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THE ROLE OF UNCERTAINTY ON U.S. OBESITY: AN APPLICATION OF CONTROL THEORY

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The Role of Uncertainty on U.S. Obesity: An Application of Control Theory

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Abstract

This paper considers the problem of a consumer that cares about her health, which we proxy by deviations from current weight to ideal weight, and derives utility from eating and disutility from performing physical activity while taking into account the uncertainty associated with calorie consumption and physical activity. Using U.S. data, we find that uncertainty regarding the effectiveness of physical activity produces a larger cautionary response. Moreover, it is harder to learn and is more important to the agent than the uncertainty regarding the calorie content of food. These results can help policymakers design more cost effective policies.

JEL Classification: C61, I18.

Key Words: Obesity, Control, Multiplicative Uncertainty, Kalman Filter.

1 Introduction

During the past twenty years there has been a dramatic increase in obesity rates in the United States.[1] According to the Center for Disease Control and Prevention (CDC) the prevalence of obesity rates almost doubled from about 15% in 1980 to 27% in 1999. This trend continues to grow with an estimated 66% of Americans being diagnosed as either

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[1]The standard definition of obesity is a body mass index or BMI (weight divided by height squared) over 30 kg/m^2. BMI is a routinely used indirect measure for body fatness, specifically obesity, in epidemiological research and is highly correlated with other direct measures like Dual-energy x-ray absorptiometry (DEXA) for older populations.

overweight or obese for the period 2003-2004.[1] In 2008, only one state (Colorado) had a prevalence of obesity less than 20%. Thirty-two states had a prevalence equal to or greater than 25%; six of these states (Alabama, Mississippi, Oklahoma, South Carolina, Tennessee, and West Virginia) had a prevalence of obesity equal to or greater than 30%.

Individual behaviours, environmental factors, and genetics all contribute to the complexity of the obesity epidemic. Economics can shed light on how people make choices when improving health outcomes is a main objective. This approach allows rational behaviour to be brought into focus which can help understand the causes associated with the rise in obesity.

The role of technological change in modifying individual eating and exercise patterns has been the main channel used to explain the rise in obesity. In particular, the literature has explored how technological change caused food prices to fall. Lower food prices increase the consumption of food, which translates into higher calorie consumption.[2] The literature has also examined how technological change transformed the type of work people perform, from physically demanding to sedentary jobs, and how affects the time devoted to cooking activities. The increase in sedentary jobs implies a reduction in calories spent and less cooking time. These factors further increase the consumption of food away from home which tends to have more calories.[3] Thus, understanding why economic agents make choices that result in higher obesity rates is crucial in shaping policies to reverse the obesity epidemic.

In this paper we analyse an aspect of the obesity debate that has not been fully explored by the literature. The role of information on obesity will be considered in light of how choices regarding net calorie intake are affected by different types of uncertainty. The sources of uncertainty that an economic agent faces when controlling her weight are biological (how effective is physical activity in burning calories) and regulation (labelling policies regarding the calorie content of food away from home). Given such an uncertain environment, the goal herein is to determine the sources of uncertainty that most reduce the economic welfare of agents. This in turn, will help identify policy actions, labelling policies or campaigns promoting exercise, that are more valuable.

Regarding calorie intake uncertainty, in 1994 the National Labelling and Education Act (NLEA) required manufacturers to include a nutrition information panel on the label of almost all packaged foods, however this policy did not require any disclosure for foods purchased at restaurants. Refer as food away from home herein. The current NLEA imposed significant changes in the information about calories and nutrients that manufacturers of packaged foods must provide to consumers. This change in regulation could affect health outcomes. In this spirit, Variyam and Cawley (2006) find that the NLEA labels had a beneficial impact on health outcomes. The authors find that as a result of the new labels introduced by the NLEA, the BMI and probability of obesity among white female label users were significantly lower than they would have been in the absence of the new labels.[4]

Given that the current NLEA does not require nutrition information for foods away from home, consumers may increase the chances of misjudging the nutrient content of meals eaten out, inadvertently consuming more calories.[5] This lack of calorie information in foods away from home is quite relevant to the obesity epidemic since these types of foods represent a large and increasing share of total food expenditures. Americans spent about 46% of their total food budget on food away from home in 2002, compared to 27% in 1962. Moreover, USDA's food intake surveys show that between 1977-78 and 1994-96, the share of daily caloric intake from food away from home increased from 18% to 32%. These are important factors to consider when studying obesity issues and motivates our work.

With respect to the biological uncertainty, the amount of calories spent while exercising varies depending on the activity undertaken, the intensity level and individual specific characteristics. The two different sources of uncertainty we consider in this paper can yield significant different calorie expenditures across activities and intensity levels. The goal of this paper is to determine which source of uncertainty has the greater impact.

The analytical framework we consider is based on a representative consumer that cares about her health, which we proxy by minimizing the deviations from what is considered a healthy weight and her actual weight, and derives utility from eating and disutility from exercising.[6] We analyse a modified version of the traditional Linear Quadratic Tracking model with one control and two state variables while facing two different sources of uncertainty. Thus, in order to maximize utility, the representative agent chooses: the amount of exercise and calories to consume, which are the controls of the problem. Finally, a situation where the agent can learn something regarding the underlying sources of uncertainty is considered. The model is calibrated to U.S. data.

In this paper we find that biological uncertainty produces a larger cautionary reaction in physical activity and food consumption than uncertainty due to regulation. Thus, our results suggest that it may be more beneficial to implement policy that provides more information regarding the amount of calorie expenditure from physical activity rather than a labelling policy for foods away from home. In terms of welfare, eliminating small levels of biological uncertainty represents a utility gain equivalent to 9% more food intake or 36% less exercise. Similarly, eliminating small degrees of calorie intake uncertainty represents a utility gain equal to 8% more food intake or 35% less physical activity. These results provide estimates that reflect the willingness of the representative agent to pay to implement policies that reduce uncertainty.

When learning is introduced into this proposed framework, our results show that the representative agent obtains higher welfare when learning about biological uncertainty than when learning about calorie intake. However, learning about calorie intake is easier. These results suggest that information policies regarding calories burnt during physical activity may be more useful and valued by consumers than a labelling policy.

The following section presents a formal model of weight control, the analytical solution and the calibration of the parameters. Section III calibrates the parameters in the model. Section IV provides the optimal response to changes in each type of uncertainty. Section V analyses the cost of the different types of uncertainty while Section VI incorporates learning into the framework. Finally, Section VII offers some concluding remarks.

2 A Formal Framework

While genes are important in determining a person's susceptibility to weight gain, from an accounting point of view, people gain weight if calories consumed are greater than calories expended. Thus, being overweight is a result of energy imbalance for a given period of time.

Body weight is measured in pounds and evolves according to the following transition equation:

$$x_{k+1} = x_k - \gamma BMR_k + \gamma c_k \tag{1}$$

where x_k denotes actual weight in period k, BMR is the basal metabolic rate, which represents the amount of calories used in supporting the essential human activities, such as breathing and heart movements, and c_k is the net added calories in period k. Finally, the parameter γ is a conversion factor that translates calories into pounds.

Following Ladkawalla and Philipson (2003), we assume that BMR is a linear function of the weight which is given by: BMR=$\lambda x_k + \mu_k$; where μ is an additive noise that captures the approximation error as well as the uncertainty about the relation between the basal metabolic rate and actual weight, and λ is the fraction of actual weight required for agents to satisfy the basic metabolic functions. Hence, Eqn. (1) can be written as follows:

$$x_{k+1} = \alpha x_k + \gamma c_k - \gamma \mu_k; \tag{2}$$

where $\alpha \equiv (1 - \gamma\lambda) \in (0, 1)$ denotes the effective fraction of weight an individual carries over from the previous period. Moreover, net calorie intake, c_k, can be defined as follows:

$$c_k = \pi Z_k - E_k; \tag{3}$$

where Z_k is the gross intake of calories through food consumption during period k, $\pi \in (0, 1)$ represents the fraction of calories consumed during digestion and E_k is the amount of calories spent during physical activity. Following Cutler, Glaeser and Shapiro (2003), E_k is a function of weight and amount of time devoted to physical activity, which is given by:

$$E_k = \tau \langle x \rangle u_k + \psi_k; \tag{4}$$

where $\langle x \rangle$ is the average weight of the agent, u_k is the number of hours of physical activity, τ measures the average effectiveness of physical activity in burning calories and ψ_k is the error term that captures the non-linearities between weight and physical activity as well as any uncertainty.

