A Theory of the Supply of Inside Money

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Abstract

This paper advances a theory of the supply of inside money that is squarely based on optimisation, and which sets out from the question, ‘As outside money has an opportunity cost that a mere promise to pay outside money does not, why is outside money used at all?’. The theory identifies the nominal rate of return on capital as the key determinant of the supply of inside money. So just as the nominal rate of return on capital is the *cost* of *demanding* money, so the nominal rate of return is identified here as the *reward* for *supplying* (inside) money. And just as the demand for money is negatively related to the nominal rate of return on capital, so the supply of inside money is positively related to the nominal rate of return on capital.

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This paper advances a theory of the supply of inside money (see Coleman 2007 for an expanded treatment). The theory is one that is squarely based on optimisation, is impelled by Hicksian themes of the competition between inside and outside money, and which sets out from the question, 'As outside money has an opportunity cost that a mere promise to pay outside money does not, why is outside money used at all?'.[1]

The theory of inside money advanced here identifies the nominal rate of return on capital, ι, as the key determinant of the supply of inside money. So just as the nominal rate of return on capital is the *cost* of *demanding* money, so the nominal rate of return is identified here as the *reward* for *supplying* (inside) money. And just as the demand for money is negatively related to the nominal rate of return on capital, so the supply of inside money is positively related to the nominal rate of return on capital.

1 The benefits of the supply of inside money.

We suppose that money provides a benefit by reducing the frequency of costly liquidations of capital; a benefit that is represented by the appearance of real money holdings in the utility function.

$$U = u(C, C_1, \ldots C_T; h, h_1, \ldots h_T)$$

C = consumption

h = holdings of real money balances

T = final period; the current period is indexed as zero

We make the assumption that money holdings, h, can consist of either *outside money* or *inside money*.

Outside money is whatever is universally accepted, without cost, as tender. This is typically state money today; 'fiduciary' notes and coin.

Inside money is a (credible) promise to pay outside money. More precisely, it is a credible promise to pay outside money to the bearer of the promise, on the demand of the bearer, and at no cost to the bearer.[2] These promises will circulate within that network of people who have been persuaded of their credibility. We will usually think in terms of 'individuals' issuing these promises, but we can think - Kaldor style (Kaldor 1970) – of 'firms', who pay their workers and suppliers in 'chits', which circulate within that network of businesses who have been persuaded of their credibility.

As inside money is a credible promise to pay the bearer outside money (at no cost), the benefit of an extra unit of inside money is the same as the benefit of an extra unit of outside money, U_h.

Given this perfect substitutability of inside and outside money, there is from the point of view of the money holder, just 'money', and the money holder will hold money until the marginal utility of money relative to the marginal utility of consumption equals the nominal rate of return on capital.

$$\frac{U_h}{U_C} = \iota$$

U_h = marginal utility of real balances

U_C = marginal utility of consumption

ι = nominal rate of return on physical capital[3]

2 The costs of the supply of inside money.

From benefits of inside money, we turn to its cost.

The principal cost of the supply of inside money will be assumed to arise from making any promise to pay a *credible* promise to pay. Anyone can promise; but not everyone's promises are credible. There are costs in making a promise a believable, namely the costs of providing evidence of the solvency and honesty of the issuer of the promise. Evidence of solvency includes audited accounts, and perhaps investment in 'conspicuous capital' (e.g. ostentatious buildings). Evidence of honesty might include demonstrations of the willingness of persons of known honesty to associate with, and speak for, the issuer of the promise. These evidences are costly, and we will call these costs 'credibility costs'.

There is a second cost of the supply of inside money that we will sometimes consider. This turns on our assumption that the promise to pay money is a promise to pay money at no cost to the bearer, where 'cost' includes inconvenience and time loss to the bearer in being paid. The provision of honouring a promise in way that is both convenient and timely to the bearer presumably also involves cost to the issuer. We might call these 'convenience costs'.

There is, thirdly, the matter of 'operational costs'. It may cost money to produce the physical embodiment of promises, and to produce them in a way that is not worth the while of a forger successfully forging them.

