THE IMPOSSIBILITY OF A NEUTRAL RESOURCE RENT TAX

Ben Smith
Department of Economics
Australian National University

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Faculty of Economics and Commerce
and
Economics Program, Research School of Social Sciences
Australian National University

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BEN SMITH

Faculty of Economics and Commerce
Australian National University

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Abstract: The proposition that the Resource Rent Tax can have a neutral effect on the level of mining investment has re-surfaced in recent literature. This paper shows that the proposition is essentially false. The models used to generate it involve a single dimension of investment decision making. The introduction of realistic additional dimensions, such as determining how much information to seek about the mineral deposit before mining and deciding upon the appropriate timing and rate of mining investment and extraction, makes a neutral RRT impossible to achieve.

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Key Words: Minerals Taxation, Resource Rent Tax

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I. Introduction

The Resource Rent Tax (RRT) was proposed by Garnaut and Clunies-Ross (1975) as a less distorting (and potentially more stable) means of taxing mining rents than the forms of royalty generally used. The RRT is a tax on the net cash flows of a mining project, in which negative net cash flows in any period are carried forward with interest (at a rate called the *threshold rate)* and are deductible against subsequent positive net cash flows in the calculation of tax liability. Thus, a project subject to the RRT would not pay any tax until the original investment had been recovered with interest equal to the threshold rate. The *quid pro quo* would be that the tax rate applying once the threshold had been exceeded would be high relative to normal royalty rates.

An important strand of the subsequent literature on mining taxation was concerned with the appropriate balance between conditional profits taxes of the RRT variety and up-front auctioning of rights to exploit mineral leases as means of obtaining government revenues.[1]

A separate discussion, to which the present paper contributes, has focussed on the possibility, suggested by Garnaut and Clunies-Ross, that the RRT could have a (nearly) neutral effect on investment decisions. Following critical comments by Swan (1976, 1978) and Dowell (1978), Garnaut and Clunies-Ross (1979) elaborated on the requirement to balance the investment deterring effects of the tax against its investment promoting effects. Fane and Smith (1986) used a simple example, in which the effects of the RRT were compared with those of a Pure Rent Tax (PRT) levied at the same rate, in order to clarify the nature of the investment deterring and investment promoting effects of the former.[2]

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[1] Contributions to this literature include Dowell (1978), Emerson and Lloyd (1983), Garnaut and Emerson (1984), and Fane and Smith (1986).

[2] The Pure Rent Tax is alternatively referred to as the ‘Brown’ Tax, after its original proponent, E.Carey Brown (see Swan, 1976).

More recently, Fraser (1993, 1998) has given detailed consideration to conditions under which the opposing incentive effects of the RRT might exactly cancel out. These two papers employ the same simple model of mining investment but the investing firm is assumed to be risk-neutral in the first paper and risk-averse in the second. Both papers use numerical examples to explore the possibility that RRT parameters can be set so as to cause the firm to choose the same level of investment as would have been chosen in the absence of the tax.

In summary, Fraser's (1993) results for the case of a risk-neutral investor are:

- the neutrality of the RRT is independent of the rate of tax, so the issue is whether there exists a threshold rate that provides no (net) incentive for either under- or over-investment;
- there is a range of parameter values, relating to expected profitability and levels of uncertainty, at which the neutral threshold rate does exist and where the government obtains positive expected tax revenue; but
- with low enough expected profitability and/or high enough levels of uncertainty, the RRT will have a net investment deterring effect at any value of the threshold rate that is low enough for the government ever to obtain any tax revenue; and
- with high enough expected profitability and/or low enough uncertainty, the RRT will have a net investment encouraging effect even if the threshold rate is set equal to zero.

It is straightforward to show analytically that the last of these results is incorrect and this is done in passing in the next section of the paper.

Fraser's (1998) results for the case of the risk-averse investor are:

- neutrality depends both on the tax rate and on the threshold rate and, for any set of profitability / uncertainty parameters, there exists a continuum of neutral combinations of tax and threshold rates in which a higher tax rate is associated with a higher value of the threshold rate;

- progression from a lower neutral tax / threshold rate pair to a higher neutral pair increases the government's expected tax revenue; but
- if the RRT is combined with an auction system of allocating mineral leases, the combined sum of up-front bid plus expected tax payments will not necessarily increase as the tax (and threshold) rate are increased.

The last result is again incorrect, as shown in Appendix 1. No result equivalent to the third dot point in the risk neutrality case is reported. That result must also apply with risk aversion and its absence reflects the limited range of parameter values over which the numerical analysis of Fraser (1998) was conducted.