Finally, we assume that there is some measurement error when accounting for the actual amount of calories consumed.[7] In particular, we define Z_k as the actual amount of calories consumed and F_k as the assumed amount of calories consumed. The following equation shows their relationship:

$$Z_k = HF_k + \theta_k \tag{5}$$

where H is the measurement parameter and θ_k denotes the measurement error. The parameter H is unknown to the consumer. Hence, H and θ_k represent the uncertainty that is related and unrelated to the actual amount of calorie content of food, respectively. Eqn. (5) explicitly reflects the fact that the agent imperfectly accounts the total amount of calories consumed. Substituting Eqns. (3)-(5) into Eqn. (2), we obtain the following transition equation:

$$x_{k+1} = \alpha x_k + \beta u_k + \phi F_k + \varepsilon_k \tag{6}$$

where $\beta \equiv -\gamma \tau \langle x \rangle$, $\phi \equiv \gamma \pi H$ and $\varepsilon_k \equiv \gamma \pi \theta_k - \gamma(\mu + \psi)$.

An important feature that has not been fully explored so far in the literature is the role of uncertainty on weight control. In order to do so, we let β and ϕ be stochastic to consider the biological and labelling sources of uncertainty, respectively. This approach allows clear identification of the underlying uncertain parameters of the model, τ in β (in the case of burning calories), and H in ϕ (in the case of consuming calories) which allows for the study of their impact on the eating and physical activity decision of agents.[8] These two different sources of uncertainty can potentially yield different calorie expenditures across activities.

The goal of this paper is to determine which source of uncertainty has the greater impact on welfare.

Given the evolution of body weight, the problem of a representative consumer that cares about her health and derives utility from eating and disutility from performing physical activity while taking into account net calorie uncertainty will now be considered. In particular, the consumer problem can be expressed as minimizing the difference between the observed and desired weight, which is a proxy for health, as well as maximizing the utility gains from eating and disutility from exercising subject to the transition weight equation. The representative agent chooses a strictly positive sequence of exercise and calories, $\{u_k, F_k\}_{k=0}^{N-1}$ knowing the mean and variance of the stochastic terms, when making her net calorie decision. Moreover, since the Riccati equations for the Linear Quadratic Problem emerges from the first-order conditions alone, then this problem can also be expressed as finding the controls $\{u_k, F_k\}_{k=0}^{N-1}$ that minimize the objective function J of the form:

$$\underset{\{u_k,F_k\}_{k=0}^{N-1}}{min} J = E\left\{\delta^N \frac{W_N}{2}[x_N - x_N^{\#}]^2 + \sum_{k=0}^{N-1} \delta^k \left\{\frac{W_k}{2}[x_k - x_k^{\#}]^2 + aF_k^2 + b[D - u_k]^2\right\}\right\} \tag{7}$$

subject to

$$x_{k+1} = \alpha x_k + \beta u_k + \phi F_k + \varepsilon_k \tag{8}$$

$$\varepsilon \sim (0, \sigma)\ , \beta \sim (\langle\beta\rangle, \sigma_\beta)\ , \phi \sim (\langle\phi\rangle, \sigma_\phi)\ ;$$

where $x_k^{\#}$ is the desired weight, W_k represents the penalty matrices for period k for deviations between the actual and desired weight, and δ denotes the discount factor. a and b are positive constants that denote the relative importance of the utility gains from eating food and disutility from exercising, respectively, and D represents the total number of minutes available to exercise. Note that W_k also denotes the importance of good health relative to food consumption and exercise. The initial condition x_0 is given and represents the initial weight of the agent. Finally, since x_k cannot take negative values, the distribution of the uncertain parameters and the additive noise are restricted to be non-asymptotic.

2.1 Analytical Solution.

The solution to representative agent's problem, Eqns. (7) and (8), is given by the following feedback rule:[9]

$$\begin{pmatrix} u_k \\ F_k \end{pmatrix} = \mathbf{G_k} x_k + \mathbf{g_k} \tag{9}$$

where G_k and g_k are the feedback matrices given by the following equations:

$$\mathbf{G_k} = \begin{pmatrix} G_{u_k} \\ G_{F_k} \end{pmatrix} = -\frac{1}{det}\begin{pmatrix} \alpha\langle\beta\rangle k_{k+1}(\delta k_{k+1}\sigma_\phi^2 + a) \\ \alpha\langle\phi\rangle k_{k+1}(\delta k_{k+1}\sigma_\beta^2 + b) \end{pmatrix} \tag{10}$$

$$\mathbf{g_k} = \begin{pmatrix} g_{u_k} \\ g_{F_k} \end{pmatrix} = -\frac{1}{det}\begin{pmatrix} \langle\beta\rangle\rho_{k+1}(a + \delta k_{k+1}\sigma_\phi^2) - 2bD(a + \delta k_{k+1}(\sigma_\phi^2 + \langle\phi\rangle^2) \\ \langle\phi\rangle\rho_{k+1}(b + \delta k_{k+1}\sigma_\beta^2) + 2\delta bD\langle\beta\rangle\langle\phi\rangle k_{k+1} \end{pmatrix} \tag{11}$$

where $\langle\beta\rangle$ and $\langle\phi\rangle$ are the expected values of β and ϕ, k_{k+1} and ρ_{k+1} are the corresponding Riccati matrices of the control problem and det is the determinant of the following matrix.

$$\mathbf{\Theta} = \begin{bmatrix} \delta k_{k+1}[\sigma_\beta^2 + \langle\beta\rangle^2] + b & \delta k_{k+1}\langle\beta\rangle\langle\phi\rangle \\ \delta k_{k+1}\langle\beta\rangle\langle\phi\rangle & \delta k_{k+1}[\sigma_\phi^2 + \langle\phi\rangle^2] + a \end{bmatrix}$$

which is part of the solution of $\mathbf{G_k}$ and $\mathbf{g_k}$. The analytical solution to the Riccati matrices, k_{k+1} and ρ_{k+1}, are shown in Appendix A and the value of the determinant is given by the following expression:

$$det = \delta k_{k+1}\{\delta k_{k+1}(\sigma_\beta^2\sigma_\phi^2 + \sigma_\beta^2\langle\phi\rangle^2 + \sigma_\phi^2\langle\beta\rangle^2) + (\sigma_\beta^2 + \langle\beta\rangle^2)a + (\sigma_\phi^2 + \langle\phi\rangle^2)b\} + ba. \tag{12}$$

The complexity of the analytical solution does not allow us to make conclusive predictions regarding the optimal decisions since they depend on underlying parameter values of the model. Thus, in order to have sharper predictions regarding the optimal strategy for representative consumer the model needs to be parameterized.

3 Calibration of the Model

We calibrate the model by matching the observables derived from the model to the corresponding U.S. data counterparts. Hence, the number of calories per pound of weight is 3,500, which implies $\gamma = 1/3500 = 0.0002857$.

We simplify the calibration by normalizing the "nutritional" parameters and set $H = 1$. Regarding net calories, Burke and Heiland (2004) report values of π=0.9, λ=3.19 and μ=844. This allows us to compute ϕ=0.0002571, α=0.999089 and BMR=1450.8. Moreover, we assume the traditional discount rate of 3% per year which implies the following daily discount factor: δ=$exp(-0.03/365)$=0.999918. Thus, in our model k represents one day and N=365.[10]

Metabolic differences between males and females exist. Thus, given the information available we decide to calibrate our model to a representative female. The average weight of a woman in the U.S. in 2004 was 164 pounds according to the CDC. This weight is used as the initial value of the control variable and $\langle x\rangle$ in Eqn. (4). Dong, Block and Mandel (2004) report an average energy expenditure of 37.8 calories per day per kilogram for women in the U.S. According to the average weight of 164 pounds, this corresponds to a total energy expenditure of 2,812 calories per day. Nevertheless, this last estimate includes the energy expenditure through the BMR. In order to calibrate $\tau\langle x\rangle$ we subtract the BMR to the total energy expenditure and divide it by the total number of minutes in a day. Hence, we obtain $\tau\langle x\rangle = 0.94527778$ which implies that $\beta = -0.0002700794$.

The desired weight target in pounds assumes a BMI of 22, which is the mean of what is considered healthy. Given that the average height of women is 5 feet and 4 inches, the resulting target weight is then 128 pounds. That is, a weight loss of 36 pounds or 22% in a year. A linear path is set for the desired weight from 164 to 128 in a year. This implies that the individual looses about 0.0987 pounds per day or 0.7 pounds per week. This level of weight loss is below the maximum safe value of two pounds per week, according to the Food an Drug Administration.[11]

Since there are no direct estimates of a and b, we calibrate them so that the initial values of calorie intake and energy expenditure match the values reported or implied in the literature. We find that a=7.5E-07 and b=9.8E-07, in the absence of any uncertainty, produce an initial value of F_k equal to 1747 calories per day and an energy expenditure above BMR of 1476 calories per day.[12] The energy expenditure for the initial condition of 1476 calories per day is consistent with the calorie expenditure of physical activity above BMR implied by the values reported on Burke and Heiland (2004) and Dong, Block and Mandel (2004).

All parameter values used in our numerical examples are given by Table 1.

