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PUBLIC CAPITAL SPILLOVERS AND GROWTH: A FORAY DOWNUNDER

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1. INTRODUCTION

IN THIS PAPER, we extend the deterministic growth model of Glomm and Ravikumar (1994) to a stochastic growth version with endogenous public capital spillovers. By introducing simple shocks to production technology, private capital and public capital investment, we can derive testable time series properties of the analytical model along the lines of Lau and Sin (1997) who first investigated a similar question for the US. We allow growth of per capita income to be generated exogenously via Harrod-neutral technical progress and/or endogenously by aggregate public infrastructure spillovers. The postulation of strict endogenous growth is tested empirically for Australia using a constructed annual data set for the period 1959/60–2003/04.

To the best of our knowledge, there has been no work that takes the approach in Lau and Sin (1997), of using a theory-consistent approach to test for endogenous growth effects, with respect to Australian data. We show that the hypothesis of strict endogenous growth due to public capital spillovers cannot be statistically rejected. Unlike Lau and Sin (1997) we also explore the short-run causal links between public capital accumulation and growth. Since the time-series causal chain running from public capital accumulation to private outcomes is obvious in the theoretical model, we investigate whether this short-run effect is plausible using a vector error correction model. The idea here is to let the data speak as much as possible with respect to the short-run dynamics while imposing the estimated long-run

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relationship implied by the theory. We find further short-run evidence of public capital contributing to permanent increases in the levels of per capita income and private capital.

The role of public capital as distinct from private capital in fostering growth has received attention from the economics profession as early as Arrow and Kurz (1970). This hypothesis became known in the empirical literature as the public capital debate, which began with the seminal work of Aschauer (1989a). Aschauer's method of estimating a single aggregate production function (which incorporates public capital stock) was first adapted for Australian studies by Otto and Voss (1994). Both papers found that there was a significantly large elasticity of output (in the order of 0.40) with respect to public capital. Their methodology was not without criticism. The critiques range from claims of possible endogeneity of the public capital variable to the ad hoc nature of imposing a production function.[1]

Nevertheless, the production function approach is still valid and not ad hoc, albeit subject to a different interpretation. The production function can be interpreted as a long run relationship between output, and the private and public inputs, as in Flores de Frutos, Gracia-Díez, and Pérez-Amaral (1998). Furthermore, this time series property of the variables can be derived from a stochastic growth framework with sound microfoundations.

The paper is thus arranged. A stochastic growth version of the Glomm and Ravikumar model and the time series (cointegration) basis of the production function framework is derived in Section 2. Section 3 contains the estimation and test of strict endogenous growth within the cointegrating relationship. The short-run and impulse response analysis using the VEC(2) structure is enumerated in Section 4. The paper concludes with Section 5.

2. The theoretical model

A stochastic growth version of the Glomm and Ravikumar model is presented in this Section. A representative household-worker chooses an optimal consumption or investment path to maximise expected lifetime utility, given resource constraints and taking government policy as given. The fiscal policy is assumed to be a Ramsey planning problem subject to technological constraints and a periodic balanced budget à la Barro (1990).

2.1 Technology and household choice

Let Y be aggregate output, K be aggregate private capital stock, L be the total number of workers or population and $\tilde{G}$ be a measure of congestion-adjusted public capital stock to be defined later. The Harrod-neutral rate of technological progress is denoted by x. We would like public capital to enter aggregate production so that potentially there would be

[1] See Sturm (1998, pp. 57-65) for a survey. See e.g. Lynde and Richmond (1992) and Berndt and Hansson (1991).

a spillover effect.[2] Assume that the aggregate production function takes the Cobb-Douglas form

$$Y_t = AK_t^{\alpha}\left[(1+x)^t L_t\right]^{1-\alpha}\tilde{G}_t^{\theta}\epsilon_t^P; \qquad \alpha, \theta \in (0,1) \tag{1}$$

which yields the production function in per worker terms as

$$y_t = A(1+x)^{(1-\alpha)t}k_t^{\alpha}\tilde{G}_t^{\theta}\epsilon_t^P; \qquad \alpha, \theta \in (0,1) \tag{1'}$$

where the lower case variables, *y* and *k*, denote per worker output and private capital respectively. Thus, the model nests the possibilities of exogenous and/or endogenous growth. The production technology is subject to random shocks, ϵ_t^P, assumed to be multiplicative in this model. Aggregate public capital, $\tilde{G}_t$, enters as an input into production (implying the spillovers or externality effect) and it is taken by the representative agent as given. Further, aggregate public capital is subject to congestion from its use by private production

$$\tilde{G}_t = \frac{G_t}{K_t^{\phi}\left[(1+x)^t L_t\right]^{1-\phi}}; \qquad \phi \in (0,1) \tag{2}$$

where G_t is the aggregate stock of public infrastructure investment and ϕ and $(1-\phi)$ denote the degree of congestion arising from private capital stock and labor force, respectively. This is contrary the usual notion that public goods are non-exclusive and non-rival.

We can further detrend equation (1'). Let $\hat{y}_t := y_t/(1+x)^t, \hat{k}_t := k_t/(1+x)^t$, and $\hat{g}_t := G_t/L_t(1+x)^t$. Thus equation (1') can be written in per efficiency unit worker terms as

$$\hat{y}_t = A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P; \qquad \alpha, \theta, \phi, (\alpha-\theta\phi) \in (0,1) \tag{3}$$

Assume that there is 100 per cent depreciation at the end of each period for private capital. Then private per capita investment will give the following period's capital stock per efficiency unit worker

$$k_{t+1} = i_t\epsilon_{t+1}^K \tag{4}$$

where *i* is investment per efficiency unit worker and ϵ_{t+1}^K is a random shock and k_0 is given. Similarly, aggregate public infrastructure investment is assumed to depreciate fully at the end of the period

$$G_{t+1} = I_t^G\epsilon_{t+1}^G \tag{5}$$

[2]This effect would depend on the parameters and is the object of empirical testing later.

where I^G is aggregate public expenditure on infrastructure and G_0 is given.

Let τ be the uniform income tax rate. The household solves

$$V(\hat{k}_0, \epsilon_0^P) = \max_{\{\hat{c}_t, \hat{k}_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(\hat{c}_t); \qquad \beta \in (0,1) \tag{6}$$

subject to

a) $\hat{k}_{t+1} = (1-\tau_t)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P - \hat{c}_t$

b) $\hat{k}_0, \hat{g}_0$ given

c) $\hat{c}_t, \hat{k}_{t+1} \geq 0$

for all $t \in \mathbb{N}$.

It is shown in Appendix A, by restating the problem in equation (6) as a dynamic program, that the solution to the household problem taking government policy $\{\tau_t, \hat{g}_t\}_{t=0}^{\infty}$ as given, yields the optimal paths of consumption and private capital as

$$\hat{c}_t = (1-\tau_t)\left[1-(\alpha-\theta\phi)\beta)\right] A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{7}$$

$$\hat{k}_{t+1} = (1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{8}$$

for all states and dates $t \in \mathbb{N}$, given k_0. The analytical solutions were obtainable by assuming logarithmic utility, Cobb-Douglas technology, 100 per cent depreciation of private and public capital, a uniform tax structure and a balanced budget. This also simplifies the cointegrating properties of the variables.