We suppose that all three costs increase as the issue of promises requires. This is fairly obvious with respect to convenience and operational costs. With respect to 'credibility costs' there are two reasons one has to spend more on credibility to increase issue;

1. 'Credibility Deepening'. As the magnitude of these liabilities rise there must be more scrutiny to establish whether the issuer can and will meet these expanded liabilities ('John can pay $1,000, but can he pay $10,000? More evidence is needed').[4]

2. 'Credibility Widening'. If the issue is to expand, the network amongst which these promises are accepted must expand. More persons must be persuaded that the issuer is solvent and faithful to his promises.

The increasing costs of issuing premises can be represented by letting Z be the total costs of issuing n of inside money.

$$Z = Z(n) \qquad\qquad Z' > 0 \qquad\qquad (1)$$

$n \equiv$ issue of inside money in real terms [5]

The marginal cost of issue will prove to be significant, and we symbolise it z.

$$\zeta \equiv \frac{\partial Z}{\partial n} = Z'(n) > 0 \qquad\qquad (2)$$

ζ (n) = Z'(n) is the cost of establishing the credibility of the nth dollar promised.

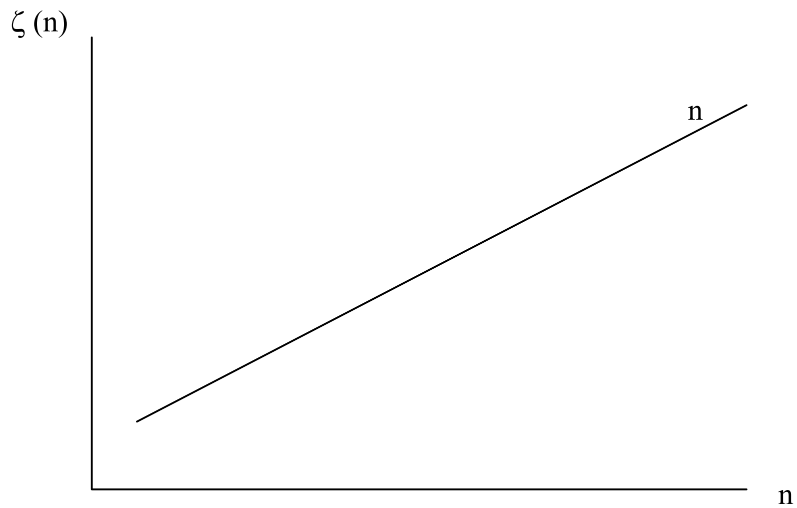
Several points about z should be noted.

- ζ is dimensioned like the interest rate. It's a cents per period per dollar type variable. But instead of being a rate of return, it is a rate of outlay.

- ζ will be positive at n = 0. There is a minimum marginal cost of inside money.

- $\frac{\partial \zeta}{\partial n} > 0$. The marginal cost of establishing credit worthiness is increasing in n ('increasing marginal costs'), at least for 'low' and 'high' magnitudes of n. This assumption is required for the existence of a maximum in the households' inside money issue problem.

Figure 1: The marginal cost of inside money rises with inside money



Two more issues merit airing at this point.

Credibility decay

There is the question of how long credibility lasts, once it has been acquired. Once a promise has been made credible, how long will it be credible for? Forever? This period? A finite number of periods? We will begin by making the fairly extreme assumption that credibility lasts only ‘one period’. Promises made this period are credible for redemption at the opening of the following period, but are otherwise have zero credibility in the following periods.

This assumption will be relaxed later.

Heterogeneity

These cost functions may differ from person to person. The cost is presumably lower for persons both wealthy and trusted, than for persons that are both poor and distrusted. The cost may be so high that it is prohibitive to issue any.