With the qualifications stated, Fraser's conclusions are consistent with those of earlier literature. For a specified investment opportunity there can exist RRT parameters such that the tax will not affect the level of investment undertaken. This does not mean that a taxing authority can intentionally apply a neutral RRT to any given investment, since that would require it to be as well-informed about the profitability / uncertainty variables as the investors themselves. The more modest proposition would be that there exists a set of RRT parameters that, if applied by the taxing authority (by accident or design), would have a neutral effect on a given investment decision.

The central argument of the present paper is that this proposition is essentially false. More precisely, it is an artifact of the uni-dimensional nature of the examples employed to generate it and these do not appropriately represent the sorts of investment decisions faced by an entity to which the RRT might ever apply. As soon as we allow even the simplest deviations from these kinds of examples, the proposition breaks down. There is, in fact, no neutral set of RRT parameters that the taxing authority could apply.

The plan of the paper is as follows. In the next section Fraser’s basic model is revisited for the case of the risk-neutral firm and a clearer exposition of the nature of the RRT and its incentive effects is provided. A very simple example is used to demonstrate the possibility of a neutral RRT in that model. Subsequent sections maintain the same simple framework but introduce practically relevant additional dimensions to the firm's investment decision problem. These involve giving the firm the opportunity to acquire additional information about its mineral deposit before committing to mining, giving it freedom to decide on the timing of production (but not investment), and giving it freedom to decide on the timing of the investment itself. The RRT that would be neutral in the absence of these additions is, in each case, shown not to be neutral once we allow for them. More important, there is no longer any set of RRT parameters that would have a neutral effect. The paper concludes with brief observations about the implications of this and about the broader context in which the appropriate design of mining taxation mechanisms should be considered.

II. Neutrality of the RRT in the Baseline Model

Fraser's example is of a mining company that has to choose the scale of its capacity investment before the size of the deposit to be extracted is known. The model is single period: the company invests in capacity, k, and then immediately produces output equal to the minimum of the actual size of the deposit, x, and the level of capacity installed. Thus, with a constant cost per unit of' capacity (c) and constant net price per unit of output (p), the firm's profit in the absence of tax is:

$$p = px - ck \text{ if } x \leq k \quad \text{and} \quad p = pk - ck \text{ if } x > k$$

where x is assumed to be a continuous, positive random variable with probability distribution $f(x)$ and cumulative probability distribution $F(x)$.

These features are retained in the following, except that investment is undertaken in a 'first' period while output is produced in a subsequent 'second' period. This allows a more standard treatment of the RRT threshold as a rate of interest at which negative net cash flows are carried forward over time until they can be set against positive net cash flows. The introduction of a fixed interval between investment and production makes no difference to the results. Later, though, we allow the length of this interval to be a choice variable of the firm.

For a risk-neutral firm, and in the absence of taxation, the first order condition for determining the optimal level of capacity investment is

$$\frac{d\mathrm{E}(\pi)}{dk} = \frac{p}{1+r}[1-F(k)]-c = 0 \qquad (1)$$

where E(π) is the expected level of profit, r is the per period riskless rate of interest, and 1-$F(k)$ is the probability that $x > k$ (and, therefore, that a 'unit' expansion in capacity will result in a 'unit' expansion in output and sales).

Denoting the optimal level of capacity as k*, (1) can be rearranged to give

$$F(k^*) = \frac{p-c(1+r)}{p} \qquad (2)$$

which indicates that k^* is positive for any value of $p > c(1+r)$, increases as p rises relative to $c(1+r)$, and is equal to the median of $f(x)$ when $p = 2c(1+r)$.

(*i*) *Neutrality of'the Pure Rent Tax (PRT)*

The PRT involves the government taking a share of net cash flows, positive or negative, equal to the tax rate, *t*. In our example, this would involve the government contributing the share *t* of the first period investment cost and receiving the share *t* of second period net revenues. Since this is equivalent to the government and the firm entering a joint venture, with shares equal to *t* and 1-*t* respectively, it has no effect on the optimal investment decision of the risk-neutral firm.[3] The first order condition for optimal capacity investment with the PRT in place is

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3 When the investing firm is risk-averse, it has been argued that the PRT will encourage a higher level of investment than would otherwise be chosen since it reduces the firm's exposure to project risk (for example, see Emerson and Lloyd, 1983, and Hinchy, Fisher, and Wallace, 1989). However, as Smith (1997) notes, this advantage of having the government as a joint venture partner is reduced to the extent that the government's

$$\frac{dE(\pi)}{dk} = \frac{p(1-t)}{1+r}[1-F(k)] - c(1-t) = 0 \tag{3}$$

which has the same solution for k as (1). The necessarily neutral character of the PRT makes it a useful yardstick against which to evaluate the Resource Rent Tax.