2.2 *Public sector*

The government budget is such that public investment demand each period is exactly financed by income tax revenue:

$$I_t^G = \tau_t Y_t \tag{9}$$

We assume a government policy to be one that implements a Ramsey optimal fiscal plan. The government maximizes the same objective function as households but it also takes into account the optimal behavior of private agents with respect to the policy plan in a competitive equilibrium. The optimal policy is determined by solving

$$v(\hat{k}_0, \hat{g}_0, \epsilon_0^P) = \max_{\{\tau_t, \hat{k}_{t+1}, \hat{g}_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln\left\{(1-\tau_t)\left[1-(\alpha-\theta\phi)\beta)\right] A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P\right\} \tag{10}$$

subject to

a) $\tau_t \in (0,1)$

b) $\hat{g}_{t+1} = \tau_t A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P$

c) $\hat{k}_{t+1} = (1-\tau_t)(\alpha - \theta\phi)\beta A \hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P$

d) $\hat{k}_0, \hat{g}_0$ given

for all $t \in \mathbb{N}$. Notice that we have replaced per period consumption in the objective with the competitive behavior in (7) and also encoded households' optimal capital investment decision (8) into constraint c) above.

It is assumed that the benevolent government maximises household welfare when it maximises household consumption growth. A further assumption is that the sequence $\{\tilde{G}_t\}_{t=0}^{\infty}$ is bounded above by $\{\eta^t \tilde{G}_t\}_{t=0}^{\infty}$ for some value $\eta \geq 1$, to ensure that the infinite horizon household objective is bounded above for all feasible consumption paths. In other words, the optimal paths in equations (7) and (8) will be unique: Glomm and Ravikumar (1994).

2.3 Optimal public policy

Solving the government's problem by dynamic programming (Appendix B), it is found that the optimal tax rate is a function of constants.Specifically, the optimal tax rate is defined by the function

$$\tau_t = \theta\beta \tag{11}$$

for all states and dates $t \in \mathbb{N}$. Thus, the optimal tax rate is equal to the one-period discounted share of public capital in output, where the government faces the same subjective discount rate, β, as the household.

Second, given the optimal choice of public policy, the evolution of private capital per efficiency unit worker in equation (8) can be described by the first-order stochastic difference equation

$$\hat{k}_{t+1} = (1-\theta\beta)(\alpha - \theta\phi)\beta A \hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{12}$$

The evolution of public capital per efficiency unit worker is

$$\hat{g}_{t+1} = \theta\beta A \hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{13}$$

Consequently, the ratio of the optimal paths for private and public capital stays constant over time. This can be observed by taking the ratio of equation (13) to equation (12), which yields

$$\frac{\hat{g}_{t+1}}{\hat{k}_{t+1}} = \frac{\theta}{(1-\theta\beta)(\alpha-\theta\phi)} \tag{14}$$

for all states and dates $t \in \mathbb{N}$.

2.4 *Long-run growth*

Substitution of equation (14) into (12) gives the essential difference equation for the evolution of private capital

$$\hat{k}_{t+1} = [(1-\theta\beta)(\alpha-\theta\phi)]^{1-\theta}\,\theta^{\theta}\beta A\hat{k}_t^{\alpha+(1-\phi)\theta}\epsilon_t^P \tag{15}$$

Under an assumption of constant returns to scale to reproducible factors, where $\alpha + (1 - \phi)\theta = 1$, the steady-state ($\epsilon_t^P = 1;\ \forall t$) growth rate of private capital will be given by $[(1-\theta\beta)(\alpha-\theta\phi)]^{1-\theta}\,\theta^{\theta}\beta A$, which is perpetual and non-explosive. Also, output and public capital will grow at the same rate as private capital, with constant returns to scale Cobb-Douglas technology.

2.5 *Testable time series properties of the model*

Equations (12) and (13) can be written in natural logarithm and substitution of these into the stochastic investment equations in (4) and (5) yields

$$\ln\hat{k}_{t+1} = \ln[(1-\theta\beta)(\alpha-\theta\phi)\beta A] + (\alpha-\theta\phi)\ln\hat{k}_t + \theta\ln\hat{g}_t + \ln\epsilon_t^P + \ln\epsilon_{t+1}^K \tag{16}$$

and

$$\ln\hat{g}_{t+1} = \ln(\theta\beta A) + (\alpha-\theta\phi)\ln\hat{k}_t + \theta\ln\hat{g}_t + \ln\epsilon_t^P + \ln\epsilon_{t+1}^G \tag{17}$$

Multiplying equation (16) by $(1-\theta L)$ on both side, where L is the lag operator, and substituting for $(1-\theta L)\ln\hat{g}_t$ from equation (17) yields an equilibrium dynamic equation of the log of per capita private capital expressed in terms of its own lags and the external shocks

$$\begin{aligned}&\{1-[\alpha+(1-\phi)\theta]\,L\}(\ln k_t + xt)\\ &\quad= \{(1-\theta)\ln[(1-\theta\beta)(\alpha-\theta\phi)\beta A] + \theta\ln(\theta\beta A)\}\\ &\qquad+ \theta L\ln\epsilon_t^G + L\ln\epsilon_t^P + (1-\theta L)\ln\epsilon_t^K\end{aligned} \tag{18}$$

Multiplying equation (17) by $[1-(\alpha-\theta\phi)L]$ and substituting for $[1-(\alpha-\theta\phi)L]\ln\hat{k}_t$ from equation (16) yields the equilibrium path for aggregate public capital

$$\begin{aligned}&\{1-[\alpha+(1-\phi)\theta]\,L\}(\ln g_t - xt)\\ &\quad= \{[1-(\alpha-\theta\phi)]\ln(\theta\beta A) + (\alpha-\theta\phi)\ln[(1-\theta\beta)(\alpha-\theta\phi)\beta A]\}\\ &\qquad+ [1-(\alpha-\theta\phi)L]\ln\epsilon_t^G + L\ln\epsilon_t^P + (\alpha-\theta\phi)L\ln\epsilon_t^K\end{aligned} \tag{19}$$

Also, taking logs of the equation for the private production function in equation (3), multiplying this by $\{1-[\alpha+(1-\phi)\theta)]L\}$ and expressing this in per worker terms, yields

$$\begin{aligned}\{1-[\alpha+(1-\phi)\theta]\,L\}\,(\ln y_t - xt) \\ = \{1-[\alpha+(1-\phi)\theta]\}\ln A + (\alpha-\theta\phi)\ln[(1-\theta\beta)(\alpha-\theta\phi)\beta A] + \theta\ln(\theta\beta A) \\ + (\alpha-\theta\phi)\ln\epsilon_t^K + \theta\ln\epsilon_t^G + \{1+[\alpha+(1-\phi)\theta]\,L\}\ln\epsilon_t^P \quad (20)\end{aligned}$$

This equation describes the equilibrium path of the log of output per worker, $\ln y_t$.

