi = \gamma Q_1 Q_2 + \frac{1}{2}Q_1^2 + \frac{1}{2}Q_2^2. \quad (4)$$

In the next section, we illustrate how the two fundamental effects – the selection and the internalisation of the Cournot externality effects – drive the strategic interaction among firms in the three market structures.

[7] Note that in this case, $Q_2 = q_D > 0$. Only the multi-product firm exits market two. For $Q_2 = 0$, the critical value required would be the same as a_B.

[8] The computation of the equilibrium outcome is standard. The proofs are available upon request.

	$a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$	$\frac{(3-\gamma)(1-\gamma)}{3+\gamma^2} \leq a \leq 1-\gamma$	$1-\gamma \leq a$
B	$q_A = q_B = \frac{3-2\gamma(1-a)}{(3+2\gamma)(3-2\gamma)}$; $q_C = q_D = \frac{3(1-a)-2\gamma}{(3+2\gamma)(3-2\gamma)}$; $Q_1 = 2q_A$; $Q_2 = 2q_C$; $\pi_A = \pi_B = \frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2}$; $\pi_C = \pi_D = \frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}$.		
P	$q_{A1} = \frac{4a\gamma-4\gamma+\gamma^2+3}{(\gamma+1)(3-\gamma)(\gamma+3)(1-\gamma)}$; $q_{A2} = \frac{(3+\gamma^2)(1-a)-4\gamma}{(1+\gamma)(1-\gamma)(3-\gamma)(3+\gamma)}$; $q_B = \frac{3-\gamma(1-a)}{(3+\gamma)(3-\gamma)}$; $q_D = \frac{3(1-a)-\gamma}{(3+\gamma)(3-\gamma)}$; $Q_1 = q_{A1} + q_B$; $Q_2 = q_{A2} + q_D$; $\pi_A = \frac{2(1-\gamma)(3-\gamma)^2(1-a)+9a^2+7a^2\gamma^2}{(3+\gamma)^2(3-\gamma)^2(1+\gamma)(1-\gamma)}$; $\pi_B = \frac{(3-\gamma(1-a))^2}{(3-\gamma)^2(\gamma+3)^2}$; $\pi_D = \frac{(3-3a-\gamma)^2}{(\gamma+3)^2(3-\gamma)^2}$.	$q_{A1} = q_B = \frac{2-\gamma(1-a)}{2(3-\gamma^2)}$; $q_{A2} = 0$; $q_D = \frac{3(1-a)-2\gamma}{2(3-\gamma^2)}$; $Q_1 = \frac{2-\gamma(1-a)}{(3-\gamma^2)}$; $Q_2 = \frac{3(1-a)-2\gamma}{2(3-\gamma^2)}$; $\pi_A = \pi_B = \frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2}$; $\pi_D = \frac{(3a+2\gamma-3)^2}{4(3-\gamma^2)^2}$.	
F	$q_{A1} = q_{B1} = \frac{1-\gamma(1-a)}{3(\gamma+1)(1-\gamma)}$; $q_{A2} = q_{B2} = \frac{1-a-\gamma}{3(1-\gamma)(\gamma+1)}$ $Q_1 = 2q_{A1}, Q_2 = 2q_{A2}$; $\pi_A = \pi_B = \frac{2-2\gamma(1-a)-2a+a^2}{9(1+\gamma)(1-\gamma)}$		$q_{A1} = q_{B1} = \frac{1}{3}$; $q_{A2} = q^*_{B2} = 0$; $Q_1 = \frac{2}{3}, Q_2 = 0$; $\pi_A = \pi_B = \frac{1}{9}$.

Table 1: Summary of the market equilibrium.

2.2 Prices and Welfare

In this subsection we present price and welfare comparison across the three market structures. The lemma below shows that prices for both goods are always higher under full multi-market contact. This is intuitive since the multi-market firm has less incentive to expand its output. Output expansion in one market hurts not only its profitability in the given market, but also its profitability in the other market. In the tables, we use superscript B (P, F) to denote variables for market structure B (P, F).

Lemma 1 *The ranking of market prices is* $P_1^B \leq P_1^P \leq P_1^F$ *and* $P_2^B \leq P_2^P \leq P_2^F$ *whenever* P_2 *is defined.*

Proof. With market outputs given in Table 1, market prices are computed through inverse demand functions derived in Equation 2, $P_i = \alpha_i - Q_i - \gamma Q_j$, where $i = \{1, 2\}$, and $i \neq j$. For example, in market structure B,

$$\begin{aligned} P_1^B &= \alpha_1 - Q_1 - \gamma Q_2 \\ &= \alpha_1 - 2\frac{3 - 2\gamma^2 + \gamma(1-a)}{(3+2\gamma)(3-2\gamma)}. \end{aligned}$$

All other prices are derived accordingly with the outputs given in Table 1 for different parameter ranges. The price comparison is straightforward, and the proof is available upon request. ■

While Lemma 1 is not surprising, the welfare comparison is less straightforward as indicated in the following proposition:

Proposition 1 *The welfare ranking is summarised in Table 2.*

$a \leq \frac{(8\gamma^3-20\gamma^2-24\gamma+45)(3-2\gamma)}{135-12\gamma^2-16\gamma^4}$	$TS^B > TS^P > TS^F$
$\frac{(8\gamma^3-20\gamma^2-24\gamma+45)(3-2\gamma)}{135-12\gamma^2-16\gamma^4} \leq a \leq \frac{(9-3\gamma-4\gamma^2)(3-2\gamma)}{3(9-2\gamma^2)}$	$TS^P > TS^B > TS^F$
$\frac{(9-3\gamma-4\gamma^2)(3-2\gamma)}{3(9-2\gamma^2)} \leq a \leq \frac{16\gamma^3-12\gamma^2-78\gamma+81}{3(27-4\gamma^2)}$	$TS^P > TS^F > TS^B$
$a \geq \frac{16\gamma^3-12\gamma^2-78\gamma+81}{3(27-4\gamma^2)}$	$TS^F \geq TS^P \geq TS^B$

Table 2: Welfare rankings.