Table 1: Parameter Values

δ	0.9988	α	0.99894
a	8.1E-07	β	-0.00027
b	1.1E-06	ϕ	0.000257
D	1440	x_0	164

Finally, the penalty matrices W_k represent the value of health to the individual relative to consuming food and exercising. Moreover, the evolution over time of these matrices can also be interpreted as some sort of commitment that the individual has toward losing weight and improving her health. In this paper, we confine the analysis to cases where the individual is committed to losing weight.[13]

We choose values of W_k that produce an optimal weight close to the desired level and at the same time, rule out unhealthy or unfeasible levels of calorie intake and exercise, particularly in the first and last period. That is, we dismiss values of W_k that generate trajectories where the weight loss occurs in the first and last couple of days by starvation and/or amounts of exercise that are not feasible. Hence, we first choose a value of W_N that does not produce an unhealthy decrease of calorie intake and an unattainable increase in exercise in the last period. The rest of the trajectory is obtained by smoothly decreasing W_N until W_0. The increasing value of W_k as $k \to N$ reflects the commitment of the individual to achieve the desired weight loss. Thus, we use the following sequence for the penalties: $W_k = 40 * (k+1)$ for $k = 0, ..., N$.

The difference in the magnitude of W_k with respect to a and b is explained by the units of the variables that each of them multiplies in the utility function. The values of W_k are much larger than a and b. However, the deviations between the actual and the target weight are close to zero, whereas the values of calorie consumption (F_k) and minutes of exercise over 1440 ($D - u_k$) are significantly greater than zero. Expanding the quadratic term on exercise in Eqn. (7) we obtain that in terms of utility the effect of exercise is given by $-2bDu_k + bu_k^2$. Thus, using the suggested parameter values, the effect of calorie consumption on utility in steady state for the certainty equivalence case is 31% greater than the effect of physical activity.[14]

4 Results

Given our calibration strategy, we can interpret our problem as having a representative agent in the year 2004 that tries to control her weight over one year by choosing the amount of food and the time devoted to physical activities.

In this section we explore the role of different sources of uncertainty. Our benchmark model considers the case when there is no uncertainty in β or ϕ; that is, the certainty equivalence solution (CE). We then compare how the different sources of uncertainty affect the optimal behaviour of the representative agent. Moreover, since the additive noise does not impact the optimal policy rule (as shown in Section 2.1) and its mean value is zero, we set it equal to zero in the numerical results with exception to the solution to the Kalman filter in Section 5.

To simplify notation throughout the rest of the paper we normalize u_k by the total number of minutes in a day, $u^n = u_k/1440$. Hence, u_k^n represents the amount of physical activity with respect to a maximum of physical activity. Thus, changes in u_k^n represent changes in either intensity or time devoted to physical activity or both.

4.1 Certainty Equivalence Results.

In this section, we consider a situation where the expected values of the state variables of the economy are given and the optimal policy is independent of all higher moments. That is, in the certainty equivalence case we consider an environment where the underlying variance of the uncertain processes do not matter; i.e, we set $\sigma_\beta^2 = \sigma_\phi^2 = 0$. Hence, the values of physical activity and food intake in the certainty equivalence results are defined as follows:

$$u_k^{n*} = (u_k|_{\sigma_\beta^2=\sigma_\phi^2=0})/1440 \qquad F_k^* = F_k|_{\sigma_\beta^2=\sigma_\phi^2=0} \qquad x_k^* = x_k|_{\sigma_\beta^2=\sigma_\phi^2=0} \tag{13}$$

Figure 1 depicts the optimal trajectory of weight, normalized physical activity and gross calories intake through food consumption in this environment.

Figure 1: Certainty Equivalence Weight, Calorie intake and Physical Activity

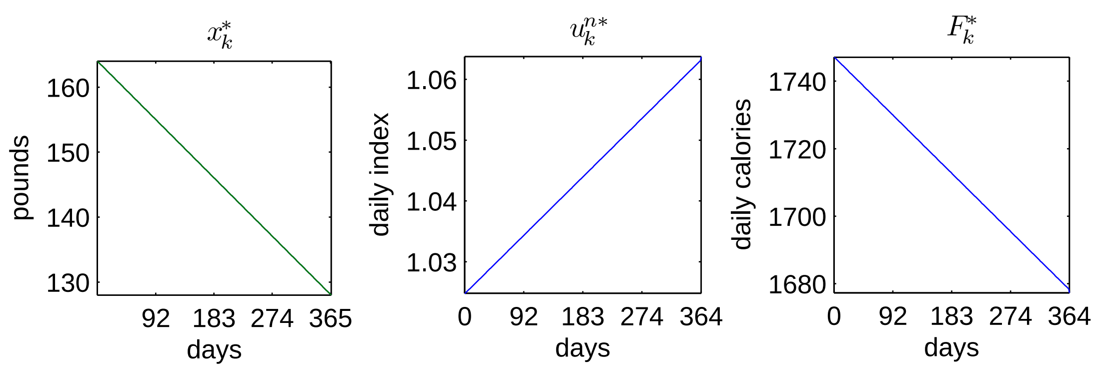


Figure 1 shows that physical activity increases while food consumption decreases. The combination of lower calorie intake and higher physical activity allows the agent to reach the desired weight daily.

Since our representative agent is far from her ideal body weight, it is optimal to lower her calorie intake and increase her physical activity. In particular, the disutility she derives from actually "being" unhealthy is much larger than the disutility she obtains by eating fewer calories and exercising more. Thus, the agent increases daily physical activity by 3.6% during the entire year which corresponds to an increase in daily calories burnt of 52.6 from the first to the last day of the year. On the other hand, the amount of daily calorie intake decreases monotonically 69.2 calories or 4.1% from beginning to end.

While changes in both physical activity and calorie intake achieve the desired weight and maximize overall utility, a more substantial adjustment comes from the calorie intake when the initial condition is far from her ideal weight. As our results indicate, the representative agent has an asymmetric response in her optimal decisions when higher moments of the underlying uncertainty are ignored. In particular, the representative agent decides to consume less calories rather than exercise more in order to achieve her ideal weight.

The next section examines the robustness of these results once higher moments of the different sources of uncertainty are considered.

4.2 Multiplicative Uncertainty Results.

In this subsection the role of multiplicative uncertainty is considered. In particular, we examine how different types of uncertainty due to the current labelling policies, ϕ, or biological, β, impact the optimal level of physical activity and food consumption of the representative consumer. We note than when there is uncertainty in the environment the overall utility of the agent will decrease relative to the CE solution.

We introduce the importance of higher moments in the different sources of uncertainty by finding the appropriate variance values for ϕ and β so that we obtain 1%, 0.95% and 0.90% for the coefficient of variation (CV). Since β is negative we define CV as follows:

$$CV_\beta = \sigma_\beta/|\beta| \qquad \text{and} \qquad CV_\phi = \sigma_\phi/|\phi|. \tag{14}$$

By forcing the CV to a certain value across both types of uncertainty, the impact on welfare with similar magnitudes of uncertainty due to labelling policies or biological factors can be compared.

4.2.1 Multiplicative Uncertainty in Calorie Intake.

In this subsection we study the effects of higher uncertainty in the calorie intake, which is captured by ϕ, on the optimal response of the representative agent. Relative to the CE this new environment attempts to capture the effects of the current NLEA that do not require nutritional information for foods away from home. Thus consumers may misjudge the nutrient content of meals eaten out, inadvertently consuming more calories.

In order to perform this experiment, we set σ^2_β=0 and choose values of σ_ϕ that yield a CV_ϕ of 1%, 0.95% and 0.90%.[15] The resulting values for optimal food consumption and physical activity are given by $F_{\phi,k}$ and $u_{\phi,k}$:

$$u^n_{\phi,k} = u^n_k|_{\sigma^2_\phi>0,\sigma^2_\beta=0} \qquad F_{\phi,k} = F_k|_{\sigma^2_\phi>0,\sigma^2_\beta=0} \tag{15}$$

To quantify the impact of the different sources of uncertainty, we compute the percentage change in $u^n_{\phi,k}$, $F_{\phi,k}$ and $x_{\phi,k}$ relative to the corresponding values in certainty equivalence case. Thus we define the following relative measures:

$$u'_{\phi,k} = \left(u^n_{\phi,k}/u^{n,*}_k - 1\right) \qquad F'_{\phi,k} = \left(F_{\phi,k}/F^*_k - 1\right) \tag{16}$$

A positive value of $u'_{\phi,k}$ or $F'_{\phi,k}$ implies that higher uncertainty in ϕ increases the amount of physical activity or food intake. Therefore, in this case, higher uncertainty in the calorie content of food would produce an aggressive response of physical activity and food intake. Similarly, negative values would imply a precautionary response relative to the CE case.

The results of our experiments are summarized in Figure 2 which displays the values for $u'_{\phi,k}$, and $F'_{\phi,k}$ when $CV_\phi = 1\%$, 0.95% and 0.90%. Since we do not find any significant effect in the optimal weight with respect to its CE value, we do not report it in Figure 2.

Figure 2: Effect of σ^2_ϕ on physical activity and food consumption.