Perpetual and stable growth at steady state: In this growth model, growth in per capita output or income depends on the coefficient of the lagged output variable, $\alpha+(1-\phi)\theta$. This is also the sum of all the exponents (or what is loosely known as the factor shares in neoclassical terms) of the private and public inputs into production. There will be no perpetual growth in the per capita variables once the economy reaches the steady-state path, if $\alpha+(1-\phi)\theta<1$, since the effects of past disturbances decay successively in equation (20). Conversely, the steady-state growth path will be explosive if $\alpha+(1-\phi)\theta>1$. In this case there is increasing returns to all inputs.

To obtain perpetual growth with stability in the model, it is a requirement that $\alpha+(1-\phi)\theta=1$ and $x=0$. This is the strict endogenous growth case. Thus, even if private production displays diminishing returns to private inputs, overall it experiences constant returns to scale due to the spillover effect from public capital. Hence there are two empirical properties to be expected of the variables in the endogenous growth case. First, the sequences $\{k_t\}_{t=0}^{\infty}$, $\{g_t\}_{t=0}^{\infty}$ and $\{y_t\}_{t=0}^{\infty}$ will be exact unit root processes. Second, and consequently, the first difference of the logs of the per capita variables will be white noise processes, if the linear combinations of the shocks in (18) to (20) are stationary in levels.

Derivation of cointegrating relationships: If there are three I(1) variables in the system, there can be a maximum of two linearly independent cointegrating vectors. For non-explosive, perpetual endogenous growth, it was concluded that $\alpha+(1-\phi)\theta=1$. Using this fact in equations (19) and (20), and then subtracting the former from the latter, and performing the same again on equation (18) and (20) gives the cointegrating space as

$$\ln y_t - \ln k_t = (1-L)^{-1}\ln\epsilon_t^P - \theta\ln\epsilon_t^K + \theta\epsilon_t^G \quad (21)$$

$$\ln y_t - \ln g_t = (1-L)^{-1}\ln\epsilon_t^P + (1-\theta)\ln\epsilon_t^K - (1-\theta)\epsilon_t^G \quad (22)$$

If the cointegrating space in equation (21) and (22) is rejected, then there may be at most one cointegrating vector. This cointegrating equation is a linear combination of all the variables. This be shown by multiplying equation (21) on both sides by $(1-\theta)$, and and equation (22) on both sides by θ, and then summing the two equations, to obtain

$$\ln y_t - (1-\theta)\ln k_t - \theta \ln g_t = (1-L)^{-1} \ln \epsilon_t^P \quad (23)$$

The cointegrating equation in (23) also represents the production function at steady state with non-explosive, perpetual growth. In general, without assuming $\alpha + (1-\phi)\theta = 1$, the single unrestricted cointegrating equation can be derived from equation (3) yielding

$$\ln y_t - (\alpha - \theta\phi)\ln k_t - \theta \ln g_t - \{1 - [\alpha + (1-\phi)\theta]x\}t - \ln A = (1-L)^{-1} \ln \epsilon_t^P \quad (24)$$

Note that (23) is a nested case of (24) where (23) was derived under the hypothesis of $\alpha + (1-\phi)\theta = 1$. These possible cointegrating relationships will be tested in Section 3 of this paper.

3. Evidence for Australia

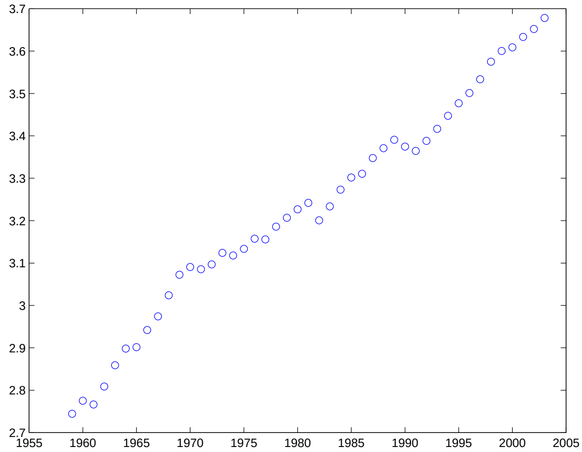
3.1 Data

The empirical analysis in this part involves annual time series from 1959/60 to 2003/04 for Australia. It is important, for the purposes of testing for cointegrating relationships, to have a longer series as opposed to a more frequently sampled series.[3] Gross domestic product (million AU$) at 2002/03 constant prices is constructed, with seasonal adjustment, from Australian Bureau of Statistics (ABS) National Accounts, Table 5204.02. From ABS National Accounts, Table 5204.70, we extracted both the year-end net government (public) capital stock (million AU$) and year-end net private capital stock (million AU$). Both types of net capital stock are generated by institutional sector with 2002/03 constant prices (Seasonally adjusted). The end-year net private capital stock involves both non-financial and financial corporations. Population data in Australia (000 persons) is obtained mainly from OECD Economic Outlook: Table C3-AUS-Y (1960–2003). Population level in 1959 originates from World Bank World Tables.

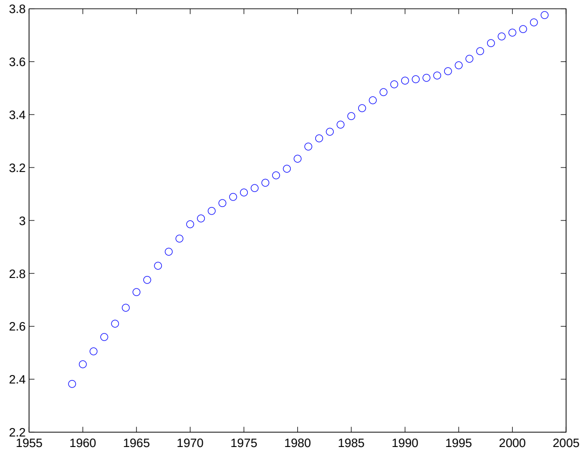
All the per-capita variables are expressed in logarithms as implied by equations (23) and (24). Not surprisingly, from figure 2, we observe the three series roughly exhibit increasing time trends in our sample period. However, the net public capital stock per capita (in logarithm) seems to suffer from a structural change after 1979 and remains roughly constant thereafter. Therefore, in our empirical estimations and tests, we will take into account this structural break.

[3]See Hakkio and Rush (1991). All data were extracted from dX Database.

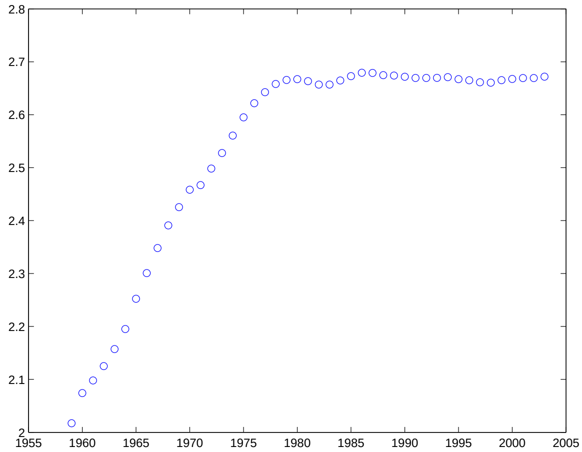
FIGURE 1. LOG PER CAPITA OUTPUT, PRIVATE CAPITAL AND PUBLIC CAPITAL, RESPECTIVELY.