Proof. See the Appendix. ■

The results in Proposition 1 reflect the tension between the selection effect (shutting down the inefficient firms is beneficial) and the internalisation of the Cournot externality effect (reducing the production of good 2 allows firms to sustain a higher price for good 1). These two effects are influenced by the degree of asymmetry (a) between markets 1 and 2 and the degree of substitutability (γ) between goods 1 and 2.

From the consumer's point of view, market structure B always yields the highest surplus since prices are the lowest. However, as a increases, the asymmetry between the two markets increases, and social welfare may increase with the presence of multi-market firms since there is more efficiency gain from reducing the production of good 2. Therefore, with a low a, the social welfare is the highest in market structure B. As a gets sufficiently large, market structure F dominates. Market structure P is the best for intermediate values of a. Note that all the critical a values listed in Table 2 decrease as γ increases. The band for market structure P to yield the highest social welfare is the widest for intermediate values of γ.

Comparing the critical values in Table 1 and Table 2, we have the following Proposition.

Proposition 2 *Market structure F gives the highest social welfare among the three market structures only if for both multi-product firms, $q_{A2} = q_{B2} = 0$. Market structure P gives the highest social welfare among the three market structures only if for the multi-product firm, $q_{A2} = 0$.*

Proof. Given that $1-\gamma \leq \frac{16\gamma^3-12\gamma^2-78\gamma+81}{3(27-4\gamma^2)}$, the critical a value required for market structure F to give the highest social welfare compared with the other two market structures is greater than the critical value required for the firms in F to cease production of good 2. Therefore, whenever market structure F gives higher social welfare, it must be the case that there is a corner solution in market 2. Similarly, as $\frac{(8\gamma^3-20\gamma^2-24\gamma+45)(3-2\gamma)}{135-12\gamma^2-16\gamma^4} \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$, market structure P yields the highest social welfare only if the multi-product firm in market structure P ceases production of good 2. ∎

The Proposition above establishes that the selection effect is a necessary condition for market structure F (and P) to yield the highest welfare.

3 Endogenous Mergers

This section examines two related questions. First, we ask what mergers are profitable in each market structure. For the full multi-market structure, there is only one merger possible – a merger from two firms producing the two goods to a single firm producing two goods. Such merger to monopoly is clearly profitable. The determination of the profitability of mergers for the two other market structures is more complex and is summarised by Propositions 3 and 4 below. These propositions also allow us to answer a second question: what market structure is more likely to arise in an environment where the benchmark firms are allowed to pursue any profitable mergers?

Proposition 3 *Conditions for profitable mergers in the benchmark market structure are summarised in Table 3. The derivation of the critical values are in the appendix with the values reproduced in the footnote here.*[9]

[9] $a^{B1} = \frac{(3-2\gamma)(4\gamma^3-25\gamma^2-12\gamma+4\gamma^4+36)}{(4\gamma^4-51\gamma^2+108)}$; $a^{B2} = \frac{(2-\gamma)(1-\gamma)}{2+\gamma^2}$;
$a^{B3} = \frac{-(2-\gamma)(3-2\gamma)(-4\gamma^2+7\gamma-7\gamma^3-4\gamma^4+24)+\sqrt{2(1-\gamma)(\gamma+1)(2-\gamma^2)(2\gamma+3)^2(2-\gamma)^2(\gamma+2)^2(3-2\gamma)^2}}{25\gamma^2-17\gamma^4+8\gamma^6-144}$;
$a^{B4} = \frac{(1-\gamma)(3-2\gamma)(6\gamma+28\gamma^2+40\gamma^3-27)+\sqrt{72(1-\gamma)(\gamma+1)(2\gamma^2-1)(2\gamma-3)^2(2\gamma+3)^2}}{\gamma(80\gamma^4-8\gamma^2+153)}$.

	$a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$	$\frac{(3-\gamma)(1-\gamma)}{3+\gamma^2} \leq a \leq 1-\gamma$	$1-\gamma \leq a$
Inter-market (e.g., A & C)	not profitable	not profitable	$\gamma \leq 0.89$ or $a \geq a^{B1}$
Intra-market (e.g., A & B)	$\gamma \leq 0.66$	$\gamma \leq 0.66$	$\gamma \leq 0.66$
Inter + intra (e.g., A B C)	$a \geq \max\left\{a^{B2}, a^{B3}\right\}$ or $\gamma \leq \sqrt{\frac{1}{2}}$ or $a^{B2} \leq a \leq a^{B4}$	$a \geq a^{B3}$	$\gamma \leq 0.89$ or $a \geq a^{B3}$

Table 3: Profitable mergers under market structure B.

Proof. See the Appendix. ■

We should note that for the merger between three firms – for example, firms A, B, and C – the merger profitability analysis is undertaken against the pre-merger profits, π_A, π_B, and π_C. This is the standard approach. Different answers may be obtained if the reference point is a two-firm merger first – for example, firms A and B – followed by the profitability analysis of adding another firm – for example, firm C – into this coalition.

Proposition 3 suggests that whether or not an intra-market merger is profitable depends only on γ. For $\gamma = 0$, the two markets are independent, and an intra-market merger is simply a merger between duopolists to form a monopolist. Such a merger is always profitable. This suggests that under our set-up with both inter- and intra- competition, a merger of the two firms within one market is only profitable if the two markets are relatively isolated. Proposition 3 also shows that an inter-market merger is profitable for large a. In particular, for a merger between A and C to be profitable, the required critical a is greater than $1-\gamma$. A two-firm inter-market merger is only profitable in the parameter range where the merged entity ceases production in market 2. In this case, in addition to the efficiency gain, it is also easier for this merger to satisfy the incentive compatibility constraints as the firm in market 2 would have lower pre-merger profit.