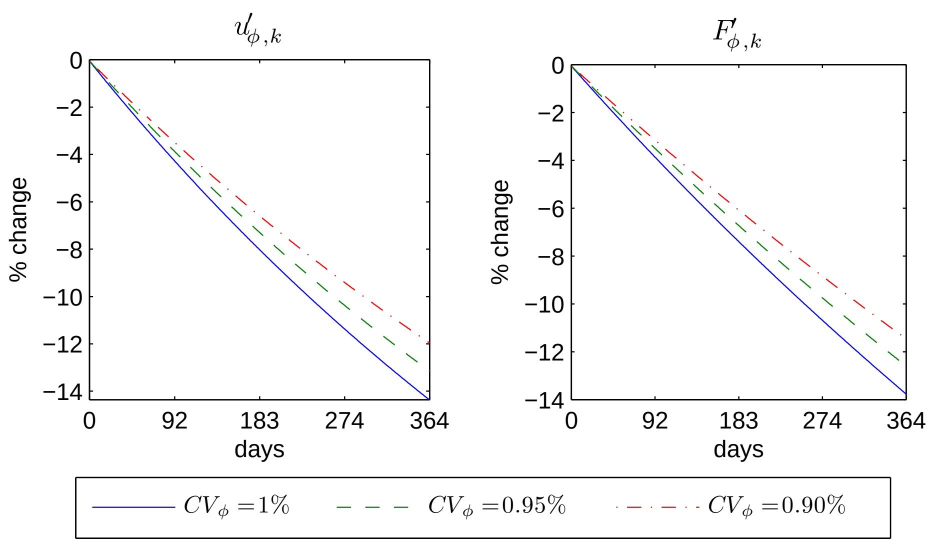


Figure 2 shows that $u'_{\phi,k}$ <0 and $F'_{\phi,k}$ <0 for all the three degrees of uncertainty considered. Moreover, the decrease in both variables is monotone throughout the year. When $CV = 1\%$, physical activity and calorie intake at the last day of the year decrease by approximately 14% in relation to the case when there is no uncertainty. Physical activity and calorie intake at the end of the year is 11% and 17% lower respectively, than in the first period.

In addition, Figure 2 indicates that higher degrees of uncertainty in ϕ produce higher reductions on the level of physical activity and food intake. This cautionary response is driven by the risk aversion of the representative agent. These results echo the traditional cautionary results of Brainard (1967). Note that relative to the CE, when there is uncertainty in the calorie content of food, the agent faces a higher variance in her body weight. A risk averse agent will reduce food consumption to decrease the variance in body weight, the precautionary response. However, this decrease in calorie consumption has also the effect of reducing the amount of physical activity required to achieve the same weight target. Notice that by lowering physical activity, the individual can attain the desired weight and also decrease the disutility from exercising.

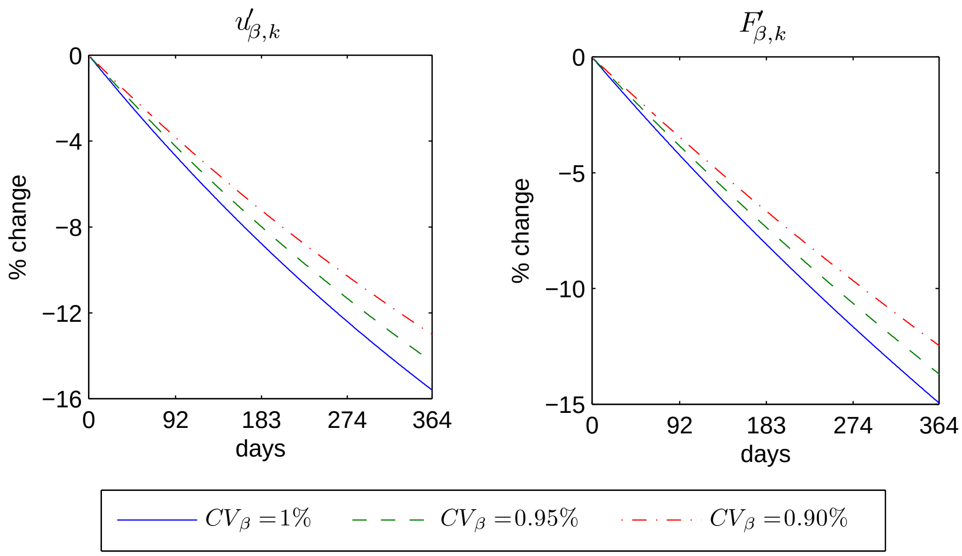
Finally, we note that even though the source of uncertainty arises from the calorie content of food, the representative agent has almost symmetric responses, in terms of magnitude, for calorie consumption and physical activity. This experiment then suggests that a comprehensive labelling policy that removes most of the nutritional uncertainty of food away from home, would result in less calories consumed and an increase in physical activity relative to the full information and certain case. In other words, dieting would be more effective which in turn would provide more incentives to exercise in order to achieve the healthy weight.

4.2.2 Multiplicative Uncertainty in Biological Factors.

In order to analyse the biological uncertainty we set $\sigma^2_\beta \neq 0$ and $\sigma^2_\phi = 0$. We then follow a similar procedure as in the previous case and examine the effect on optimal physical activity, food consumption and weight.

Figure 3 shows the results for $u'_{\beta,k}$ and $F'_{\beta,k}$ when $CV_\beta = 1\%$, 0.95% and 0.90%. Again, since we find that the optimal weight does not change with respect to its CE solution, we do not report it in Figure 3.

Figure 3: Effect of σ^2_β on physical activity and food consumption.



In general, the results in Figure 3 for higher uncertainty in β are similar to those of the uncertainty stemming from ϕ; i.e, $u'_{\beta,k}<0$ and $F'_{\beta,k}<0$. Thus, higher uncertainty in the effectiveness of physical activity to burn calories reduces the amount of food consumption and physical activity. In particular, when $CV = 1\%$ the difference in physical activity with respect to the corresponding values in the certainty equivalence case increases close to zero in the first period to 16% at the end of the year. Similarly, food consumption decreases from almost zero in the first period to 15% in the last period with respect to the corresponding values in the certainty equivalence case. Thus, relative to the beginning of the year, calorie consumption and physical activity in the last period are 19% and 12% lower, respectively.

This precautionary response of physical activity and food consumption is also a result of the risk aversion of the representative agent. Higher uncertainty in β makes the effect of physical activity on the weight more uncertain. A risk averse agent will lower the amount of physical activity to reduce the variance of the weight. Given that the representative agent is experiencing a large disutility from being away from her ideal body weight, lower physical activity requires lower food consumption. The decrease in physical activity and achieving the desired weight increases utility, whereas higher food consumption reduces utility.

Summarizing, when the source of uncertainty arises from biological factors, the representative agent has asymmetric responses, in terms of magnitude, for calorie consumption and physical activity. In particular, when faced with biological uncertainty the representative agent has a much stronger response to physical activity than in her calorie intake decision.

From the two previous experiments we can conclude that the two different sources of uncertainty are not symmetric. Decisions regarding the reduction in calorie consumption and an increase in exercise crucially depend not only on the magnitude of that uncertainty but also its source.

4.3 Relative Response to Multiplicative Uncertainty.

In this subsection, we further formalize the results of the preceding sections. In particular, we identify what type of multiplicative uncertainty biological (β) or due to labelling laws (ϕ) produces a larger precautionary response. This exercise puts forward policy actions, labelling policies or campaigns promoting exercise, that may increase welfare.

In order to identify possible actions, we compare the magnitudes of the precautionary responses to changes in σ_β^2 and σ_ϕ^2 for the same level of uncertainty (i.e. same coefficient of variation). To simplify exposition we define the following relative measures:

$$u_k^\circ = (u_{\beta,k}/u_{\phi,k} - 1) \quad F_k^\circ = (F_{\beta,k}/F_{\phi,k} - 1) \quad \text{where } CV_\beta = CV_\phi \tag{17}$$

If a variable in Eqn. (17) is positive (negative) then the response is more (less) cautionary to changes in σ_β^2 than to σ_ϕ^2 (for the same relative level of uncertainty in β and ϕ). This implies that for such a variable the uncertainty in β is more (less) important, in terms of the reaction, than the uncertainty in ϕ.

Figure 4 shows u_k° and F_k° for $CV_\beta = CV_\phi = 1\%, 0.95\%$ and 0.90%. The results in Figure 4 are the implied ratios of the results shown in Figure 2 and 3. Since the optimal weight is almost unchanged when β and ϕ are uncertain, we do not include it in Figure 4.

Figure 4: Relative response of u_k and F_k to uncertainty in β and ϕ.