3 The optimal supply of inside money.

The condition of optimisation

The individual's maximisation problem is,

Choose $C, C_1, \ldots C_T; h, h_1, \ldots h_T$

Max $U = u(C, C_1, \ldots C_T; h, h_1, \ldots h_T)$

subject to

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Period 0: $PC + H + K_1 P + PZ(n) = Pw + P\rho_{-1}K + PK + M + N$

Period 1: $P_1C_1 + H_1 + P_1K_2 + P_1Z(n_1) = P_1w_1 + P_1\rho K_1 + P_1K_1 + H + N_1 - N$

etc

(3)

M = endowment of outside money at opening of period zero

K = capital

w = real wage

ρ = rate of profit

H = holdings of nominal money balances

The period budget constraints can be consolidated into a single budget constraint,

$$C+\frac{C_1}{1+\rho}+\ldots+h\frac{[\rho+\pi+\rho\pi]}{[1+\rho][1+\pi]}+\frac{h_1}{[1+\rho]}\frac{[\rho_1+\pi_1+\rho_1\pi_1]}{[1+\rho_1][1+\pi_1]}+\ldots$$

$$=m+n\frac{\rho+\pi+\rho\pi}{[1+\rho][1+\pi]}+\frac{n_1}{[1+\rho]}\frac{[\rho_1+\pi_1+\rho_1\pi_1]}{[1+\rho_1][1+\pi_1]}-Z(n)-\frac{Z(n_1)}{1+\rho}-\ldots+w+\frac{w_1}{1+\rho}+\ldots+K[1+\rho_{-1}]$$

(4)

π = rate of inflation

m = endowment of outside money in real terms

Optimisation with respect to n implies,

$$\zeta\equiv Z'(n)=\frac{\rho}{1+\rho}+\frac{\pi}{1+\pi}-\frac{\rho}{1+\rho}\frac{\pi}{1+\pi}=\iota \qquad (5)$$

The equality says that inside money is issued until the marginal credibility cost, ζ, equals the nominal rate of return on capital, ι.

This result can be rationalised by three equally valid arguments.

The Argument from Increasing Capital Holdings.

Suppose that promises to pay are issued, but are immediately used to purchase capital. This capital is then sold next period to meet the redemption of the promise to pay.

The addition to costs is ζ. There is no benefit from any reduced frequency of capital liquidations, since the quantity of money held is no higher. But there is a benefit from the capital acquired. This is the income per dollar of capital that has been acquired, which is ι. So matching cost with benefit,

$$\zeta(n) = \iota \qquad (6)$$

Notice that the income per dollar of capital is nominal income per dollar. This reflects the fact that, if inflation is positive, not all of the proceeds of the sale of capital need go to meet redemption. Part of the sale– the nominal capital gain – is left over for the issuer.[6]

The above argument has an implication of considerable significance: there *is* an aspect of 'printing money' in issuing inside money. There is a net income to be derived from doing it. The issuer gains a capital asset, and receives it income, less the costs of establishing credibility. Plainly, allowing people to build their own printing presses, and spend the outside money they print, is socially wasteful. And similarly there is waste in persons issuing inside money: resources are devoted to establishing credibility for the purpose of avoiding the opportunity cost of outside money. But the opportunity cost of outside money is purely a private cost, and involves no social cost. Thus the reduction in the holding of outside money, that the issue of inside money permits, produces no reduction in social costs, but it does involve costs. Inside money *is* socially wasteful.

The Argument from Increasing Money Holdings. Consider a person who issues an extra quantity of promises to pay, holding consumption and capital holdings constant. The addition to costs is ζ. The additional benefit lies in the reduced frequency of capital liquidations, allowed by the higher quantity of money held. This benefit is the marginal utility of money, U_h.[7] Thus

$$\text{Net Benefit of Marginal Issue of Inside Money} = U_h - \zeta(n)U_C \qquad (7)$$

Inside money is issued until the net benefit of an extra issue is zero,

$$U_h - \zeta(n)U_C = 0$$

or,

$$\zeta(\mathrm{n}) = \frac{\mathrm{U_h}}{\mathrm{U_C}} \qquad (8)$$

But utility maximisation also implies,

$$\frac{U_h}{U_C} = \frac{\rho}{1+\rho} + \frac{\pi}{1+\pi} - \frac{\rho}{1+\rho}\frac{\pi}{1+\pi} = \iota \qquad (9)$$

Substituting the optimisation condition for money demand into the optimisation condition for inside money supply yields the result,

$$\zeta = \iota$$

An argument from consumption. Suppose the issue of money was used to purchase one unit of consumption in the current period. This would have a credibility cost, z. It would also entail a reduction in consumption in the next period to meet redemption. The net benefit is,

$$U_C - \zeta U_C - \frac{U_{C1}}{1+\pi}$$

Ensuring this is zero, and recalling $U_C = [1+\rho]U_{C1}$, yields the same result.