There are a few cases where a merger between three firms (A, B, and C) can be profitable. The first possibility is a large a. A lower γ reduces the threshold a required. A three firm merger is also profitable if the markets are relatively isolated, $\gamma \leq \sqrt{\frac{1}{2}}$. Finally, a three firm merger can also be profitable if a is small. In this parameter range ($a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$), the asymmetry between the markets is small and efficiency gains are therefore low. The merged entity continues to produce both goods. For a merger to be profitable, it must then involve firms with a large combined output in the market. This result is analogous to the classic result of Salant, Switzer and Reynolds (1983) that a merger among symmetric firms is not profitable unless it involves 80% of the firms in the industry.

The proposition below summarises the profitability analysis for mergers under partial multi-market contact.

Proposition 4 *Conditions for profitable mergers with partial multi-market contact are summarised in Table 4. The derivation for the critical values are in the appendix with the values reproduced in the footnote here.*[10]

	$a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$	$\frac{(3-\gamma)(1-\gamma)}{3+\gamma^2} \leq a \leq 1-\gamma$	$1-\gamma \leq a$
Inter-market B & D	Not profitable	$\gamma \leq 0.77$ and $a \geq a^{P1}$	$\gamma \leq 0.77$ or $a \geq a^{P2}$
Intra + inter A & B	$\gamma \leq 0.77$ and $a \geq a^{P3}$	$\gamma \leq 0.77$	$\gamma \leq 0.77$
Intra + inter A & D	$\gamma \leq 0.6$ and $a \leq a^{P4}$	$\gamma \leq 0.77$ and $a \geq a^{P5}$	$\gamma \leq 0.77$ or $a \geq a^{P2}$
$\pi_{AB} > \pi_{AD}$	if $a \geq a^{P6}$	if both mergers are profitable	

Table 4: Profitable mergers under market structure P.

Proof. See the Appendix. ■

Since a merger in this market structure involves both intra- and inter-market mergers, in general, profitable mergers require γ to be small and a

[10] $a^{P1} = \frac{(1-\gamma)\left(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45\right)-\sqrt{8\gamma^2(1-\gamma)(\gamma+1)\left(3-2\gamma^2\right)\left(3-\gamma^2\right)^2}}{\left(45-13\gamma^4-48\gamma^2\right)}$;
$a^{P2} = \frac{2\gamma^3+3\gamma^2-30\gamma+27}{3\left(\gamma^2+9\right)}$; $a^{P3} = \frac{5\gamma-3}{5\gamma}$; $a^{P4} = \frac{3-5\gamma}{3}$;
$a^{P5} = \frac{-3\left(1+\gamma^2\right)(1-\gamma)^2+\sqrt{(1-\gamma)(\gamma+1)\left(\gamma^2+1\right)\left(3-\gamma^2\right)^2}}{6\gamma\left(\gamma^2+1\right)}$;
$a^{P6} = \frac{3(1-\gamma)(2-\gamma)\left(8-\gamma^3+2\gamma^2+8\gamma\right)-\sqrt{16\left(5-\gamma^2\right)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3\left(5\gamma^4-12\gamma^2+16\right)}$.

to be large. An exception is the profitable AD merger for the parameter range, $a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$. In this case, the asymmetry is small between the two markets, there is no corner solution, and the merged entity continues to produce both products. Thus, A and D would only have the incentives to merge if a is small and firm D also has significant pre-merger output share.

These propositions also allow us to consider the following question. Starting with the benchmark, if firms A and C were to merge, would firms B and D find it profitable to merge (resulting in structure F) or would B and D prefer to stay separate (resulting in structure P)? From the benchmark, firms A and C only have the incentive to merge for high values of a. Furthermore, the critical a value for profitable AC merger in market structure B is higher than the critical a value for profitable BD merger in market structure P. Therefore, if AC merger is profitable, firms B and D would always have the incentive to merge. For high values of a, market dynamics might naturally result in a market structure where firms operate in multiple markets. Importantly, for high values of a, market structure F yields the highest social welfare.[11] Note that for intermediate values of a, structure P maximises social welfare but this structure is unlikely to emerge given that the associated merger is not profitable.

With the inclusion of both inter- and intra-market competition, first, there exists endogenous mergers. Even with the presence of the outsider firms, some firms would still have the incentives to merge. The optimal market structure depends on both a and γ. The welfare effects of merger thus also depend on both a and γ.

4 Conclusion

This paper examines a simple model of multi-market competition. It shows that when firms compete in quantity, although full multi-market contact might lead to higher prices, the welfare consequences are more complex and depend on how two fundamental forces play out. The first is the selection effect, which works towards increasing welfare as shutting down the inefficient

[11] Recall that this follows from the portfolio effect (that is, producing less of the inefficient good 2) rather than from lower prices.

firm is beneficial. The second opposing effect is the internalisation of the Cournot externality effect; reducing the production of good 2 allows firms to sustain a higher price for good 1. This works towards increasing prices and, therefore, decreasing the consumer surplus (but increasing the producer surplus). These two effects are influenced by the degree of asymmetry (a) between markets 1 and 2 and the degree of substitutability (γ) between goods 1 and 2. The higher a is, the more relatively inefficient market 2 is, and the stronger is the selection effect. The higher γ is, the more closely linked the two markets are and the stronger the externality effect would be. A merger internalises the effects more when γ is large. This makes the merged entity a lot less aggressive and hence unlikely to raise profits for the merged entity. On top of this, a lower γ implies more isolated markets and makes intra-market merger more profitable. Therefore, the general result is that merger is more likely to be profitable when γ is low.