(a) $\ln y$



(b) $\ln k$



(c) $\ln g$

Weak stationarity of the series: From Table 1, the Augmented Dickey-Fuller (1979, 1981) unit root test reveals all the variables appear to be non-stationary in levels (contain a unit root), but will be stationary after taking first differences.[4] Thus, for latter estimation of cointegrating relationship as well as vector error correction model, we will not result in imbalanced regressions since the levels of variables are integrated with order 1, or I(1), but with order 0, or I(0), after taking first differences.

TABLE 1
UNIT ROOT TESTS FOR $\ln y$, $\ln k$ AND $\ln g$ IN LEVELS AND FIRST DIFFERENCES

	Endogenous variables in levels			
	$\ln y$	$\ln k$	$\ln g$	$\ln g$
Sample period	1960–2003	1961–2003	1964–1978	1979–2003
Lags of ADF test	0	1	4	2
Exogenous	constant and trend	constant and trend	constant and trend	constant
ADF statistics	−2.1479	−2.1439	−0.2497	−2.5309
5% critical value	−3.5155	−3.5181	−3.7597	−2.9862
	Endogenous variables in first differences			
	$\Delta \ln y$	$\Delta \ln k$	$\Delta \ln g$	$\Delta \ln g$
Sample period	1961–2003	1961–2003	1964–1978	1979–2003
Lags of ADF test	0	0	3	1
Exogenous	constant	constant	constant and trend	constant
ADF statistics	−5.6132	−3.0520	−4.5334	−4.7051
5% critical value	−2.9314	−2.9314	−3.7597	−2.9862

Note: the lag length of each dependent variable is optimally determined by Schwarz information criterion (SC).

Cointegration and output elasticity estimates: The long-run cointegrating relationships between $\ln y$, $\ln k$ and $\ln g$ depends on their non-stationary fluctuation along the time trend. Firstly, the cointegrating relationships in equation (21) and (22) are rejected as their linear combination of residuals fail to be a stationary process. Therefore, we cannot have a maximum of two linearly independent cointegrating vectors in out sample. These results are shown in Table 2.

Next, we consider whether there exists one cointegrating vector between the three variables in this system. To motivate this possibility, consider the three-dimensional scatter plot of $\{\ln y, \ln k, \ln g\}$ in Figure 2. This raw and informal plot suggests that all three series occur along some common vector at least in the long run. In other words, there is some informal evidence of a single cointegrating vector for all three variables. There appears to be a little kink in the scatter plot (in the "north-eastern" direction) possibly due to the apparent structural break in the time series for $\ln g$.

As previously mentioned, we allow the structural break of $\ln g$ to augment the regression

[4] We modify the unit root test for $\ln g$ by examining two separated period to avoid its structural change after 1979.

TABLE 2
UNIT ROOT TESTS FOR $(\ln y - \ln k)$ AND $(\ln y - \ln g)$

	Endogenous variables in levels	
	$(\ln y - \ln k)$	$(\ln y - \ln g)$
Sample period	1960–2003	1961–2003
Lags of ADF test	0	1
Exogenous	with constant and trend	with constant
ADF statistics	-1.4650	0.9674
5% critical value	-3.5155	-2.9134

Note: the lag length of each dependent variable is optimally determined by Schwarz information criterion.

for the theoretical relationship in equations (23) and (24). Therefore, the long-run relationship between the three series is determined by

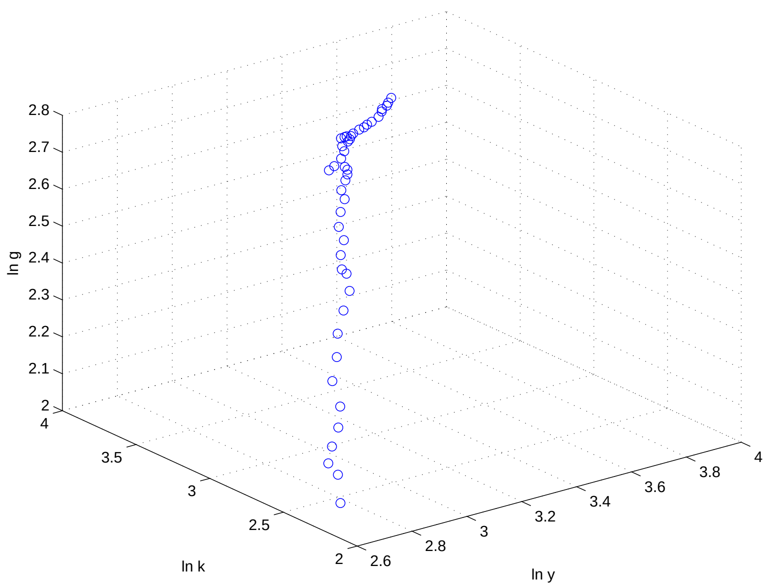
$$\ln y_t = \beta_0 + \beta_1 \ln k_t + \beta_2 \ln g_t + \beta_3 t + \beta_4 D_{79} + \beta_5 D_{79} t \tag{25}$$

where D_{79} is the dummy variable that captures the obvious structural change in the series $\{\ln g_t\}$ such that

$$D_{79} = \begin{cases} 1 & \text{if } t \in \{1979, 1980, ..., 2003\} \\ 0 & \text{if } t \in \{1959, 1960, ..., 1978\} \end{cases}$$

and the β_i's are reduced-form coefficients to be estimated.

FIGURE 2. SCATTER PLOT OF LOG PER CAPITA OUTPUT, PRIVATE CAPITAL AND PUBLIC CAPITAL.



The existence of cointegrating relationship amongst the three variables requires the OLS residuals from equation (25) to be stationary otherwise we would have a spurious regression. The reduced-form coefficients β_1, β_2 and β_3 in equation (25), respectively, denote $(\alpha - \theta\phi)$, θ and $\{1 - [\alpha + (1-\phi)\theta]x\}$ originating from equation (24). Specifically, if the estimated coefficients follow the conditions that $\beta_1 + \beta_2 = 1$ and $\beta_3 = 0$, the engines of economic growth are endogenously determined by both net private and public capital (that is, equation (23) holds such that the production function in equation (3) exhibits constant returns to scale). Conversely, if $\beta_1 + \beta_2$ is estimated to be less than one (i.e. $\beta_3 > 0$ and equation (24) holds in this system), the economy will grow exogenously along the time trend.

The OLS estimation provides the following coefficients embedded in equation (25) and the parentheses below estimated coefficients denote respective standard errors.

$$\ln y_t = \underset{(0.4176)}{0.6070} + \underset{(0.1885)}{0.0927} \ln k_t + \underset{(0.3865)}{0.9520} \ln g_t - \underset{(0.0070)}{0.0126} t - \underset{(0.1868)}{0.6516} D_{79} + \underset{(0.0099)}{0.0306} D_{79} t \quad (25')$$

From the above estimation results, we cannot reject the null hypotheses of $\beta_1 + \beta_2 = 1$ and that $\beta_3 = 0$ for 5% level of significance. This suggests that the hypothesis of strict endogenous growth due to public capital spillover into private production is the main driving force in Australia during our sample period. Moreover, the net government capital plays a more significant role (with very high input share 0.9520) than net private capital does (with insignificant input share 0.0927). Our result contrasts with the results from Lau and Sin (1997), who used US data for a similar regression and estimated the shares of $\ln g$ and $\ln k$ to be 0.11 and 0.43, respectively. The result of Lau and Sin (1997) suggested the dominance of private capital in the process of economic growth in the US. Another study for Australia was carried out by Otto and Voss (1994) using a shorter sample period of 1966/67–1989/90. Although their findings revealed a higher share of public capital in the production process than Otto and Voss (1994), the share of private capital was estimated to be unreasonably negative.