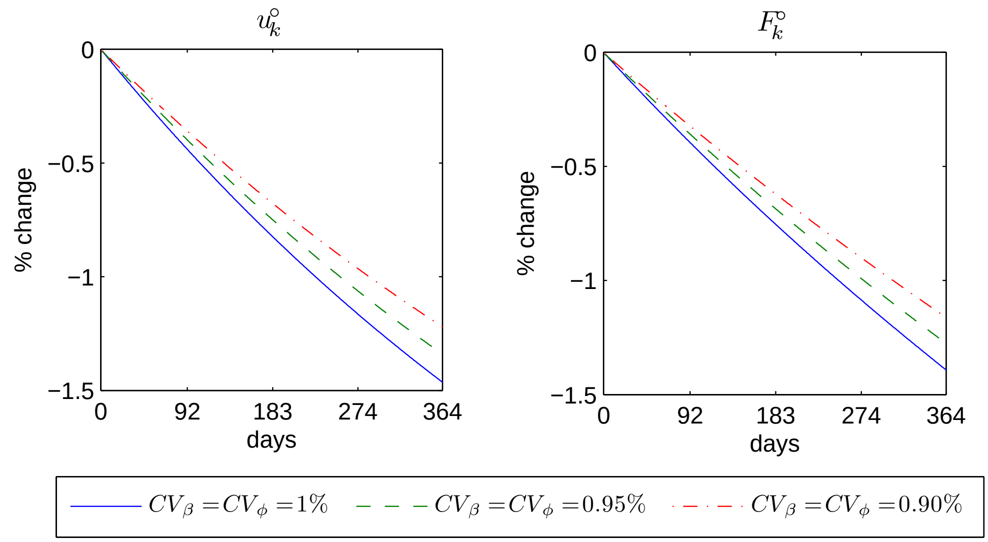


Figure 4 shows that the physical activity and food consumption responses are more cautious when the source of uncertainty is due to biological factors rather than labelling. That is, uncertainty about the effectiveness of physical activity to burn calories produces a larger cautionary reaction in physical activity and food consumption than the uncertainty about the calorie content of food.

Figure 4 also shows that the difference in physical activity and food consumption in the case of $CV_\beta = 1\%$ compared to $CV_\phi = 1\%$ is almost zero in the first period. This difference increases at a decreasing rate until the last period where $u_{\beta,k}$ is 1.5% lower than $u_{k,\phi}$ and $F_{\beta,k}$ is 1.4% lower than $F_{\phi,k}$. The two lines above the case where $CV_\beta = CV_\phi = 1\%$ indicate two interesting results. First, the difference in the amount of physical activity and food consumption increase with higher degrees of uncertainty. Second, the disparity between the different degrees of uncertainty is lower for the initial periods than for the later periods.

These results highlight the fact that the agent responds asymmetrically to the different sources of uncertainty. This in turn suggests that it may be more beneficial to have a policy that provides more information regarding the amount of calorie expenditure when exercising than a labelling policy for foods away from home. In order to precisely determine which type of policy would be more valued for consumers it is important to know how much agents are willing to give up in terms of calorie consumption when faced with different sources of uncertainty.

5 Cost of Uncertainty

Taking into account the resources required to reduce uncertainty, it is important to estimate the benefit of reducing the different sources of uncertainty in terms of food consumption or physical activity. This consideration allows policy makers to compare the benefits and costs of potential information policies.

In this section, we obtain the cost of multiplicative uncertainty for the consumer in terms of food consumption and physical activity. In particular, we compute the additional amount of calorie intake the individual has to consume, in the presence of multiplicative uncertainty, to achieve the same level of utility as in the case where there is no uncertainty (i.e. the certainty equivalence case). Similarly, in a separate experiment, we compute the additional amount of physical activity that the individual has to undertake in the presence of multiplicative uncertainty to achieve the same level of utility as in the certainty equivalence case.

5.1 Cost of Uncertainty in Calorie Intake.

In this subsection we want to compute how much it would cost to have a comprehensive labelling policy. In other words, what would be the cost to expand the current NLEA so that nutritional information requirements for foods away from home are also included. Rather than computing this cost in terms of welfare we compute it in terms of calories consumed or burnt through exercise.

In order to do so, we undertake the following steps. First, we obtain the utility level for each period in the certainty equivalence case, $(J_{ce,k})$. Second, we introduce uncertainty in ϕ and obtain the corresponding level of weight $(x_{\phi,k})$, physical activity $(u_{\phi,k})$ and calorie intake $(F_{\phi,k})$ for each period. Third, we set the utility in the certainty equivalence equal to the expression of the utility in the multiplicative case plus the percentage change in either the calorie intake or physical activity. The expression where the physical activity is the variable allowed to adjust is the following:

$$J_{ce,k} = \beta^k(0.5(x_{\phi,k} - x_k^{\#})^2 W_k + aF_{\phi,k}^2 + b(D - u_{\phi,k}(1 - \Delta u_{\phi,k}))^2) \tag{18}$$

where $\Delta u_{\phi,k}$ is the percentage change in physical exercise that makes the utility when ϕ is uncertain equal to the utility in the certainty equivalence case. Given that multiplicative uncertainty reduces utility and physical activity generates disutility, the latter has to decrease to increase utility, i.e. $\Delta u_{\phi,k} < 0$. The expression where food consumption changes to equate utility is given by the following:

$$J_{ce,k} = \beta^k (0.5(x_{\phi,k} - x_k^{\#})^2 W_k + a(F_{\phi,k}(1 - \Delta F_{\phi,k}))^2 + b(D - u_{\phi,k})^2) \tag{19}$$

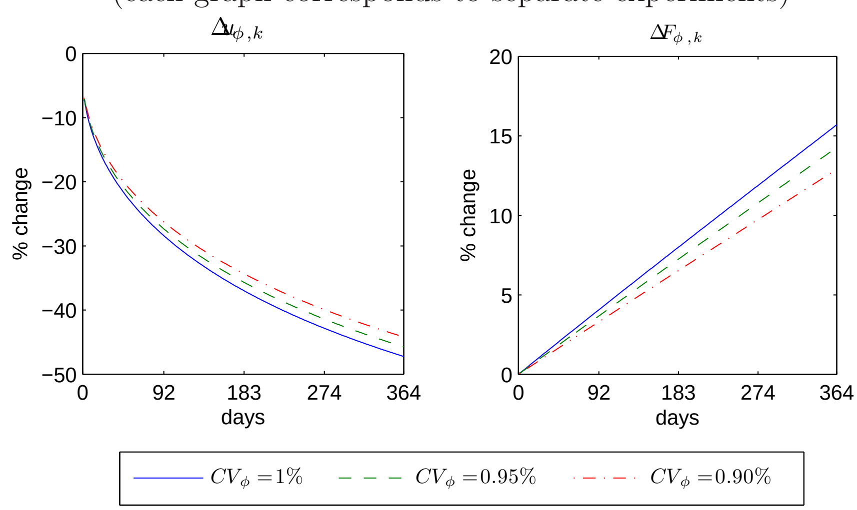
where $\Delta F_{\phi,k}$ is the percentage change in calorie intake that makes the utility when ϕ is uncertain equal to the utility in the certainty equivalence case. Given that multiplicative uncertainty reduces utility, food consumption has to increase to equate utilities, i.e. $\Delta F_{\phi,k} > 0$. Finally, we solve $\Delta u_{\phi,k}$ and $\Delta F_{\phi,k}$ from Eqns. (18) and (19) which yield

$$\Delta u_{\phi,k} = D/u_{\phi,k} - \left(J_{ce,k}/b\beta^k u_{\phi,k}^2 - (0.5(x_{\phi,k} - x_k^{\#})^2 W_k + aF_{\phi,k}^2)/bu_{\phi,k}^2\right)^{0.5} - 1 \tag{20}$$

$$\Delta F_{\phi,k} = \left(J_{ce,k}/a\beta^k F_{\phi,k}^2 - (0.5(x_{\phi,k} - x_k^{\#})^2 W_k + b(D - u_{\phi,k})^2)/aF_{\phi,k}^2\right)^{0.5} - 1. \tag{21}$$

Thus, the uncertainty in biological factors costs the individual in terms of welfare the equivalent of $\Delta u_{\phi,k}$ in physical activity or $\Delta F_{\phi,k}$ in food consumption. We introduce uncertainty levels in ϕ equal to $CV_\phi = 1\%, 0.9\%, 0.95\%$ and compute $\Delta u_{\phi,k}$ given by Eqn. (20). In a separate experiment, we introduce the same levels of uncertainty in ϕ and compute $\Delta F_{\phi,k}$ given by Eqn. (21). The results are shown in Figure 5.

Figure 5: Percentage Change in $u_{\phi,k}$ and $F_{\phi,k}$.
(each graph corresponds to separate experiments)



The left panel of Figure 5 shows the results when physical activity adjusts and food consumption remains at the CE level. The right panel displays the results when food consumption is the variable allowed to change and physical activity remains at the same value as in the CE case.

The left panel of Figure 5 displays three important results. First, as expected the individual needs to decrease physical activity to achieve the same level of utility as in the CE. Second, higher levels of uncertainty in ϕ produce larger decreases in physical activity. Third, the percentage change in physical activity decreases at a decreasing rate from the first to the last day of the year. That is, when $CV_\phi = 1\%$ the individual is willing to exercise 8% less in the first period in order to obtain the same level of utility as in the CE. In contrast, the agent is willing to undertake around 50% less physical activity in the last period.