This analysis should be viewed as a preliminary step towards understanding the dynamics of multi-market competition. It simply illustrates that mergers can increase welfare under multi-market competition. Although this result is not new *per se* [12], its novelty arises from the fact the increase in welfare might not originate from the market (as strictly defined from a competition analysis perspective) in which the merger takes place, but instead from a related market. This raises important issues for merger analysis under competition law.

This framework, however, can be generalised in a number of ways. First, it is important to understand how the two effects identified in the paper – the selection and internalisation of externality effects – play out when there are more than two firms in both markets. It is important to understand how an increase in the number of competitors affects their impacts on both inter- and intra-markets competition. Second, one can explicitly consider the existence of common fixed costs across markets (synergies). This will strengthen the selection effect and may also mean greater gains under full multi-market contact. Third, we can extend the framework to consider other pricing schemes. For example, we can allow firms that offer the two

[12] See, for example, Perry and Porter (1985) and Farrell and Shapiro (1990).

products to compete by offering bundles. We conjecture that this can lead to very fierce competition under full multi-market contact. Fourth, we can use this simple framework to consider the scope for a firm that offers the two goods to behave anti-competitively in order to exclude rivals from one of the markets. Finally, we use the quadratic utility function and linear demand function to obtain closed form merger profitability conditions and to facilitate comparison with the literature. Extending the results along the lines of Farrell and Shapiro (1990), Cheung (1992), and Faulí-Oller (1997, 2002) to establish general conditions with differentiated goods and multi-product firms would be a valuable contribution to the literature. We are currently working on this set-up.

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5 Appendix

Proof. of Proposition 1: We use the definition of the total surplus in Equation 3 for the welfare analysis.

Case 1 ($1-\gamma \leq a$): Given $\left\{Q_1^B, Q_2^B, Q_1^F, Q_2^F\right\}$, $TS^B \geq TS^F$ if $a \leq \frac{\left(9-3\gamma-4\gamma^2\right)(3-2\gamma)}{3(9-2\gamma^2)}$. Note that $1-\gamma \leq \frac{\left(9-3\gamma-4\gamma^2\right)(3-2\gamma)}{3(9-2\gamma^2)} \leq \frac{3-2\gamma}{3}$.

Given $\left\{Q_1^B, Q_2^B, Q_1^P, Q_2^P\right\}$, $TS^B \geq TS^P$ if $a \leq \frac{\left(8\gamma^3-20\gamma^2-24\gamma+45\right)(3-2\gamma)}{135-12\gamma^2-16\gamma^4}$. Note that $\frac{\left(8\gamma^3-20\gamma^2-24\gamma+45\right)(3-2\gamma)}{(135-12\gamma^2-16\gamma^4)} \leq \frac{3-2\gamma}{3}$. Finally, $TS^F \geq TS^P$ if $a \geq \frac{16\gamma^3-12\gamma^2-78\gamma+81}{81-12\gamma^2}$. Note that $1-\gamma \leq \frac{16\gamma^3-12\gamma^2-78\gamma+81}{3(27-4\gamma^2)} \leq \frac{3-2\gamma}{3}$. Also, we have $\frac{16\gamma^3-12\gamma^2-78\gamma+81}{3(27-4\gamma^2)} \geq \frac{\left(9-3\gamma-4\gamma^2\right)(3-2\gamma)}{3(9-2\gamma^2)} \geq \frac{\left(8\gamma^3-20\gamma^2-24\gamma+45\right)(3-2\gamma)}{(135-12\gamma^2-16\gamma^4)}$.

Case 2 ($\frac{(3-\gamma)(1-\gamma)}{3+\gamma^2} \leq a \leq 1-\gamma$): Given $\left\{Q_1^B, Q_2^B, Q_1^F, Q_2^F\right\}$ in this case, $TS^B \geq TS^F$ if $a \leq \frac{-(\gamma+3)(1-\gamma)(3-2\gamma)^2+\sqrt{(1-\gamma)(\gamma+3)(3-\gamma)(\gamma+1)(3-2\gamma)^2(2\gamma+3)^2}}{2\gamma(27-2\gamma^2)}$. Note that $\frac{-(\gamma+3)(1-\gamma)(3-2\gamma)^2+\sqrt{(1-\gamma)(\gamma+3)(3-\gamma)(\gamma+1)(3-2\gamma)^2(2\gamma+3)^2}}{2\gamma(27-2\gamma^2)} \geq 1-\gamma$.

Similarly, $TS^P \geq TS^F$ if
$\frac{(1-\gamma)\left(45-45\gamma+42\gamma^2-4\gamma^4\right)-\sqrt{16\gamma^2(1-\gamma)(1+\gamma)(15-\gamma^2)(3-\gamma^2)^2}}{45+87\gamma^2-4\gamma^4} \leq a$
$\leq \frac{(1-\gamma)\left(45-45\gamma+42\gamma^2-4\gamma^4\right)+\sqrt{16\gamma^2(1-\gamma)(1+\gamma)(15-\gamma^2)(3-\gamma^2)^2}}{45+87\gamma^2-4\gamma^4}$. Note that
$\frac{(1-\gamma)\left(45-45\gamma+42\gamma^2-4\gamma^4\right)-\sqrt{16\gamma^2(1-\gamma)(1+\gamma)(15-\gamma^2)(3-\gamma^2)^2}}{45+87\gamma^2-4\gamma^4} \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$ and
$\frac{(1-\gamma)\left(45-45\gamma+42\gamma^2-4\gamma^4\right)+\sqrt{16\gamma^2(1-\gamma)(1+\gamma)(15-\gamma^2)(3-\gamma^2)^2}}{45+87\gamma^2-4\gamma^4} \geq 1-\gamma$. Finally, note that $\frac{\left(8\gamma^3-20\gamma^2-24\gamma+45\right)(3-2\gamma)}{(135-12\gamma^2-16\gamma^4)} \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.