Economic relevance of long-run estimates: Since one of the parameters $\{\alpha, \phi\}$ cannot be identified from the estimates of β_1 and $\beta_2 = \theta$, we perform the following informal exercise to check whether our estimates provide some sensible economic parameterization. We use the following guideline. Since we know very little about the congestion parameter ϕ in the model, except that it must be constrained within the open set $(0, 1)$, we calibrate α to two scenarios such that $\alpha = 1/3$ in one scenario and $\alpha = 1/4$ in the other. The latter calibration is motivated by the argument that the usual share of private capital stock in levels to be lower than the stylized fact of $1/3$ in the presence of endogenous growth effects. Table 3 demonstrates a comparison of estimated results between the two previous studies and our findings. It should noted that when considering the congestion effect of aggregate public infrastructure in equation (2), the estimated coefficients of $\ln g$ and $\ln k$ from Lau and Sin (1997) seem to result in a theoretically infeasible congestion parameter ϕ when we set the private capital share α as a reasonable scale (either $1/3$ or $1/4$). However, in our calculations of ϕ, based on our reduced-form estimates and the assumptions on α, we do not have such problems.

TABLE 3
THE DIFFERENCES OF ESTIMATED RESULTS BETWEEN THE US AND AUSTRALIA

	Lau and Sin (1997)		Otto and Voss (1994)		This paper	
Estimated coefficients	US data		Australia data		Australia data	
$\beta_1 = (\alpha - \theta\phi)$	0.43		-0.0870		0.0927	
$\beta_2 = \theta$	0.11		0.4303		0.9520	
Comparison	$\alpha = 1/3$	$\alpha = 1/4$	$\alpha = 1/3$	$\alpha = 1/4$	$\alpha = 1/3$	$\alpha = 1/4$
Congestion parameter ϕ	-0.88	-1.63	0.9768	0.7832	0.25	0.17

Using again the Augmented Dickey-Fuller unit root test for the OLS residuals from regression (25'), we found that they are stationary for 5% level of significance.[5] This evidence suggests that $\ln y$, $\ln k$ and $\ln g$ are cointegrated with intercept and time trend in equation (25').

4. SHORT-RUN DYNAMICS AND IMPULSE RESPONSE

Aschauer (1989a) pointed out the possibility of reverse causation between the level of public capital expenditure and production. That is, $\ln g$ responds to rises in $\ln y$. This example of Wagner's Law arises if expenditure on public infrastructure or public goods is a superior good. Furthermore, there may also be interactions between $\ln g$ and $\ln k$. On the one hand, public capital expenditure may be seen as the springboard for private investment. This runs counter to standard elementary macroeconomic argument that government expenditure tends to crowd out private investment. However, it may be that public capital increases the marginal product of private capital. An obvious example is the provision of better highways, which results in less wear and tear of private vehicles while goods are transported more efficiently. On the other hand, public capital expenditure may be seen as responding to private investment demands.

The time-series causal chain running from public capital accumulation is obvious in the theoretical model. Here we would like to investigate whether this short-run effect is empirically plausible by letting the data speak as much as possible while imposing the long-run relationship estimated earlier. Thus we conduct impulse response analysis of a vector

[5] We also reject the null hypotheses of first- and second-order serial correlation in errors, for 1% level of significance, using Breusch-Godfrey serial correlation LM test. This provides a robustness check for our estimation.

error correction model with no theoretical restrictions on the short run dynamics to let the data inform us of the short-run effect of $\ln g$ on $\ln y$ and $\ln k$.

4.1 Impulse Response Analysis

Our optimally chosen VEC(2) model incorporating (25) with exogenous variables t, D_{79} and $D_{79}t$, can be written as a restricted vector autoregression model with lag length 3 (VAR(3)). The restriction arises from the cointegration structure imposed by the long run behavior. The multivariate error structure from this is decomposed into lower triangular matrices such that the restricted VAR can be written, theoretically, as an infinite vector moving average (VMA) utilising the orthogonalised error structure. This then, ensures that a shock to a variable will have no contemporaneous correlation with other residuals.

In performing an impulse response of VEC(2) model, a simulation period of up to fifty years and an ordering of ($\ln y$, $\ln k$ and $\ln g$) is used. This ordering is determined by VAR(3) Pairwise Granger Causality/Block Exogeneity Wald Tests. Since the variables are in logarithms, each 0.01 unit change in the response functions denotes an 1 per cent change.

From Figure 3, $\ln y$ responds positively to a one-period (positive) shock from $\ln g$ and this effect fluctuates and diminishes a little over time. The response of $\ln y$ to $\ln g$ (permanently) reaches 0.6 per cent level after 40 years. The $\ln g$ shock has permanent effects on $\ln y$ due to $\ln y$ being close to a unit root.

Also, $\ln k$ responds positively to $\ln g$ from the beginning and reaches the highest response above 0.6 per cent. Thus, there is evidence of public infrastructure "crowding in" private investment, which affirms Aschauer (1989b). Similar to the response of $\ln y$ to $\ln g$, the effect on $\ln k$ resulting from a positive shock to $\ln g$ is also declining across time and roughly constant at 0.4% after 40 years. However, there does appear to be evidence of Wagner's Law as ln $\ln g$ responds positively to $\ln y$.

The influence of $\ln g$ can further been seen in the variance decomposition of the fifty-period forecast error of the variables in the system in Figure 4. It can be observed that about 30 per cent of the forecast error in $\ln y$ is due to the innovation to $\ln g$ and about 20 per cent of the forecast error of $\ln g$ is due to its own innovation. There contribution of $\ln g$ to the forecast error of $\ln k$, of about 10 per cent, is slightly lower but nevertheless substantial. Therefore, it can be concluded from the impulse response analysis and variance decompositions that public infrastructure investment does impact positively on output and private investment in the short to medium term.

5. Conclusions

It was the aim in this paper to study the effect of public infrastructure on the aggregate economy in terms of long-run growth and short-run effects. In particular, the issue was whether growth was determined in the long run, in part, by the accumulation of the stock of public infrastructure. A simple stochastic growth model nesting exogenous and endogenous growth with public capital spillovers was considered in Section 2 of the paper.

The long-run implication of this model was tested empirically for Australia in Section 3. It was found that there was evidence of cointegration between per capita output, per capita net private capital and per capita net public capital. A nested test of the strictly exogenous growth model was rejected in favour of the endogenous growth model with public infrastructure spillovers.

Lastly, the cointegrating relationship was incorporated into a VEC(2) model to study the short-run behaviour of the variables in Section 4. It was found that innovations to public infrastructure induce permanently higher levels of output and private investment in the short to medium run.