The expression for supply

The optimisation condition can be inverted to relate n to ι; that is, to derive a supply equation for inside money.

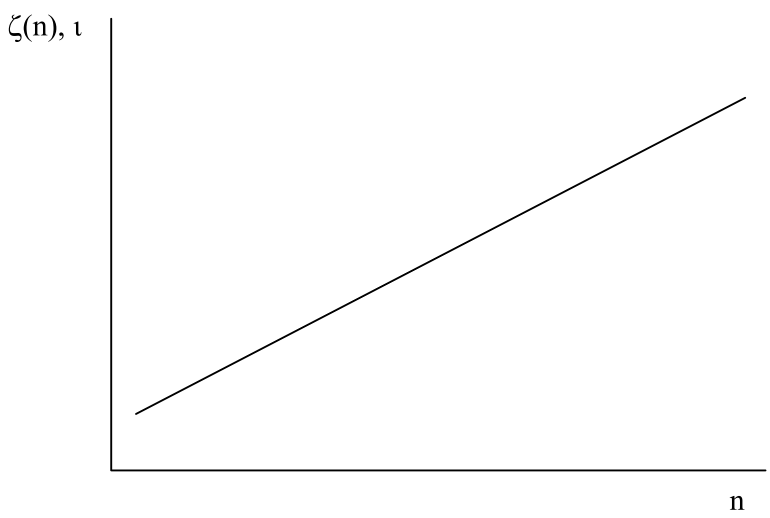
$$\zeta(n) = \iota$$

$$n = \zeta^{-1}(\iota) \tag{10}$$

$$\frac{\partial n}{n \partial \iota} \equiv \upsilon = \frac{1}{\zeta ' \zeta^{-1}(\iota)} > 1 > 0 \tag{11}$$

In terms of a figure,

Figure 2: The supply of inside money is a positive function of ι



The positive relation between inside money and the nominal rate of return can be rationalised in several ways. It can be understood in terms of the depreciation, in the real terms, of the issuer of inside money's liability that occurs under higher inflation, and so higher ι. The more one's liability declines in real terms, the more incentive one has have to issue the liability in order to acquire some capital.

The positive relation between inside money and the nominal rate of return can also be understood in terms of the Argument From Increased Money Holdings, we may reason: 'An increase in the rate of return on capital reduces the amount of money held, and increases the implicit rate of return on extra money, $\eta \equiv \frac{U_h}{U_C}$, including extra inside money. That rate of return now exceeds the marginal cost of circulation of a dollar, ζ. Therefore the supply of inside money will be expanded until it rises so high that ζ reaches η (= ι). To put it another way, as the rate of return on capital rises, the implicit rate of return on money rises. This amounts to the benefit of persuading people of your credit rises; so you do persuade more people.

4 Banknotes, Bank Deposits and Inside Money

How does the theory of inside money developed here relate, if at all, to the categories of inside money typically observed?

The inside money we have dealt with is not bank money - banks have not been mentioned. But, historically, inside money has been commonly bank money: either commercial banknotes (in the 19^{th} c) or bank deposits (in the 20^{th}). So there is a distance between the categories deployed in the theory, and the categories of commercial life. Can the theoretical categories - promises to pay the bearer be related – be related to bank money? Can the analysis of promises to pay be construed to be an implicit analysis of bank money?