(ii) On the Neutrality of the Resource Rent Tax

The RRT is also a tax on net cash flows but, instead of the government directly contributing a share of negative net cash flows when they occur, these are accumulated with interest (at the threshold rate) and deducted from future positive net cash flows before calculating tax liabilities. A useful characterisation is that the RRT is equivalent to a PRT where the government borrows from the firm to finance its contribution to initial investment expenditures and then repays the loan from its share of subsequent net revenues. Since this repayment is conditional on the uncertain future revenue flows of the project, the firm requires a premium interest rate to compensate for the possibility of partial or complete default on the 'debt'.[4] Neutrality of the RRT requires that this (threshold) rate be sufficiently high that the firm is prepared to hold the amount of 'debt' implied by the level of investment that would be optimal in the absence of tax, but not so high that the firm wishes to expand its 'debt' holding beyond this (which it could do by investing a greater amount).

This characterisation provides intuition for the earlier discussion of Fraser's results. For a risk-neutral firm the *rate* of tax is not relevant to the neutrality of the RRT because, for a given

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participation simply displaces private sector joint venture arrangements which are a common feature of large mining projects.

4 This characterisation builds on the observation by Fane and Smith (1986) that, if the government *guarantees* to meet its share of initial expenditures, with accumulated interest at some future date, the appropriate threshold rate would be the riskless rate of interest. The RRT would then be equivalent to a PRT in which the government issues the firm with a bond equal in value to its share of the initial investment cost.

level of investment, a higher tax rate increases both the size of the debt that the firm is holding and the expected capacity of the government to repay that debt in the same proportion, so that the expected return per unit of debt is unchanged.[5] Clearly, it is possible that, to induce the firm to hold the amount of debt required for a neutral investment outcome, the threshold rate would need to be so high that the government's share of all possible future net revenues would be wholly exhausted in debt repayment. Then, the RRT is the same as having no tax. At the other extreme, if future net revenues were certain to be large enough to provide full debt repayment, the neutral value of the threshold rate would simply be equal to the riskless rate of interest.

Returning to our example; with the threshold rate denoted as r^*, the firm investing ck in the first period will have a second period deduction of $(1+r^*)ck$. However, it will only get the full tax credit if the second period net revenue is at least as large. Letting x_0 be the minimum realisation of x at which the firm's net revenue is sufficient to use its deduction fully, we have:

$$px_0 = (1+r^*)ck \qquad \text{or} \qquad x_0 = \frac{c}{p}(1+r^*)k \tag{4}$$

where $x_0 < k$ is necessary for the government to have any possibility of collecting tax revenues since, otherwise, the deduction available to the firm will be at least equal to the maximum possible second period net revenue. Thus, we can restrict attention to values of r^* such that $x_0 \leq k$: that is, to cases where $r^* \leq (p\text{-}c)/c$, with $r^* = (p\text{-}c)/c$ being the limit case in which no tax revenue can be raised.

For a marginal increase in investment, expected after-tax marginal revenue depends both on the probability $[1\text{-}F(k)]$ that the stock of the resource exceeds k, so that additional pre-tax revenue

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5 We can also see why the tax rate *does* matter when the firm is risk averse. Then the firm will care about the fact that a higher tax rate requires it to hold a larger amount of risky 'debt' and will require a higher expected return (*via* a higher threshold rate) in compensation.

will be earned, and on the probability [1-$F(x_0)$] that the resource stock is large enough for the firm to be able to make use of the additional tax deduction. Thus, the first order condition for optimal investment in the presence of the RRT is

$$\frac{d\mathrm{E}(\pi)}{dk} \;=\; \frac{p(1-t)}{(1+r)}[1-F(k)]+tc\frac{(1+r^*)}{(1+r)}[1-F(x_0)] \;-c \;=\; 0 \tag{5}$$

The investment incentive effects of the RRT can be identified by comparing this first order condition to that applying with the neutral PRT levied at the same tax rate. Subtracting (3) from (5) gives the difference between the two first order conditions as

$$tc\left[\frac{(1+r^*)}{(1+r)}[1-F(x_0)]-1\right] \tag{6a}$$

which measures the excess of the present value expected tax credit provided by the RRT on a marginal unit of investment over that provided by the PRT. Neutrality of the RRT requires this to be equal to zero at the level of investment that would be chosen in the absence of tax (or under the PRT). Rearranging (6a) gives

$$tc\frac{(r^*-r)}{(1+r)}[1-F(x_0)] \;-\; tcF(x_0) \tag{6b}$$

where the first term is the expected subsidy to marginal investment that the RRT provides because the additional tax credit is inflated by setting $r^* > r$, but is only received with probability [1-$F(x_0)$], while the second term is the expected tax on investment that arises because, with probability $F(x_0)$, the firm will not receive any additional tax credit for its marginal expenditure. If $r^* = r$, the subsidy element is eliminated and the RRT must have a net taxing effect on investment whenever $F(x_0) > 0$. Hence, Fraser's claimed result that it may be impossible to avoid the net investment enhancing effect of the RRT without (in his notation) setting a negative value of the threshold rate (equivalent to setting $r^* < r$) is clearly incorrect.