Similarly, the right panel of Figure 5 also highlights three crucial findings. First, in contrast with physical activity, the representative agent increases calorie intake to obtain the same level of utility when ϕ is uncertain as in the no-uncertainty case. Second, higher levels of uncertainty in ϕ generate large increases in the cost of uncertainty in terms of food consumption. Third, the amount of food consumption increases at a constant rate throughout the year. That is, in the first period the agent does not increase food consumption by a large amount. However, in the last period the agent increases food consumption by about 15% when $CV_\phi = 1\%$.

The first period shows an interesting contrast between food consumption and physical activity. When physical activity is the variable allowed to vary higher uncertainty produces an immediate change in this variable (around 8%). However, when food consumption is the variable allowed to vary, the change in the first period is almost negligible.

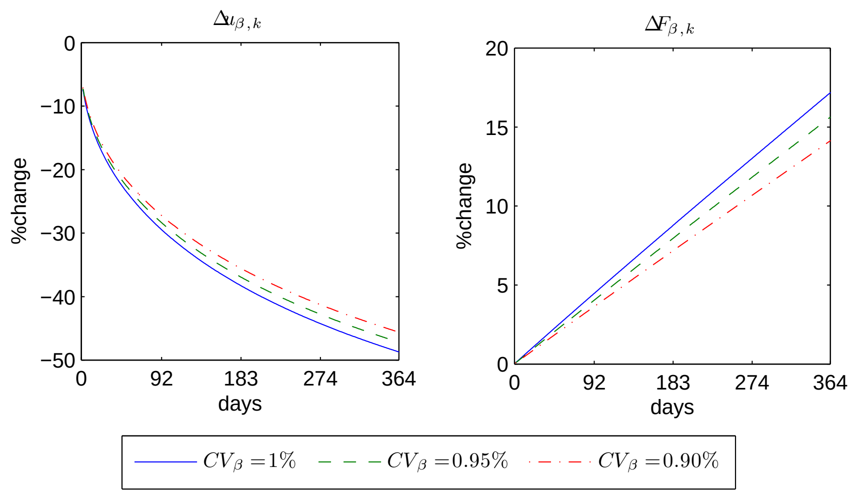
This results show that, in terms of consumption, eliminating an uncertainty level of $CV_\phi = 1\%$ is equivalent in terms of utility to an average 35% less physical activity or an average increase in food consumption of around 8%. An agent is willing to reduce physical activity by an average of 35% to obtain the same level of utility as in the certainty equivalence case. This implies that the cost of the uncertainty in this case is the equivalent to 35% more exercise as in the multiplicative uncertainty case. The equivalent cost in food consumption is an increase of almost 8%.

The numerical results in this subsection indicate that in terms of physical activity and food consumption the cost of misinformation about the precise calorie content of food can be significant, especially in the last periods. Therefore, policies aimed to reduce this type of uncertainty will certainly provide a benefit to the agent.

5.2 Cost of Uncertainty in Biological Factors.

In the case of multiplicative uncertainty in β, we follow a similar procedure as in the case of ϕ. We introduce uncertainty in β that corresponds $CV_\beta = 1\%, 0.95\%, 0.90\%$ and compute the corresponding values of $\Delta u_{\beta,k}$ and $\Delta F_{\beta,k}$. The results are shown in Figure 6.

Figure 6: Percentage Change in $u_{\beta,k}$ and $F_{\beta,k}$.
(each graph corresponds to separate experiments)



The left panel in Figure 6 shows what happens when physical activity adjusts and food consumption remains at the CE level. On the other hand, the right panel displays the results when food consumption is the variable allowed to change and physical activity remains at the CE level.

Figure 6 shows similar patterns to those found when ϕ is uncertain. The left panel shows three main results. First, the amount of physical activity decreases to achieve the same level of utility as in the CE case. Second, higher degrees of uncertainty in β produce larger decreases in physical activity. Third, the difference in physical activity in relation to the corresponding value in the CE case increases from the first to the last period. For example, at $CV_\beta = 1\%$, physical activity decreases from 7% in the first day to almost 50% in the last day of the year.

The right panel in Figure 6 shows also a similar pattern as when ϕ is uncertain. First, the consumer increases food consumption relative to its CE value by a negligible amount in the first periods. This is an important difference with physical activity where this variable decreases by 7% in the first period. Second, higher degrees of β uncertainty produce larger changes in food consumption when compared to the CE values. Third, the difference in the amount of food consumed relative to the CE value increases throughout the year at a constant rate.

The results in this section show that, in terms of consumption, eliminating an uncertainty level of $CV_\beta = 1\%$ is equivalent in terms of utility to an average 36% less physical activity or an average increase in food consumption of around 9%. An agent is willing to reduce physical activity by an average of 36% to obtain the same level of utility as in the certainty equivalence case. This implies that the cost of the uncertainty in this case is the equivalent to 36% more exercise as in the multiplicative uncertainty case. The equivalent cost in food consumption is an increase of around 9%.

In order to compare the relative benefits of reducing one source of uncertainty while keeping the other source fixed, we compute the relative adjustments in terms of calories and physical activity as seen in Figure 7.

Figure 7: Difference in costs from β and ϕ (ratio of the previous two figures) (each graph corresponds to separate experiments)

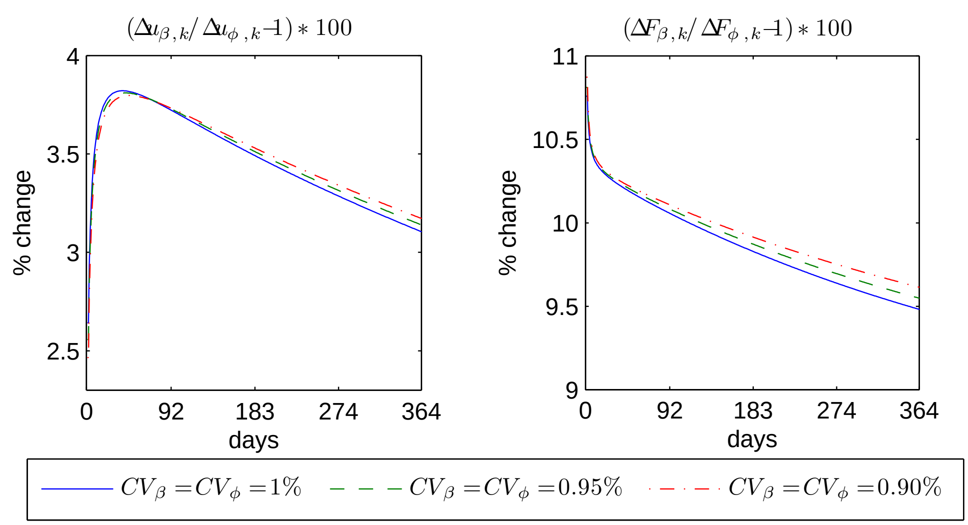


Figure 7 shows that Δu_k and ΔF_k increase more when β is uncertain than when ϕ is the uncertain variable. Thus, our results suggest that it may be more valuable for the individual to have a policy that provides more information regarding the benefits of calorie expenditure when exercising than a labelling policy that require nutritional information for foods away from home. This phenomena is shown to be the case on the basis that the same amount of resources are devoted to these two policies. Notice that implicitly here we are assuming that effectiveness to reduce the uncertainty of these two policies would be the same. We assume this to be the case as there is no evidence regarding this point. Finally, we note that, however, our methodology could easily incorporate the relative effectiveness of these two different policies when assessing the underlying costs.

6 Learning

In this section we consider the possibility that the representative agent learns about: i) the true amount of calories consumed in food, and ii) the effectiveness of physical activity to burn calories. That is, we let the agent improve her knowledge about the true value of β and ϕ. The possibility of learning is incorporated by assuming that the learning process of the agent follows a Kalman filter. This means that new data are collected each time period and the value of the uncertain parameters is updated. The advantage of the Kalman filter is that combines all the available information with knowledge about the system to produce an estimate of the variables that statistically minimizes error, see Maybeck (1979).

The learning procedure under the Kalman filter consists of two parts: prediction and correction. At each period, the agent arrives with a priori estimate of the variables, uncertain parameters, and covariances. In the prediction phase the agent uses these a priori values to predict the value of the variables. The update phase consists of improving the predicted values using the actual measurements. This generates a posteriori estimates used to generate the a priori estimates for the following period. In order to isolate the effect of learning about each parameter, our experiments assume that the agent learns only about one parameter, either β or ϕ and perform 1000 monte carlo runs for each experiment. We describe below the Kalman filter procedure when the agent learns only about β. The procedure when the agent learns only about ϕ is obtained by substituting β for ϕ and F_k for u_k.