Case 3 ($a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$): Given $\left\{Q_1^B, Q_2^B, Q_1^P, Q_2^P\right\}$ in this case, $TS^B \geq TS^P$ if
$a \leq \frac{-(\gamma+2)(1-\gamma)(2\gamma-3)^2(\gamma-3)^2+\sqrt{(1-\gamma)(2-\gamma)(\gamma+2)(\gamma+1)(2\gamma+3)^2(\gamma+3)^2(\gamma-3)^2(2\gamma-3)^2}}{\gamma(405-32\gamma^4+27\gamma^2)}$.
Note that $\frac{-(\gamma+2)(1-\gamma)(2\gamma-3)^2(\gamma-3)^2+\sqrt{(1-\gamma)(2-\gamma)(\gamma+2)(\gamma+1)(2\gamma+3)^2(\gamma+3)^2(\gamma-3)^2(2\gamma-3)^2}}{\gamma(405-32\gamma^4+27\gamma^2)} \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$. Given $\left\{Q_1^P, Q_2^P, Q_1^F, Q_2^F\right\}$ in this case, $TS^P \geq TS^F$ if
$a \leq \frac{-2(\gamma+6)(1-\gamma)(\gamma-3)^2+\sqrt{4(1-\gamma)(\gamma+6)(6-\gamma)(\gamma+1)(\gamma-3)^2(\gamma+3)^2}}{2\gamma(9-\gamma)(\gamma+9)}$. Note that
$\frac{-2(\gamma+6)(1-\gamma)(\gamma-3)^2+\sqrt{4(1-\gamma)(\gamma+6)(6-\gamma)(\gamma+1)(\gamma-3)^2(\gamma+3)^2}}{2\gamma(9-\gamma)(\gamma+9)} \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$. ■

Proof. of Proposition 3: **Case 1** ($\frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)} \leq a$): (i) A merges with C: If A merges with C, and B and D remain separated, the market structure becomes that of market structure P. The merger is profitable if $\frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2} \geq \frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2} + \frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}$. This holds for $a \geq \frac{(3-2\gamma)\left(4\gamma^3-25\gamma^2-12\gamma+4\gamma^4+36\right)}{(4\gamma^4-51\gamma^2+108)}$. Note that $\frac{3-2\gamma}{3} \geq \frac{(3-2\gamma)\left(4\gamma^3-25\gamma^2-12\gamma+4\gamma^4+36\right)}{(4\gamma^4-51\gamma^2+108)} \geq 1-\gamma$.

(ii) A merges with B: The merged firm has the best response: $q_{AB} = \frac{1-\gamma(q_C+q_D)}{2}$. For firm i in market 2, the best response is $q_i = \frac{1-a-\gamma q_{AB}-q_j}{2}$. This gives the interior solutions: $q_{AB} = \frac{3-2\gamma(1-a)}{2(3-\gamma^2)}$ and $q_C = q_D = \frac{2(1-a)-\gamma}{2(3-\gamma^2)}$. The merger is profitable if $\frac{(3-2\gamma(1-a))^2}{4(3-\gamma^2)^2} \geq 2\frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2}$. This holds for $\gamma \leq \frac{\sqrt{6-3\sqrt{2}}}{2} \approx 0.66$.

(iii) Compare the AC merger and AB merger: For the parameter range where both mergers are profitable, AB merger always gives higher profit.

(iv) C mergers with D: The best responses are $q_A = \frac{1-q_B-\gamma q_{CD}}{2}$, $q_B = \frac{1-q_A-\gamma q_{CD}}{2}$, and $q_{CD} = \frac{1-a-\gamma(q_A+q_B)}{2}$. The interior solution is $q_A = q_B = \frac{2-\gamma(1-a)}{2(3-\gamma^2)}$ and $q_{CD} = \frac{3(1-a)-2\gamma}{2(3-\gamma^2)}$. The merger is profitable if $\frac{(3(1-a)-2\gamma)^2}{4(3-\gamma^2)^2} \geq 2\frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}$. This holds for $\gamma \leq \frac{\sqrt{6-3\sqrt{2}}}{2}$.

(v) A merges with B and C: The best responses are $q_{ABC1} = \frac{1-2\gamma q_{ABC2}-\gamma q_D}{2}$, $q_{ABC2} = \frac{1-a-2\gamma q_{ABC1}-q_D}{2}$, and $q_D = \frac{1-a-\gamma q_{ABC1}-q_{ABC2}}{2}$. For the given parameter range, the firm ABC would cease to offer good 2. In equilibrium, $q_{ABC2} = 0$, $q_{ABC1} = \frac{2-\gamma(1-a)}{4-\gamma^2}$ and $q_D = \frac{2(1-a)-\gamma}{4-\gamma^2}$. The merger is profitable if $\frac{(2-\gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2} \geq 2\frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2} + \frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}$.This holds for

$$a \geq \frac{-(2-\gamma)(3-2\gamma)\left(-4\gamma^2+7\gamma-7\gamma^3-4\gamma^4+24\right)}{25\gamma^2-17\gamma^4+8\gamma^6-144} + \frac{\sqrt{2(1-\gamma)(\gamma+1)(2-\gamma^2)(2\gamma+3)^2(2-\gamma)^2(\gamma+2)^2(3-2\gamma)^2}}{25\gamma^2-17\gamma^4+8\gamma^6-144}.$$

Note that $\frac{-(2-\gamma)(3-2\gamma)\left(-4\gamma^2+7\gamma-7\gamma^3-4\gamma^4+24\right)+\sqrt{2(1-\gamma)(\gamma+1)(2-\gamma^2)(2\gamma+3)^2(2-\gamma)^2(\gamma+2)^2(3-2\gamma)^2}}{25\gamma^2-17\gamma^4+8\gamma^6-144} \leq \frac{3-2\gamma}{3}$ and $\frac{-(2-\gamma)(3-2\gamma)\left(-4\gamma^2+7\gamma-7\gamma^3-4\gamma^4+24\right)+\sqrt{2(1-\gamma)(\gamma+1)(2-\gamma^2)(2\gamma+3)^2(2-\gamma)^2(\gamma+2)^2(3-2\gamma)^2}}{25\gamma^2-17\gamma^4+8\gamma^6-144} \leq 1-\gamma$ for $\gamma \leq 0.89$. For most of the parameter range in this case, the merger is profitable.