It is easy to 'tell stories' in which the category used here– a promise to pay the bearer on demand –can be related to banking categories, and in a way such that the bank is merely a superstructure. We could suppose, for example, that every individual owns their own bank, BankofMyself. Instead of issuing to the public promises to pay, each individual borrows from their respective BankofMyself, which issues BankofMyself banknotes to the same value that they spend. Banknotes are now the form of inside money, but absolutely nothing of any substance has changed.

BankOfMyself do not exist, but only because of economies of scale, just as backyard steel foundries do not exist on account of economies of scale. We can imagine a group of persons reducing the costs of obtaining credibility for their promises to pay by grouping together, and forming a 'bank'. Instead of an individual issuing a $100 of promise to the bearer, they issue $100 of promise to intermediary – a 'bank' - that in turn issues them $100 of promises of 'the bank' to pay to the bearer. These promises to pay the bearer are 'bank notes', of the kind that circulated in the 19thc. It is the bank that does the promising, but the foundation of that promise lies in the promises made by the set of persons who have issued promises to the bank. The bank note, then, is just a way of tying together the promises of that set of persons. The credibility of the note still turns on the credibility of the persons. Nothing essential has changed.

Can deposit money also be rationalised in terms of the concepts of this paper?

To think about deposit money, imagine that bank notes do not circulate, but are left in the custody of the bank. To be quite concrete about it, we might imagine a series of labelled bins, each bin pertaining to the certain person nominated on its label. In any given bin are placed all the notes, issued by whatever bank, that are the possession of the person named on the bin's label. The amount in a given bin might be thought of the 'deposit' of the person bin pertains to. When this person wishes it, they may instruct the bank to transfer part of their note holdings to a different bin. This captures the situation of chequable deposits. The key point is that the acceptability of this payment depends on the credibility of the persons who are ultimately the banks debtors.

5 Model extensions

Scale Effects

It is very likely that the cost of an issue of any given size is reduced by the scale of the wider economy. To issue $10,000 is presumably more costly when money demand is $10m than when it is $10B. [8]

One convenient way of capturing this phenomenon is to suppose an x percent increase in issue implies an only x percent increase in the cost of issue as long as total moneu demand also increases by x percent. Equivalently, the average cost of issue per unit of issue is purely a function of the ratio of issue to total money demand.

$$Z^j = z(\frac{n^j}{h})h \qquad \frac{z'}{z}n/h > 1 \qquad (12)$$ [9]

Then

$$\zeta = z'(\frac{n^j}{h}) \qquad (13)$$

so

$$\iota = z'(\frac{n^j}{h}) \qquad (14)$$

This implies,

$$n^j = z'^{-1}(\iota)h \qquad (15)$$ [10]

This extension involves two significant revision of the theory.

1. The supply of inside money is now unit elastic to total demand for money. This unit elasticity also implies a unit elasticity between h-n and h. That is, a unit elasticity between m and h. To illustrate, if $Z = n^{1+\varphi}h^{-\varphi}$ then $\frac{n}{h} = [\frac{\iota}{1+\varphi}]^{1/\varphi}$, and $\frac{m}{h} = 1-[\frac{\iota}{1+\varphi}]^{1/\varphi}$. A 'money multiplier' like relativity

between total money and outside money emerges, although with a completely different rationale.

2. The elasticity of the supply of inside money to the nominal rate of return on capital, is no longer necessarily positive. A large enough (negative) sensitivity of total money demand to the rate of return can outweigh any (positive) sensitivity of inside money to the rate of return. More formally, the semi-elasticity of the supply of inside money will be negative if the positivised semi-elasticity of total money demand exceeds the partial semi-elasticity of inside money supply. [11]

3. n as a proportion of h may be subject to a maximum. If $Z = n^{1+\varphi}h^{-\varphi}$ then $\frac{n}{h} = [\frac{\iota}{1+\varphi}]^{1/\varphi}$. The maximum magnitude of the RHS is $\frac{n}{h} = \frac{1}{[1+\varphi]^{1/\varphi}}$.

Credibility Decay

We have supposed that outlays on credibility only secure credibility for a single period. But credit does not decay 100 percent period. The proofs of credibility (that amount to proofs of solvency and honesty) must have some endurance over time. Increasing credibility today may favourably impact on credibility tomorrow. Therefore, the higher the issue yesterday, the less the cost of accrediting a certain magnitude of n today.