First, we generate the random vector $\boldsymbol{\varepsilon}$ that contains an additive noise for each period. A different random vector is generated for each monte carlo run. Since, as mentioned in Section 2, the distribution of ε_k cannot be asymptotic, we assume that ε_k is uniformly distributed with support $(-0.5, 0.5)$. The support of ε_k implies that the highest daily additive shock is plus/minus half a pound.[16] Second, we solve the problem from period k onward as shown in the analytical solution subsection and obtain the value of the weight and for that period, x_k. Third, we obtain the a priori value for the next period of the weight and the covariance matrices using the following equations:

$$x^p_{k|k-1} = \alpha x^p_{k-1|k-1} + \beta_{k-1|k-1} u_{k-1} + \phi F_{k-1} \tag{22}$$

$$\sigma^2_{x,k|k-1} = u_{k-1}(\sigma^2_{\beta,k-1|k-1}) u_{k-1} + \sigma^2 \tag{23}$$

$$\sigma^2_{x\beta,k|k-1} = \sigma^2_{\beta,k-1|k-1} u_{k-1} \qquad \text{and} \qquad \sigma^2_{\beta,k|k-1} = \sigma^2_{\beta,k-1|k-1} \tag{24}$$

where the superscript p denotes the predicted value of the weight. The subscript $k|k-1$ represents the a priori value, that is the value at the beginning of period k. Moreover, $k-1|k-1$ is the a posteriori value from previous period.[17] Fourth, we correct the estimated values of the weight, β and σ^2_β from the previous step using the following equations:

$$\sigma^2_{\beta,k|k} = \sigma^2_{\beta,k|k-1} - \sigma^2_{x\beta,k|k-1}(\sigma^2_{x,k|k-1})^{-1}\sigma^2_{x\beta,k|k-1} \tag{25}$$

$$x^p_{k|k} = x^p_{k|k-1} + (x_k - x^p_{k|k-1}) \tag{26}$$

$$\beta_{k|k} = \beta_{k|k-1} + [\sigma^2_{x\beta,k|k-1}\sigma^2_{x,k|k-1}](x_k - x^p_{k|k-1}) \tag{27}$$

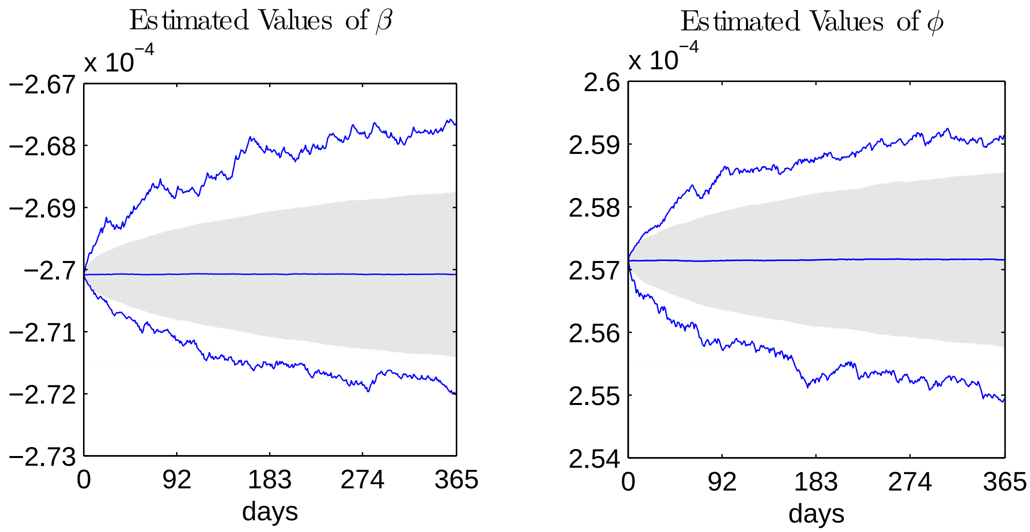
The values of $\sigma^2_{\beta,k|k}$, $x^p_{k|k}$ and $\beta_{k|k}$ are then used as the initial values for the next period and this process continues until $k = N$, upon which time a new monte carlo run can be started. The prediction part of the Kalman filter is given by Eqns. (22)-(24) whereas the correction phase is represented by Eqns. (25)-(27).

The Kalman filter procedure outlined above produces a linear, unbiased and minimum error algorithm, see Maybeck (1979). This generates, at time k, an optimal estimate of the weight. It is optimal in that the spread of the estimate-error probability density is minimized, see Zhang (1997). The term $(x_k - x^p_{k|k-1})$ in Eqns (26)-(27) is known as the *innovation* and represents the difference between the predicted and the actual value of the weight. If the innovation is zero then x and β are not updated. Another important term is given by $[\sigma^2_{x\beta,k|k-1}\sigma^2_{x,k|k-1}]$ in Eqn. (27). This term is known as the *gain* and reflects the importance that is placed in the innovation. Appendix B shows that as variance of the uncertain parameter decreases ($\sigma^2_\beta \to 0$), the importance of the gain in updating the uncertain parameter decreases and the predicted parameter value is trusted more. Conversely, when the variance of the additive noise decreases the predicted value is less important and the gain is weighted more heavily.

In the numerical experiments, we assume the initial variances to be $\sigma^2_\beta = 7.29E-12$ and $\sigma^2_\phi = 6.61E-12$, which correspond to an initial coefficient of variation of 1%. Figure 8 shows the maximum, minimum, average and standard deviation of the uncertain parameters for each time period and for both experiments.[18]

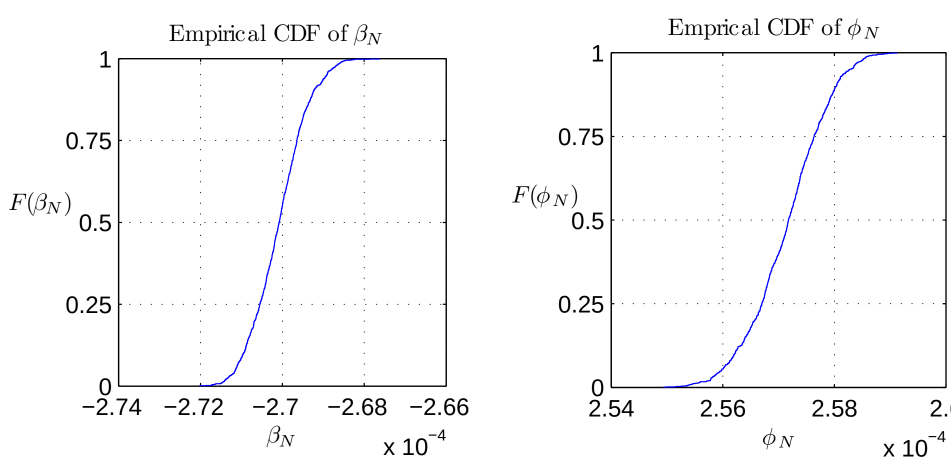
The left and right panel of Figure 8 show the maximum, minimum and average value of β and ϕ, respectively, for each period over all the monte carlo runs. The shaded areas represent one standard deviation of the updated parameter value across the monte carlo runs. Notice that this standard deviation is different from the standard deviation of ϕ and β (σ_β and σ_ϕ) that the agent updates every period.[19] In both cases, the average value of β and ϕ remain roughly unchanged across time periods. However, the extreme values and

Figure 8: Summary statistics of the updated values of β_k and ϕ_k
(each graph corresponds to separate experiments)



the standard deviations across monte carlo runs increase in the later periods. This is not surprising since the later periods estimates of β and ϕ are based on the estimates of the earlier periods which tend to increase their variability across monte carlo runs. Thus, the last period estimates incorporate learning from previous periods and therefore can be considered as the final estimates of β and ϕ. Figure 9 shows the empirical cumulative distribution function (CDF) across the monte carlo runs for the last period of β and ϕ.

Figure 9: Last period cumulative density function of β_N and ϕ_N
(each graph corresponds to separate experiments)



The CDF on the left panel of Figure 9 shows that the values of β_N are concentrated between -0.000272 and -0.000268. Similarly, the CDF on the left illustrates that the value of ϕ_N are located mostly between 0.000256 and 0.000258.

We also analyze the importance to the agent of learning about each parameter. In particular, we determine if learning about ϕ is better than learning about β. Results from Section 5 show that uncertainty about ϕ is more important than uncertainty about β. Thus, we compare the utility when the agent learns about each parameter. We find that the utility when agent learns only about ϕ is higher than the utility when learning only about β in 72.5% of the 1000 monte carlo runs. Moreover, learning only about β produces higher average utility and a lower utility variance than when learning only about β.

Finally, we address which type of uncertainty is easier to learn for the agent. One possible way to measure this is by computing the change in the coefficient of variation of each parameter. Learning using the Kalman filter allows the agent to update the parameter estimates as well as their variance. Thus, we compute the new coefficient of variation by obtaining the average value of the parameters and their standard deviation in the last period $\beta_N, \phi_N, \sigma_{\beta,N}$ and $\sigma_{\phi,N}$. The initial coefficient of variation for β and ϕ is 1% and we find that the new coefficient of variation in the last period are 0.97% and 0.96%, respectively. This indicates that it is easier for the agent to learn about the calorie content for food than about the calories burnt during physical activity.