Case 2 ($a \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$): (i) AC merger is profitable since in this case $\frac{2(1-\gamma)(3-\gamma)^2(1-a)+9a^2+7a^2\gamma^2}{(3+\gamma)^2(3-\gamma)^2(1+\gamma)(1-\gamma)} \geq \frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2} + \frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}$.

(ii) AB merger is profitable if $\gamma \leq \sqrt{\frac{6-\sqrt{18}}{4}}$.

(iii) A merges with B and C: Note that $\frac{(2-\gamma)(1-\gamma)}{2+\gamma^2} \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$. For $a \geq \frac{(2-\gamma)(1-\gamma)}{2+\gamma^2}$, as analysed in Case 1, firms ABC merger is profitable if

$max\{\frac{(2-\gamma)(1-\gamma)}{2+\gamma^2},$
$\frac{-(2-\gamma)(3-2\gamma)\left(-4\gamma^2+7\gamma-7\gamma^3-4\gamma^4+24\right)+\sqrt{2(1-\gamma)(\gamma+1)(2-\gamma^2)(2\gamma+3)^2(2-\gamma)^2(\gamma+2)^2(3-2\gamma)^2}}{25\gamma^2-17\gamma^4+8\gamma^6-144}\} \leq$
$a \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$.

For $a \leq \frac{(2-\gamma)(1-\gamma)}{2+\gamma^2}$, the equilibrium output levels are $q_{ABC1} = \frac{1-\gamma(1-a)}{2(1+\gamma)(1-\gamma)}$, $q_{ABC2} = \frac{\gamma^2-3\gamma-2a-a\gamma^2+2}{6(1-\gamma)(1+\gamma)}$, and $q_D = \frac{1-a}{3}$. ABC merger is profitable if $\frac{\left(18a\gamma-18\gamma-8a+4a^2+5\gamma^2-10a\gamma^2+5a^2\gamma^2+13\right)}{36(1+\gamma)(1-\gamma)} \geq 2\frac{(3-2\gamma(1-a))^2}{(3+2\gamma)^2(3-2\gamma)^2} + \frac{(3(1-a)-2\gamma)^2}{(3+2\gamma)^2(3-2\gamma)^2}$. This holds for $\gamma \leq \sqrt{\frac{1}{2}} \approx 0.7$. For $\gamma > \sqrt{\frac{1}{2}}$, ABC merger is profitable if $a \geq \frac{(1-\gamma)(3-2\gamma)\left(6\gamma+28\gamma^2+40\gamma^3-27\right)+\sqrt{72(1-\gamma)(\gamma+1)(2\gamma^2-1)(2\gamma-3)^2(2\gamma+3)^2}}{\gamma(80\gamma^4-8\gamma^2+153)}$. ■

Proof. of Proposition 4: **Case 1** ($1-\gamma \leq a$): (i) A merges with B: The post merger market structure is the same as a ABC merger in the market structure B analysed above. The merger is profitable if $\frac{(2-\gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2} \geq 2\frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2}$. This holds for $\gamma \leq \sqrt{2-\sqrt{2}} \approx 0.77$.

(ii) A merges with D: The merged firm has best responses $q_{AD1} = \frac{1-q_B-2\gamma q_{AD2}}{2}$ and $q_{AD2} = \frac{1-a-2\gamma q_{AD1}-\gamma q_B}{2}$. For firm B, the best response is $q_B = \frac{1-q_{AD1}-\gamma q_{AD2}}{2}$. This gives the interior solution: $q_{AD1} = \frac{2-3\gamma(1-a)+\gamma^2}{6(1+\gamma)(1-\gamma)}$, $q_{AD2} = \frac{1-a-\gamma}{2(1-\gamma^2)}$, and $q_B = \frac{1}{3}$. It can be verified that the merged entity would never cease production in market 1. For market 2, if $q_{AD2} = 0$, $q_{AD1} = q_B = \frac{1}{3}$. These quantities would indeed induce $q_{AD2} = 0$ if $a \geq 1-\gamma$. This holds in this case. Therefore, firms A and D would have the incentive to merge if $\frac{1}{9} \geq \frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2} + \frac{(3a+2\gamma-3)^2}{4(3-\gamma^2)^2}$. This holds for $a \geq \frac{2\gamma^3+3\gamma^2-30\gamma+27}{3(\gamma^2+9)}$. Note that $\frac{2\gamma^3+3\gamma^2-30\gamma+27}{3(\gamma^2+9)} \leq 1-\gamma$ if $\gamma \leq \sqrt{\frac{3}{5}} \approx 0.77$.

(iii) AD merger is more profitable than AB merger if $\frac{1}{9} \geq \frac{(2-\gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2}$. This holds for $a \leq \frac{3\gamma-\gamma^2-2}{3\gamma}$. Note that $\frac{3\gamma-\gamma^2-2}{3\gamma} \leq 1-\gamma$.

(iv) B merges with D: The resulting market structure is the same as market structure F. Both firms A and BD do not offer good 2. The merger is profitable if $\frac{1}{9} \geq \frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2} + \frac{(3a+2\gamma-3)^2}{4(3-\gamma^2)^2}$. The conditions are the same as the ones for profitable AD merger.