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FIGURE 3. IMPULSE RESPONSE FUNCTIONS WITH ORDERING ($\ln y$, $\ln k$ AND $\ln g$)

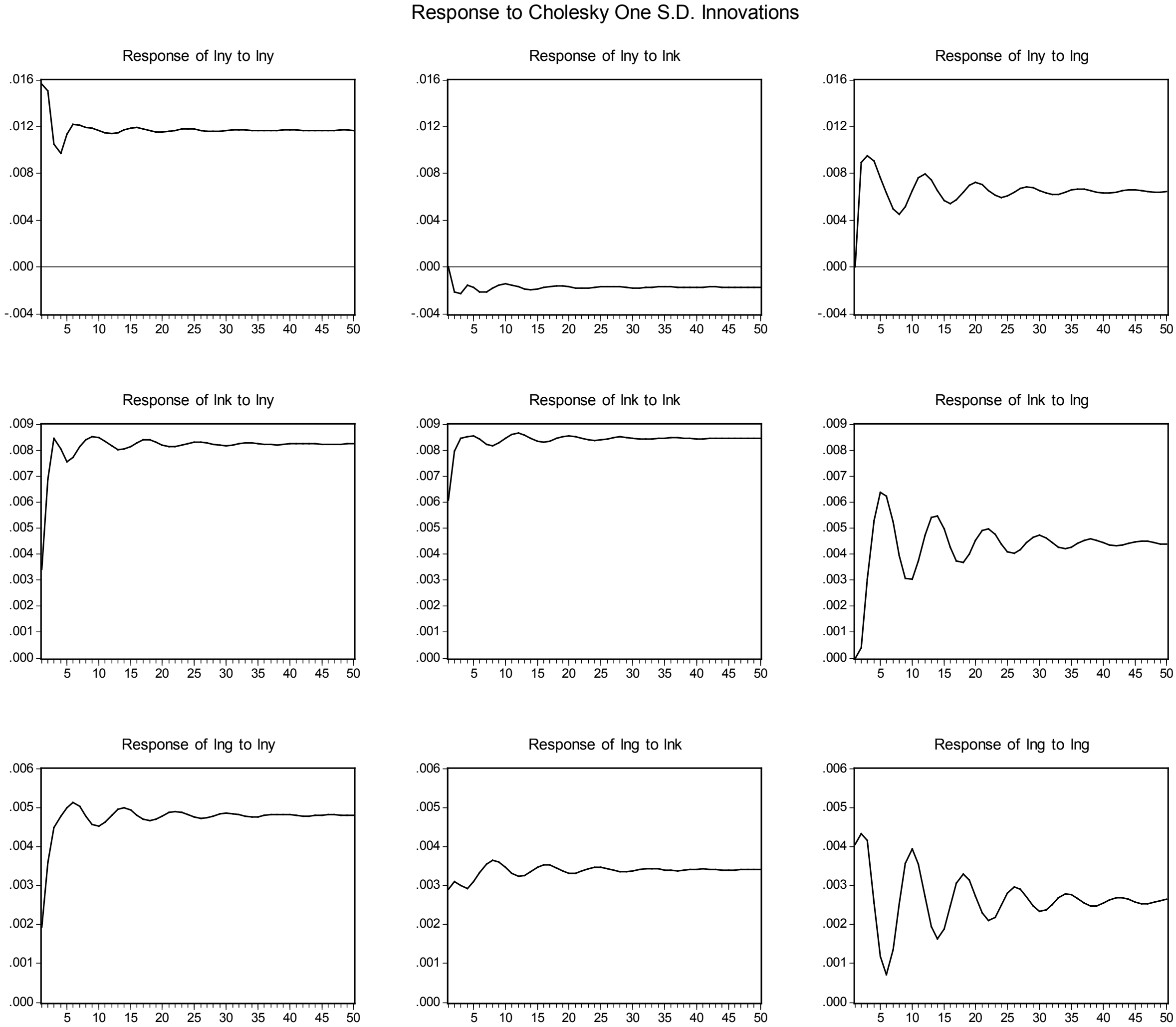
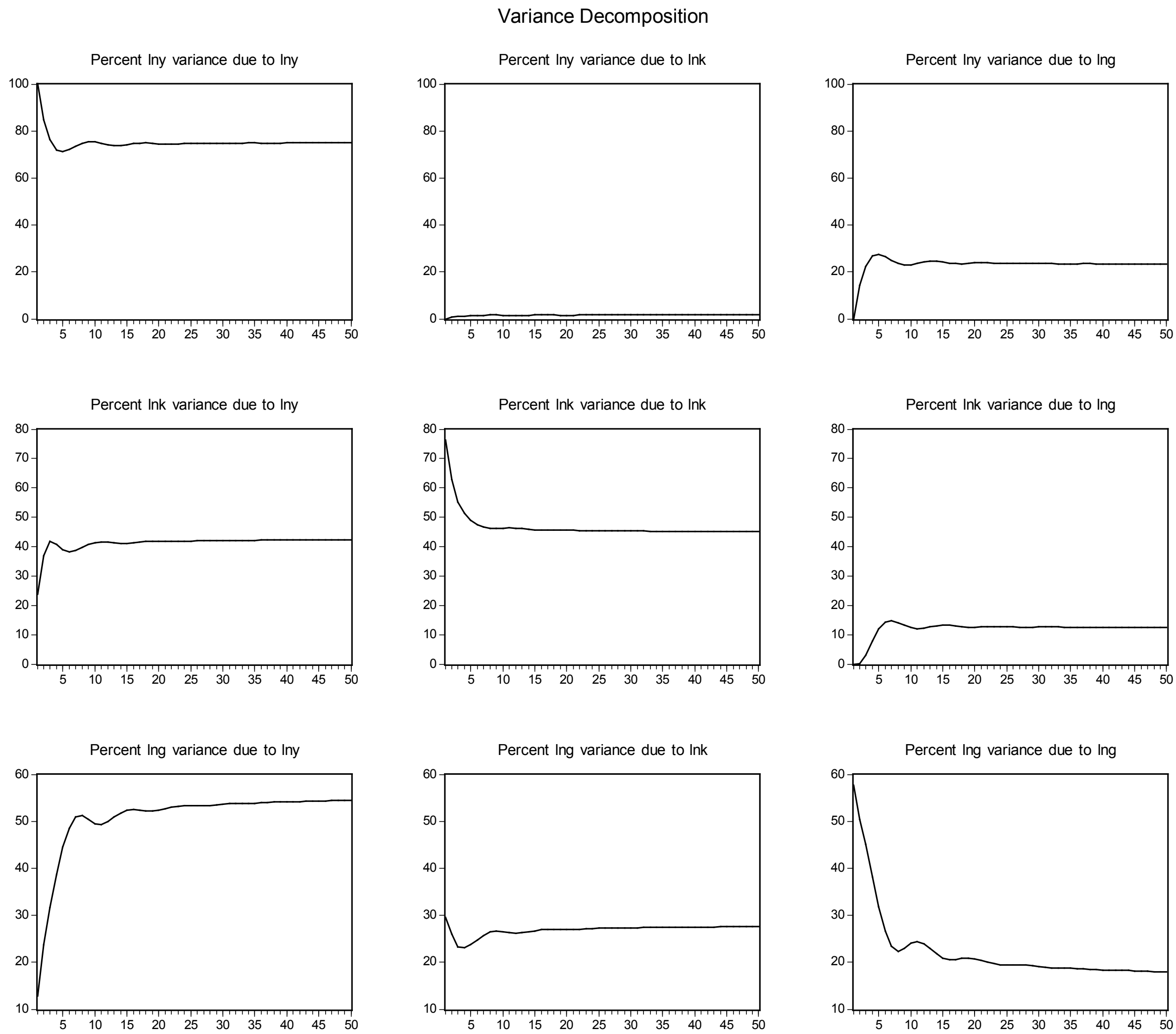