These results suggest that the agent values more learning and reducing the uncertainty of the amount of calories burnt during physical activity. However, this type of uncertainty is harder to learn than the uncertainty about the calorie content of food. Hence, policymakers may observe more substantial benefits in developing policies that provide information about the effectiveness of physical activity to achieve the desired weight.

7 Conclusions

The obesity epidemic in the U.S. of the last twenty years has drawn the attention of academics and policymakers. In this paper, we develop an analytical model to analyze the effect of information on weight control. We consider a representative agent model that is initially overweight and because of health concerns desires to loose weight. In particular, the agent chooses an amount of exercise and food intake that will minimize deviations between her actual and healthy target weight, which proxies for health, and derives utility from eating and disutility from exercising. Moreover, the agent faces uncertainty about the calorie content of food and the amount of calories burnt during exercise. We also allow the agent to learn about both types of uncertainty.

We find that higher uncertainty produces a cautionary response of the representative agent. Moreover, the agent is more sensitive to changes in the uncertainty of the effectiveness of exercise to burn calories than variations in the uncertainty of the calorie content of food. In addition, higher degrees of both types of uncertainty decrease the utility of the representative agent. Finally, learning about the calories burnt from physical activity was found to be a more difficult task than learning about calorie intake.

These results suggest that providing more information to households may prove useful in controlling the obesity epidemic. In particular, it may be more beneficial to have a policy that provides more information regarding the calorie expenditure when exercising than a labeling policy that requires nutritional information for foods away from home. Thus, providing agents with more information about the consequences of exercise may result in lower obesity rates.

[1]See the National Health and Nutrition Examination Survey for more information on this issue.

[2]See Lakdawalla, Philipson and Bhattacharya (2005), Lakdawalla and Posner (2003), Burke and Heiland (2007), and Cutler, Glaeser and Shapiro (2003) for more on this issue.

[3]See Lakdawalla, Philipson and Bhattacharya (2005), Lakdawalla and Posner (2003), Burke and Heiland (2007), and Cutler, Glaeser and Shapiro (2003) and Gomis-Porqueras and Peralta-Alva (2008) for more on these issues.

[4]The authors estimate that the total monetary benefit due to lower mortality, reduced medical expenditures, declining absenteeism, and increased productivity associated with this reduction in body weight to be about $166 billion (1991 dollars) over a 20-year period.

[5]For example, though savvy consumers may be able to infer that a dessert without a "heart healthy" logo has more cholesterol or saturated fat than one with the logo, they cannot infer any information about sugar or calorie content.

[6]Within the medical literature there is a clear standard regarding a person's healthy weight according to gender and age. For adults, a body mass index or (BMI) over 30 is considered obese, between 25 and 29.9 is considered overweight, between 18.5 and 24.9 is considered healthy and under 18.5 is consider underweight. This BMI is a measure of body fat that applies to all adults.

[7]This measurement error captures the self reporting biased typically found whe agents report the amount of calories consumed through out the day, as reported by the nutrition literature.

[8]The additive uncertainty in Eqn. (5), θ_k, is part of the additive noise of the final transition equation and consequently will not affect the optimal policy rule of the agent.

[9] See Kendrick (1981) and Ljungqvist and Sargent (2000) for more on this issue.

[10]The model can also be calibrated using k as one week and all results will be unchanged. However, the parameters in this section would need to be adjusted.

[11]Different desired weight paths can be considered. For example, a constant percentage weight loss of 0.068% per day will achieve the desired weight loss in our problem. In the numerical experiments we choose

the linear desired path but the utilization of other nonlinear desired weight paths are possible and the main results are not affected.

[12]The value of 1747 calorie intake per day for the initial day is between the values obtained by Cuttler, Glaesler and Shapiro (2003) of 1658 and 1877 (with an standard error of 23.5) reported by the CDC (2004b).

[13]We experimented with cases where the individual has a medium a low level of commitment and none of the main results in this paper change.

[14]In steady state $(0.5W_k[x - x^{\#}]^2)/(1-\delta) = 0$, $aF^2/(1-\delta) = 32536$, $(-2bDu + bu^2)/(1-\delta) = -24713$, $(bD^2)/(1-\delta) = 2$ and $J = 7823$.

[15]We also performed a grid for values of σ_ϕ that produce $CV \in (0, 20\%)$ and the qualitative aspect of the results are unchanged.

[16]The assumption of the uniform distribution of ε_k is not essential as other non-asymptotic distributions can be considered in this framework.

[17]In the initial period, Eqns. (22)-(24) are modified in that $x^p_{k-1|k-1} = x_0 = 164$ and u_{k-1} is equal to the value of u_0 in the non-learning solution.

[18]In order to obtain comparable solutions, we use the same additive noise when learning only about β as when learning only about ϕ.

[19]Learning reduces the average variance of β and ϕ from 7.29E-12 to 6.83E-12 and from 6.61E-12 to 6.10E-12, respectively.

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Appendix A

In this Appendix we show the form of the Riccati equations mentioned in the analytical solution subsection. Following Kendrick (1981) and Ljungqvist and Sargent (2000) and substituting the parameters in our problem the solution to the Ricatti equations can be expressed as follows:

$$k_k = W_k + \alpha^2 k_{k+1} - (\alpha^2 k_{k+1}^2)/(det)\Big(\delta k_{k+1}(\langle\beta\rangle^2\sigma_\phi^2 + \langle\phi\rangle\sigma_\beta^2) + a\langle\beta\rangle^2 + b\langle\phi\rangle^2\Big) \tag{A-1}$$

$$\begin{aligned}\rho_k = {} & \alpha\rho_{k+1} - W_k x_k^{\#} - (\delta^2 k_{k+1})/(det)\Big(\langle\beta\rangle(\delta k_{k+1}\sigma_\phi^2 + a)(\langle\beta\rangle\rho_{k+1} - 2bD\alpha) \\ & + \alpha\langle\phi\rangle^2\rho_{k+1}(\delta k_{k+1}\sigma_\beta^2 + b)\Big)\end{aligned} \tag{A-2}$$

where det is shown in Eqn. (12) and it is given by the following

$$det = \delta k_{k+1}\{\delta k_{k+1}(\sigma_\beta^2\sigma_\phi^2 + \sigma_\beta^2\langle\phi\rangle^2 + \sigma_\phi^2\langle\beta\rangle^2) + (\sigma_\beta^2 + \langle\beta\rangle^2)a + (\sigma_\phi^2 + \langle\phi\rangle^2)b\} + ba. \tag{A-3}$$

with $k_N = W_N$ and $\rho_N = -W_N x_N^{\#}$.

Appendix B

In this section we show some of the characteristics of the Kalman filter mentioned in Section 6. Substituting Eqns. (23)-(24) into the square bracket expression of Eqn. (27) and rearranging we obtain the following expression:

$$[\sigma_{x\beta,k|k-1}^2\sigma_{x,k|k-1}^2] = (\sigma_{\beta,k-1|k-1}^2 u_{k-1})/(u_{k-1}(\sigma_{\beta,k-1|k-1}^2)u_{k-1} + \sigma^2) \tag{B-1}$$

Taking the limit when $\sigma^2_{\beta,k-1|k-1} \to 0$ of the previous expression, we obtain the following

$$\lim_{\sigma^2_{\beta,k-1|k-1} \to 0} (\sigma^2_{\beta,k-1|k-1} u_{k-1})/(u_{k-1}(\sigma^2_{\beta,k-1|k-1})u_{k-1} + \sigma^2) = 0 \tag{B-2}$$

Hence, when $\sigma^2_{\beta,k-1|k-1} \to 0$ Eqn. (27) can be simplified to $\beta_{k|k} = \beta_{k|k-1}$ and the predicted value of β is not corrected by the gain. That is, the predicted value is trusted more as $\sigma^2_{\beta,k-1|k-1} \to 0$.

Taking the limit when $\sigma^2 \to 0$ and $\sigma^2 \to \infty$, we obtain the following expressions:

$$\lim_{\sigma^2 \to 0} (\sigma^2_{\beta,k-1|k-1} u_{k-1})/(u_{k-1}(\sigma^2_{\beta,k-1|k-1})u_{k-1} + \sigma^2) = 1/u_{k-1} \tag{B-3}$$

$$\lim_{\sigma^2 \to \infty} (\sigma^2_{\beta,k-1|k-1} u_{k-1})/(u_{k-1}(\sigma^2_{\beta,k-1|k-1})u_{k-1} + \sigma^2) = 0 \tag{B-4}$$

This implies that the gain is weighted more heavily in Eqn. (27) as the value of σ^2 decreases, i.e. the predicted value is trusted less.