One way of modelling the endurance of credibility is to suppose,

$$Z^j = z(\frac{n^j - fn^j_{-1}}{h})h \qquad (16)$$

f is a measure of the rate of survival in the 'quantity of credibility' in inside money. If f = 0, there is no survival in credibility beyond one period (as we have assumed so far). But if f =1, there is complete survival, and any outlays on credibility impact equally on the issue of inside in all future periods.

If we choose units so that h =1 then,

$$Z^j = z(n^j - fn^j_{-1}) \qquad (17)$$

For any profile of credibility outlays, to spend more on credibility in period 0, increases the supply of issue (and so holdings) in all future periods, but at a diminishing rate. Therefore,

net benefit of extra spending on credibility $= [U_h + fU_{h,1} + f^2U_{h,2} + ...]\frac{\partial n}{\partial Z} - Uc$ [12]

At the point of optimum issue the marginal net benefit is zero,

$$U_h + fU_{h,1} + f^2U_{h,2} + ... - \zeta Uc = 0 \qquad (18)$$

where $\zeta = z'(n^j - fn^j_{-1})$

This implies,

$$n^j = z'^{-1}(\iota + f\frac{\iota_1}{1+\rho} + f^2\frac{\iota_2}{[1+\rho][1+\rho_1]} + \ldots) + fn^j_{-1} \qquad (19)^{13}$$

The supply of money is now a function of the (weighted) sum of discounted future nominal rates.

The full impact of any increase of the nominal rate of return on the supply of inside money now comes with a lag.

Honouring Within Period Redemptions

We have assumed that the whole of the issue circulates within the period it is issued, and none of it is presented within that period for honouring in terms of outside money, (or presented as payment to the issuer for a debt owed to the issuer). But presumably a proportion of these notes, v, will be presented within the period. How will this affect the analysis? Using the Argument from Money Holdings, we can say that to issue $1 is not to increase money holdings by $1, but by only $[1-v]. Therefore the optimisation condition is,

$$\zeta(n) = [1- v]\ \frac{U_h}{U_C} \qquad (20)$$

or, equivalently,

$$\zeta(n) = [1- v]\ \iota \qquad (21)$$

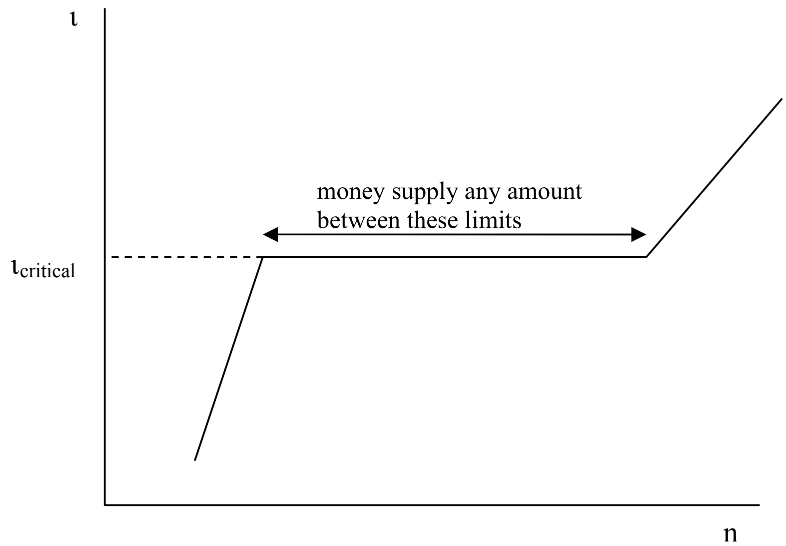
Thus n is now a function of nominal rate of return on capital factored down by the redemption rate, v.

Constant marginal costs of issue

We have assumed that the marginal cost of issue rises with issue. But optimisation is consistent with marginal costs being constant over some finite range, as long as it is rising outside that range.