FIGURE 4. FORECAST-ERROR VARIANCE DECOMPOSITIONS WITH ORDERING ($\ln y$, $\ln k$ AND $\ln g$)



APPENDIX A
DYNAMIC PROGRAMMING FOR THE HOUSEHOLD PROBLEM

The method of solving the household's intertemporal utility maximisation problem subject to given constraints and public policy in equation (6) is as follows. Bellman's (1957) principle of optimality dictates that if the sequence of $\{\hat{c}_t, \hat{k}_{t+1}\}_{t=0}^{\infty}$ is maximising, then it must also be the case that it maximises the functional over $\{\hat{c}_0, \hat{k}_1\}$ and $\{\hat{c}_t, \hat{k}_{t+1}\}_{t=1}^{\infty}$. Hence the problem in equation (6) can be written as

$$V(\hat{k}_t, \epsilon_t^P) = \max_{\hat{c}_t, \hat{k}_{t+1}} \left[\ln(\hat{c}_t) + \beta \mathbb{E}_t V(\hat{k}_{t+1}) \right]; \qquad \beta \in (0,1) \tag{A.1}$$

subject to constraints (6) (a)–(c).

A guess of the solution to (A.1) is

$$V(\hat{k}_0, \epsilon_0^P) = B_0 + B_1 \ln(\hat{k}_0) + B_2 \ln(\epsilon_0^P) \tag{A.2}$$

Substituting the form of equation (A.2) into (A.1) gives

$$V(\hat{k}_t, \epsilon_t^P) = \max_{\hat{c}_t, \hat{k}_{t+1}} \left\{ \ln(\hat{c}_t) + \beta \mathbb{E}_t V \left[B_0 + B_1 \ln(\hat{k}_{t+1}) + B_2 \ln(\epsilon_{t+1}^P) \right] \right\} \tag{A.3}$$

subject to constraints (6) (a)–(c).

At time *t*, the control variables, $\hat{c}_t$ and $\hat{k}_{t+1}$, and the state variables, ϵ_t^P and $\hat{k}_t$ are all known. Further, with the assumption that $\ln \epsilon_t$ is independently and identically distributed such that $\mathbb{E}_t \ln(\epsilon_{t+1}) = 0$, the terms in the curly brackets of equation (A.3) can be reduced to

$$\ln \hat{c}_t + \beta \mathbb{E}_t \left[B_0 + B_1 \ln(\hat{k}_{t+1}) \right] \tag{A.4}$$

Define the Lagrangian as

$$L = \ln \hat{c}_t + \beta \mathbb{E}_t \left[B_0 + B_1 \ln(\hat{k}_{t+1}) \right] + \lambda_t \left[(1-\tau_t) A \hat{k}_t^{\alpha - \theta\phi} \hat{g}_t^{\theta} \epsilon_t^P - \hat{c}_t - \hat{k}_{t+1} \right]$$

and the first-order conditions for maximisation are

$$\frac{1}{\hat{c}_t} = \lambda_t \tag{A.5}$$

$$\frac{\beta B_1}{\hat{k}_{t+1}} = \lambda_t \tag{A.6}$$

$$(1-\tau_t) A \hat{k}_t^{\alpha - \theta\phi} \hat{g}_t^{\theta} \epsilon_t^P = \hat{c}_t + \hat{k}_{t+1} \tag{A.7}$$

for all states and dates $t \in \mathbb{N}$, and the transversality condition

$$\lim_{t\to\infty} \beta^t \hat{k}_{t+1} = 0.$$

Substitute equations (A.5) and (A.6) into (A.7) to get

$$\hat{c}_t = \frac{1}{(1+\beta B_1)}(1-\tau_t)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{A.8}$$

Use the natural constraint (A.7) and (A.8) to derive the stochastic difference equation for private capital per efficiency unit worker

$$\hat{k}_{t+1} = \frac{\beta B_1}{(1+\beta B_1)}(1-\tau_t)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{A.9}$$

Substitute equations (A.8) and (A.9) into the RHS of the Bellman equation (A.3) to verify that the LHS of equation (A.3)

$$V(\hat{k}_t, \epsilon_t^P) = B_0 + B_1 \ln(\hat{k}_t) + B_2 \ln(\epsilon_t^P)$$

is equal to the RHS of equation (A.3), which is

$$\ln\left[\frac{1}{(1+\beta B_1)}(1-\tau_t)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P\right] + \beta\left\{B_0 + B_1\ln\left[\frac{\beta B_1}{(1+\beta B_1)}(1-\tau_t)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P\right]\right\}.$$

Expanding terms on the RHS

$$\begin{aligned}&\beta B_0 - (1+\beta B_1)\ln(1+\beta B_1) + \beta B_1 \ln(\beta B_1) + (1+\beta B_1)\ln\left[(1-\tau_t)A\hat{g}_t^{\theta}\right]\\&+(\alpha-\theta\phi)(1+\beta B_1)\ln(\hat{k}_t) + (1+\beta B_1)\ln(\epsilon_t^P)\end{aligned}$$

For the functional to be valid, the LHS must, *inter alia*, satisfy the condition that

$$B_1 = (\alpha - \theta\phi)(1+\beta B_1)$$

and thus, the guess in (A.4) will be correct if

$$B_1 = \frac{(\alpha-\theta\phi)}{1-(\alpha-\theta\phi)\beta} \tag{A.10}$$

Substitute (A.10) into equations (A.8) and (A.9) then we can obtain the optimal household consumption and investment paths with given public policy $\{\tau_t, \hat{g}_t\}_{t=0}^{\infty}$,

$$\hat{c}_t = (1-\tau_t)\left[1-(\alpha-\theta\phi)\beta)\right]A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{7}$$

$$\hat{k}_{t+1} = (1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{8}$$

for all states and dates $t \in \mathbb{N}$, given k_0.

APPENDIX B
THE GOVERNMENT'S PROBLEM AND OPTIMAL OUTCOMES

The government's Ramsey optimal fiscal policy problem is defined here as a dynamic program:

$$v(\hat{k}_t, \hat{g}_t, \epsilon_t^P) = \max_{\tau_t, \hat{k}_{t+1}, \hat{g}_{t+1}} \left\{ \ln\{(1-\tau_t)\left[1-(\alpha-\theta\phi)\beta\right)] A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P\} + \beta\mathbb{E}_t v(\hat{k}_{t+1}, \hat{g}_{t+1}) \right\} \tag{B.1}$$

subject to constraints (10) (a)–(d).