Case 2 ($\frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)} \leq a \leq 1-\gamma$): (i) A and B merge: As analysed in Case 1, such merger is profitable if $\gamma \leq \sqrt{2-\sqrt{2}}$.

(ii) A and D merge: In this parameter range, the merged entity would continue to produce in both markets. The merger is profitable if $\frac{18a\gamma-18\gamma-18a+9a^2+5\gamma^2+13}{36(1+\gamma)(1-\gamma)} \geq \frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2} + \frac{(3a+2\gamma-3)^2}{4(3-\gamma^2)^2}$. This holds for $a \geq$

$\frac{-3(1+\gamma^2)(1-\gamma)^2+\sqrt{(1-\gamma)(\gamma+1)(\gamma^2+1)(3-\gamma^2)^2}}{6\gamma(\gamma^2+1)}$. Note that

$\frac{-3(1+\gamma^2)(1-\gamma)^2+\sqrt{(1-\gamma)(\gamma+1)(\gamma^2+1)(3-\gamma^2)^2}}{6\gamma(\gamma^2+1)} \geq 1-\gamma$ if $\gamma \geq \sqrt{\frac{3}{5}} \approx 0.77$. Note also that $\frac{-3(1+\gamma^2)(1-\gamma)^2+\sqrt{(1-\gamma)(\gamma+1)(\gamma^2+1)(3-\gamma^2)^2}}{6\gamma(\gamma^2+1)} \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.

(iii) AD merger is more profitable than AB merger if

$\frac{18a\gamma-18\gamma-18a+9a^2+5\gamma^2+13}{36(1+\gamma)(1-\gamma)} \geq \frac{(2-\gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2}$. This holds for

$a \leq \frac{3(1-\gamma)(2-\gamma)(8+8\gamma+2\gamma^2-\gamma^3)-\sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4-12\gamma^2+16)}$ or

$a \geq \frac{3(1-\gamma)(2-\gamma)(8+8\gamma+2\gamma^2-\gamma^3)+\sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4-12\gamma^2+16)}$. Note that

$\frac{3(1-\gamma)(2-\gamma)(8+8\gamma+2\gamma^2-\gamma^3)-\sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4-12\gamma^2+16)} \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$ and

$\frac{3(1-\gamma)(2-\gamma)(8+8\gamma+2\gamma^2-\gamma^3)+\sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4-12\gamma^2+16)} \geq 1-\gamma$.

(iv) B merges with D: The market structure is the same as market structure F. In this parameter range, both firms offer both goods. The merger is profitable if $\frac{(2a\gamma-2\gamma-2a+a^2+2)}{9(1+\gamma)(1-\gamma)} \geq \frac{(2-\gamma(1-a))^2}{4(3-\gamma^2)^2} + \frac{(3a+2\gamma-3)^2}{4(3-\gamma^2)^2}$. For $\gamma \geq \sqrt{\frac{-24+\sqrt{1161}}{13}} \approx 0.88$, the inequality holds if

$a \geq \frac{-(1-\gamma)(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45)+\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(13\gamma^4+48\gamma^2-45)}$.

For $\gamma \geq \sqrt{\frac{-24+\sqrt{1161}}{13}}$,

$\frac{-(1-\gamma)(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45)+\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(13\gamma^4+48\gamma^2-45)} \geq 1-\gamma$ and the merger is not profitable.

For $\gamma < \sqrt{\frac{-24+\sqrt{1161}}{13}}$, the merger is profitable if

$\frac{(1-\gamma)(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45)-\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(45-13\gamma^4-48\gamma^2)} \leq a$

$\leq \frac{(1-\gamma)(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45)+\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(45-13\gamma^4-48\gamma^2)}$.

Note that $\frac{(1-\gamma)(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45)+\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(45-13\gamma^4-48\gamma^2)} \geq 1-\gamma$ and $\frac{(1-\gamma)(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45)-\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(45-13\gamma^4-48\gamma^2)} \geq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.

$\frac{(1-\gamma)(-39\gamma^2+9\gamma+9\gamma^3-4\gamma^4+45)-\sqrt{8\gamma^2(1-\gamma)(\gamma+1)(3-2\gamma^2)(3-\gamma^2)^2}}{(45-13\gamma^4-48\gamma^2)} \leq 1-\gamma$ if $\gamma \leq \sqrt{\frac{3}{5}} \approx 0.77$.

Case 3 ($a \leq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$**)**: (i) AB merger is profitable if $\frac{(2-\gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2} \geq \frac{2(1-\gamma)(3-\gamma)^2(1-a)+9a^2+7a^2\gamma^2}{(\gamma+3)^2(3-\gamma)^2(\gamma+1)(1-\gamma)} + \frac{(3-\gamma(1-a))^2}{(3-\gamma)^2(\gamma+3)^2}$. This holds for

$\frac{(1-\gamma)(2-\gamma)(5\gamma+3\gamma^2+8)-\sqrt{2(1-\gamma)(1+\gamma)(2-3\gamma^2)(2-\gamma)^2(\gamma+2)^2}}{(3\gamma^4-\gamma^2+16)} \leq a$

$\leq \frac{(1-\gamma)(2-\gamma)(5\gamma+3\gamma^2+8)+\sqrt{2(1-\gamma)(1+\gamma)(2-3\gamma^2)(2-\gamma)^2(\gamma+2)^2}}{(3\gamma^4-\gamma^2+16)}$. This never holds for $\gamma \geq \sqrt{\frac{2}{3}} \approx 0.82$. For $\gamma < \sqrt{\frac{2}{3}}$,