Figure 3: Supply of Inside Money with Constant Marginal Costs of Credibility

n



At the critical rate, the supply of inside money can be any magnitude, within the range of constant marginal costs.

The existence of a flat portion of the supply curve of inside money captures the kind situation speculated about by Kaldor in his critique of monetarism (Kaldor 1970). The supply of money has no unique magnitude at the critical rate. There is no such thing as ‘the money supply’.

Fixed costs

We have implicitly assumed there are no fixed cost in establishing credit. But it is plausible that there are some fixed costs in establishing credit. This means that there will be a nominal rate of return on capital that is so low, such that if the rate falls below that then there is a discontinuous jump from positive issue to zero issue. In other words, there is 'shut down' rate of return on capital, such that if the rate falls below that rate no inside money is issued.

Non-neutralities in supply

We have assumed that the real supply of inside money is completely independent of the nominal price level. But there will be such a dependence if we plausibly allow that the costs of securing the credibility of any given size of issue falls with the wealth of the issuer. Suppose

$$Z^j = Z(n^j, W^j) \qquad \frac{\partial Z^j}{\partial n^j} > 0, \ \frac{\partial Z^j}{\partial W^j} < 0 \qquad (22)$$

This reformulation does not affect the first order conditions as the wealth of person j is not j's choice variable. Therefore,

$$\zeta(n, W) = \iota \qquad (23)$$

But if $\zeta_W < 0$ then higher wealth increases supply of n for a given ι. As wealth is partly composed of outside money, we can conclude $\partial W / \partial P < 0$. Thus the supply of inside money contracts for a given P.

Interest earning Inside Money

We have assumed that the only way to induce other persons to accept one's promises to pay is to go to expense of demonstrating one's credibility to them. But there is another way of inducing others to accept one's promises to pay: pay others to accept one's promises to pay. To illustrate: if the public believes, in the absence of outlays on credibility costs, that there is a 10 percent risk that one's promise will not be honoured, then paying $10 on every $100 promise to pay that is issued to compensate them for the risk. This payment takes the appearance of 'interest'

Does this possibility of securing circulation by 'risk compensation' interest undermine the theory of this paper? To explore this, suppose that the population consists of two categories. Category One makes up a fraction 1-p of the population, and pays all of their promises. Category Twp makes up a fraction p of the population, and pays none their promises. In the absence of credibility outlays each person's category membership is private information of the person. The benefit of issuing money in these circumstances is,

$$\text{Benefit} = U_h - i_n U_c$$

where

$$i_n = p \qquad (24)$$

Let us also suppose that for the first category $\zeta < p$. Then the category 1 people will establish their credibility rather than pay risk compensating interest. The remaining persons will then be identified as persons who do not honour promises so none of their promises are accepted. We are back to the theory of this paper.

In ignoring inside money issued on the basis of risk compensating interest payments we are making some sort of implicit assumption about the cheapness of establishing probity.

Systemic Risk and 'Bank Runs'

We have ignored that the possibility (and likelihood) that the total quantity of inside money exceeds the total amount of outside money. In such a situation it is impossible for all promises for outside money to be simultaneously honoured. In other words, it is possible that no promises have credibility. This is the vicious equilibrium of bank runs.

Distinct Demands For Inside and Outside Money

In this analysis there is no 'demand for outside money' distinct from some 'demand for inside money'. Inside and outside money are perfect substitutes from the point of view of the holder, as the benefit of money to its holder regardless of whether it is 'inside' or 'outside'. There is only a 'demand for money'.

Conclusion

The paper has advanced a theory of the supply of inside money that turns on the thesis that inside money is supplied until the cost of making an extra dollar sufficiently credible to be acceptable for circulation equals the income that an extra dollar of wealth can earn. The upshot of this assumption is that the supply of inside money is a positive function of the nominal rate of return on capital.

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[1] See Hicks on the competition between inside and outside money. (Hicks 1939, 1989).

[2] Economic historians have extensively documented cases where the population at large have used as a medium of exchange such promises to pay. See Shann (1938 pp. 52-3), and O'Connell and Reid (2005).