Second, guess that the solution is of the form below

$$v(\hat{k}_0, \hat{g}_0, \epsilon_0^P) = B_0 + B_1\ln(\hat{k}_0) + B_2\ln(\hat{g}_0) + B_3\ln(\epsilon_0^P) \tag{B.2}$$

Utilising the guess in (B.2), re-write equation (B.1) as

$$\begin{aligned} v(\hat{k}_t, \hat{g}_t, \epsilon_t^P) = \max_{\tau_t, \hat{k}_{t+1}, \hat{g}_{t+1}} & \left\{ \ln\{(1-\tau_t)\left[1-(\alpha-\theta\phi)\beta\right)] A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P\} \right. \\ & \left. +\beta\mathbb{E}_t\left[B_0 + B_1\ln(\hat{k}_{t+1}) + B_2\ln(\hat{g}_{t+1}) + B_3\ln(\epsilon_{t+1}^P)\right]\right\} \end{aligned} \tag{B.3}$$

subject to (10) (a)–(d). It is also assumed here that $\ln(\epsilon_{t+1}^P)$ is white noise and therefore, $\mathbb{E}_t\ln(\epsilon_{t+1}^P) = 0$.

Define the Lagrangian as

$$\begin{aligned} L = & \left\{ \ln\{(1-\tau_t)\left[1-(\alpha-\theta\phi)\beta\right)] A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P\}\right\} \\ & + \beta\left[B_0 + B_1\ln(\hat{k}_{t+1}) + B_2\ln(\hat{g}_{t+1})\right] \\ & + \mu_t\left(\tau_t A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P - \hat{g}_{t+1}\right) + \psi_t\left[(1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P - \hat{k}_{t+1}\right] \end{aligned} \tag{B.4}$$

The necessary first-order conditions for maximisation of the Lagrangian are

$$L_{\hat{k}_{t+1}} = \frac{\beta B_1}{\hat{k}_{t+1}} - \psi_t = 0 \tag{B.5}$$

$$L_{\hat{g}_{t+1}} = \frac{\beta B_2}{\hat{g}_{t+1}} - \mu_t = 0 \tag{B.6}$$

$$L_{\tau_t} = -\frac{1}{(1-\tau_t)} + \mu_t A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P - \psi_t(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P = 0 \tag{B.7}$$

$$L_{\mu_t} = \tau_t A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P - \hat{g}_{t+1} = 0 \tag{B.8}$$

$$L_{\psi_t} = (1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P - \hat{k}_{t+1} = 0 \tag{B.9}$$

for all states and dates $t \in \mathbb{N}$, and the transversality condition

$$\lim_{t\to\infty} \beta^t \hat{k}_{t+1} = 0.$$

Express the constraints (B.8) and (B.9) in terms of $\hat{g}_{t+1}$ and $\hat{k}_{t+1}$ respectively, and substitute these into equations (B.5) and (B.6) to obtain

$$\frac{\beta B_1}{(1-\tau_t)(\alpha-\theta\phi)\beta A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P} = \psi_t \tag{B.4'}$$

$$\frac{\beta B_2}{\tau_t A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P} = \mu_t \tag{B.5'}$$

Substitute equations (B.4') and(B.5') into equation (B.7) gives

$$\tau_t = \frac{\beta B_2}{1+\beta B_1+\beta B_2} \tag{B.9}$$

Further substitution of equation (B.9) back into constraints (B.8) and (B.9) results in

$$\hat{g}_{t+1} = \left(\frac{\beta B_2}{1+\beta B_1+\beta B_2}\right) A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{B.10}$$

and

$$\hat{k}_{t+1} = (\alpha-\theta\phi)\beta\left(\frac{\beta B_2}{1+\beta B_1+\beta B_2}\right) A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P \tag{B.11}$$

Next, substitute (B.10) and (B.11) into (B.3) and compare with the form of (B.2)

$$\begin{aligned} &B_0 + B_1\ln(\hat{k}_t) + B_2\ln(\hat{g}_t) + B_3\ln(\epsilon_t^P) \\ &\quad\equiv \ln\left\{[1-(\alpha-\theta\phi)\beta]\left(\frac{1+\beta B_1}{1+\beta B_1+\beta B_2}\right)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P\right\} \\ &\qquad + \beta\left\{B_0 + B_1\ln\left[(\alpha-\theta\phi)\beta\left(\frac{1+\beta B_1}{1+\beta B_1+\beta B_2}\right)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P\right]\right\} \\ &\qquad\quad + \beta\left\{B_2\ln\left[(\alpha-\theta\phi)\beta\left(\frac{\beta B_2}{1+\beta B_1+\beta B_2}\right)A\hat{k}_t^{\alpha-\theta\phi}\hat{g}_t^{\theta}\epsilon_t^P\right]\right\} \end{aligned} \tag{B.12}$$

Expand the RHS of equation (B.12) and collect terms

$$\begin{aligned} &\beta B_0 + \ln\left\{\frac{[1-(\alpha-\theta\phi)\beta](1+\beta B_1)}{1+\beta B_1+\beta B_2}\right\} + \beta B_1\ln\left[\frac{(\alpha-\theta\phi)\beta(1+\beta B_1)}{1+\beta B_1+\beta B_2}\right] \\ &\quad + \beta B_2\ln\left[\frac{(\alpha-\theta\phi)\beta^2 B_2}{1+\beta B_1+\beta B_2}\right] + (1+\beta B_1+\beta B_2)\ln(A) \\ &\qquad + (\alpha-\theta\phi)(1+\beta B_1+\beta B_2)\ln(\hat{k}_t) \\ &\qquad\qquad + \theta(1+\beta B_1+\beta B_2)\ln(\hat{g}_t) + (1+\beta B_1+\beta B_2)\ln(\epsilon_t^P) \end{aligned} \tag{B.13}$$

For the functional to be valid, that is the LHS=RHS in (B.12), it must be that the coefficients on the LHS of (B.12) satisfy, *inter alia*

$$B_1 = (\alpha - \theta\phi)(1 + \beta B_1 + \beta B_2)$$
$$B_2 = \theta(1 + \beta B_1 + \beta B_2)$$

Solving for B_1 and B_2 yields

$$B_1 = \frac{\alpha - \theta\phi}{1 - \theta\beta - (\alpha - \theta\phi)\beta} \tag{B.14}$$
$$B_2 = \frac{\theta}{1 - \theta\beta - (\alpha - \theta\phi)\beta} \tag{B.15}$$

Substitution of equation (B.14) and (B.15) into (B.9), (B.10) and (B.11) gives the optimal tax rate and the evolutions of private and public capital

$$\tau_t = \theta\beta \tag{11}$$
$$\hat{k}_{t+1} = (1 - \theta\beta)(\alpha - \theta\phi)\beta A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \epsilon_t^P \tag{12}$$
$$\hat{g}_{t+1} = \theta\beta A \hat{k}_t^{\alpha-\theta\phi} \hat{g}_t^{\theta} \epsilon_t^P \tag{13}$$

for all states and dates $t \in \mathbb{N}$, given k_0.

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