$\frac{(1-\gamma)(2-\gamma)\left(5\gamma+3\gamma^2+8\right)-\sqrt{2(1-\gamma)(1+\gamma)(2-3\gamma^2)(2-\gamma)^2(\gamma+2)^2}}{(3\gamma^4-\gamma^2+16)} \geq \frac{(3-\gamma)(1-\gamma)}{(3+\gamma^2)}$

if $\sqrt{2-\sqrt{2}} \leq \gamma \leq \sqrt{2+\sqrt{2}} \approx 1.85$. Note that

$\frac{(1-\gamma)(2-\gamma)\left(5\gamma+3\gamma^2+8\right)+\sqrt{2(1-\gamma)(1+\gamma)(2-3\gamma^2)(2-\gamma)^2(\gamma+2)^2}}{(3\gamma^4-\gamma^2+16)} \geq \frac{(3-\gamma)(1-\gamma)}{(3+\gamma^2)}$. Also

$\frac{(1-\gamma)(2-\gamma)\left(5\gamma+3\gamma^2+8\right)-\sqrt{2(1-\gamma)(1+\gamma)(2-3\gamma^2)(2-\gamma)^2(\gamma+2)^2}}{(3\gamma^4-\gamma^2+16)} \geq \frac{(1-\gamma)(2-\gamma)}{2+\gamma^2}$ if $\gamma \geq \sqrt{\frac{1}{2}}$.

For $\sqrt{\frac{1}{2}} \leq \gamma < \sqrt{2-\sqrt{2}}$, AB merger is profitable if

$\frac{(1-\gamma)(2-\gamma)\left(5\gamma+3\gamma^2+8\right)-\sqrt{2(1-\gamma)(1+\gamma)(2-3\gamma^2)(2-\gamma)^2(\gamma+2)^2}}{(3\gamma^4-\gamma^2+16)} \leq a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.

For $\gamma < \sqrt{\frac{1}{2}}$, AB merger is profitable if $\frac{(1-\gamma)(2-\gamma)}{2+\gamma^2} \leq a \leq \frac{(3-\gamma)(1-\gamma)}{3+\gamma^2}$.

For $a \leq \frac{(1-\gamma)(2-\gamma)}{2+\gamma^2}$, the merged firm AB produces both goods. The merger is profitable if

$\frac{\left(18a\gamma-18\gamma-8a+4a^2+5\gamma^2-10a\gamma^2+5a^2\gamma^2+13\right)}{36(1+\gamma)(1-\gamma)}$

$\geq \frac{2(1-\gamma)(3-\gamma)^2(1-a)+9a^2+7a^2\gamma^2}{(3+\gamma)^2(3-\gamma)^2(1+\gamma)(1-\gamma)} + \frac{(3-\gamma(1-a))^2}{(3-\gamma)^2(\gamma+3)^2}$. This holds for $a \geq \frac{5\gamma-3}{5\gamma}$.

Note that $\frac{5\gamma-3}{5\gamma} \leq \frac{(1-\gamma)(2-\gamma)}{2+\gamma^2}$ if $\gamma \leq \sqrt{\frac{1}{2}} \approx 0.7$. Therefore, for $\gamma \leq \sqrt{\frac{1}{2}}$, AB merger is profitable if $a \geq \frac{5\gamma-3}{5\gamma}$.

(ii) AD merger is profitable if

$\frac{18a\gamma-18\gamma-18a+9a^2+5\gamma^2+13}{36(1+\gamma)(1-\gamma)} \geq \frac{2(1-\gamma)(3-\gamma)^2(1-a)+9a^2+7a^2\gamma^2}{(3+\gamma)^2(3-\gamma)^2(1+\gamma)(1-\gamma)} + \frac{(3-3a-\gamma)^2}{(\gamma+3)^2(3-\gamma)^2}$. This holds for $a \leq \frac{3-5\gamma}{3}$. Note that $\frac{3-5\gamma}{3} \geq 0$ if $\gamma \leq \frac{3}{5}$.

(iii) BD merger is profitable if $\frac{\left(2a\gamma-2\gamma-2a+a^2+2\right)}{9(1+\gamma)(1-\gamma)} \geq \frac{(3-\gamma(1-a))^2}{(3-\gamma)^2(\gamma+3)^2} + \frac{(3-3a-\gamma)^2}{(\gamma+3)^2(3-\gamma)^2}$.

This holds for $a \geq \frac{-(1-\gamma)(3-\gamma)^3+\sqrt{(1-\gamma)(\gamma+1)(3-\gamma)^3(\gamma+3)^3}}{2\gamma(5\gamma^2+27)}$. Note that

$\frac{-(1-\gamma)(3-\gamma)^3+\sqrt{(1-\gamma)(\gamma+1)(3-\gamma)^3(\gamma+3)^3}}{2\gamma(5\gamma^2+27)} \geq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$.

(iv) Comparison of AB and AD mergers: In this parameter range, both merged firms would continue to offer both products. AB merger would give higher profits compared with AD merger if $\frac{(2-\gamma(1-a))^2}{(\gamma+2)^2(2-\gamma)^2} \geq \frac{18a\gamma-18\gamma-18a+9a^2+5\gamma^2+13}{36(1+\gamma)(1-\gamma)}$.

This holds for $\frac{3(1-\gamma)(2-\gamma)\left(8-\gamma^3+2\gamma^2+8\gamma\right)-\sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4-12\gamma^2+16)} \leq$

$a \leq \frac{3(1-\gamma)(2-\gamma)\left(8-\gamma^3+2\gamma^2+8\gamma\right)+\sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4-12\gamma^2+16)}$. Note that

$\frac{3(1-\gamma)(2-\gamma)\left(8-\gamma^3+2\gamma^2+8\gamma\right)+\sqrt{16(5-\gamma^2)(1-\gamma)^2(2-\gamma)^2(\gamma+2)^2(\gamma+1)^2}}{3(5\gamma^4-12\gamma^2+16)} \geq \frac{(3-\gamma)(1-\gamma)}{(\gamma^2+3)}$.

■