[3] It proves convenient to measure the nominal rate of return on capital as the increment in nominal value, between period zero and period one, expressed as a proportion of the nominal value in period one (rather than in period zero) Thus $\iota \equiv \frac{\rho+\pi+\rho\pi}{[1+\rho][1+\pi]} = \frac{\rho}{1+\rho} + \frac{\pi}{1+\pi} - \frac{\rho}{1+\rho}\frac{\pi}{1+\pi} \approx \rho + \pi$.

[4] The magnitude of n is assumed to be known or knowable.

[5] N is the nominal issue. Z is related to the real circulation of notes, N/P ≡ n, not N. The same nominal issue may be considered either very extensive or very slight, depending on whether P is low or high.

[6] Proof:

$$-\zeta Uc + \rho U_{C1} + \frac{P_1 - P}{P_1} U_{C1} = 0$$

But $U_{C1}[1+\rho] = U_C$, so

$$-\zeta + \rho \frac{1}{1+\rho} + \frac{1}{1+\rho} - \frac{P}{P_1}\frac{1}{1+\rho} = 0$$

Thus,

$$\zeta = \rho \frac{1}{1+\rho} + \frac{1}{1+\rho} - \frac{P}{P_1}\frac{1}{1+\rho} = 1 - \frac{P}{P_1}\frac{1}{1+\rho} = \iota .$$

[7] The benefit of the additional issue of inside money equals the marginal utility of money in the current period only, as on present assumptions, credit decays at a rate of 100 percent per period. It is the higher holding of money allowed by the issue of inside money that makes it possible to honour the issue at the opening of the next period, without cost to next period consumption.

[8] The more commercial traffic there is, the easier it is to circulate promises to pay. The quantity of one's promises that any circle of creditors will bear will rise the greater their demand for money.

[9] For the $Z^j = z(\frac{n^j}{h})h$ specification to capture the greater costliness of, for example, n = \$10,000 when h = \$10m than when h =\$10B, it must be that $\frac{z'n/h}{z} > 1$. The elasticity of cost Z to n (normalised by h) must be greater than 1.

[10] If $Z^j = z(\frac{n^j}{h})h$ then $m = [1 - z^{-1}(\iota)]h(\iota)$.

[11] As $\iota = z'(\frac{n}{h})$, $d\iota = z''\frac{dn}{h} - z''\frac{n}{h}\frac{dh}{h}$. Thus,

$$\frac{dn}{nd\iota} = \frac{h/n}{z''} + \frac{dh}{hdi}$$

$\frac{dn}{nd\iota}$ = semi-elasticity of inside supply less the positivised semi-elasticity of total demand

[12] $$n = z^{-1}(Z) + fn_{-1}$$

Thus,

$$n_1 = z^{-1}(Z_1) + fn$$

etc

Consequently,

$$\frac{\partial n_1}{\partial Z} = \frac{\partial n_1}{\partial n}\frac{\partial n}{\partial Z} = f\,\frac{\partial n}{\partial Z}$$

and

$$\frac{\partial n_2}{\partial Z} = \frac{\partial n_2}{\partial n_1}\frac{\partial n_1}{\partial Z} = f\,\frac{\partial n_1}{\partial Z} = f^2\,\frac{\partial n}{\partial Z}$$

etc

[13] The optimisation condition can be written:

$$\frac{U_h}{Uc} + f\,\frac{U_{C1}}{U_C}\frac{U_{h1}}{U_{C1}} + f^2\,\frac{U_{C1}}{U_C}\frac{U_{C2}}{U_{C1}}\frac{U_{h2}}{U_{C2}} + ... - z'(n - fn_{-1}) = 0$$

Substitution of the first order condition for consumption ($U_C = [1+\rho]U_{C1}$), and money demand ($\iota = U_h / U_C$) yields,

$$z'(n - fn_{-1}) = \iota + f\,\frac{\iota_1}{1+\rho} + f^2\,\frac{\iota_2}{[1+\rho][1+\rho_1]} + ...$$