The subsidising and taxing effects on marginal investment will exactly cancel out when

$$\frac{(r^*-r)}{(1+r^*)} \;=\; F(x_0) \tag{7}$$

so neutrality of the RRT requires this condition to be met when $k = k^*$. Substituting the earlier definition of x_0, we can alternatively write the requirement as

$$\frac{(r^* - r)}{(1 + r^*)} \; = \; F\left(\frac{c}{p}(1 + r^*)k^*\right) \tag{8}$$

As already noted, setting $r^* = r$ results in a net taxing effect whenever $F(x_0) > 0$ while, with $r^* = (p\text{-}c)/c$, the RRT will be neutral by virtue of the fact that the firm can never have any tax liability. The question, then, is whether there exists some intermediate value of r^* that satisfies (8) and provides the government with positive expected tax revenue.

To proceed further requires examination of particular examples. In the following the simplest possible case, in which $f(x)$ is a uniform distribution over a defined range, is used for illustrative purposes.

(iii) The Uniform Distribution Example

Let x be uniformly distributed in the 0-1 interval [with the consequence that $F(x) = x$] and, for notational convenience, define $g = \dfrac{1 + r^*}{1 + r}$ and $q = \dfrac{p}{c(1 + r)}$ [the ‘profitability’ ratio]. Then the condition for optimal investment in the absence of tax, given by (2), reduces to

$$k^* \; = \; F(k^*) \; = \; \frac{q - 1}{q} \qquad \text{s.t. } k^* \geq 0 \tag{9}$$

while the neutrality condition, (8), becomes

$$\frac{\gamma - 1}{\gamma} = \frac{\gamma k^*}{q} \tag{10}$$

Then, substituting (9) into (10) and rearranging yields

$$\frac{g - 1}{g^2} \; = \; \frac{q - 1}{q^2} \tag{11}$$

which has two solutions for γ: namely, $\gamma = q$ and $\gamma = q/(q\text{-}1)$.

When $\gamma = q$, $r^* = (p-c)/c$ and the RRT cannot raise any revenue. At values of $q \leq 2$, $\gamma = q$ is the smaller of the two possible solutions for γ, so the alternative would yield a higher value of r^* and an even larger deduction for the firm. Thus, for values of $q \leq 2$, the neutral RRT exists but cannot raise any tax revenue.

With $q > 2$, however, $q/(q-1) < q$, so the second solution for γ provides a lower neutral value of r^* that does not eliminate all possible tax revenue. Our future interest will, therefore, be confined to cases in which $q > 2$ and where, with

$$1 + r^* \;=\; g\,(1+r) \;=\; \frac{q}{q-1}(1+r) \qquad \text{and hence} \qquad r^* \;=\; \frac{1+qr}{q-1} \tag{12}$$

the RRT has both a neutral effect on investment and generates positive expected tax revenue. As the profitability ratio, q, becomes large, γ tends to unity and the neutral value of r^* tends to equality with the riskless interest rate, r.

This simple example illustrates the proposition that the RRT threshold rate can be set so as to yield a neutral effect on a single, specific investment but that below a certain level of 'profitability' this RRT cannot raise any tax revenue. In order to apply the neutral RRT intentionally, the government would need full knowledge of the particular investment decision problem facing the firm. In contrast, the Pure Rent Tax could be applied uniformly across all investments and would be neutral in all cases.

Moreover, the neutral RRT must always generate lower expected tax revenue than a PRT levied at the same tax rate. This is because setting r^* high enough to avoid deterring the unit of investment that would be marginal in the absence of tax requires the deduction provided for intramarginal units of investment to be overly generous.[6] This difference between PRT and

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[6] The actual difference between expected tax revenue under the two taxes, for $q \geq 2$ in the uniform distribution example, is given by equation (A8) of Appendix 2. However, the general proposition that the RRT must yield less revenue can be demonstrated by reference to

RRT expected tax revenues is most extreme for projects whose profitability is sufficiently low that the neutral RRT cannot raise any revenue at all. As project profitability increases, the required excess of the threshold rate over the riskless rate of interest falls and the extent of expected revenue sacrifice relative to the PRT diminishes.

More important for the purpose of this paper, the neutrality of the RRT has been illustrated in an example in which the firm has only one decision to make: namely, *given the existing information* how much should it invest *now* in development of a resource that will be extracted *next period.* In reality, a firm holding mineral rights would need to decide how much to spend on collection of additional information (exploration) before committing to mine development, with decisions on the scale and timing of capacity investment and production being conditional on the information obtained. The following sections examine the effects of allowing for this multidimensional, and conditional, nature of the firm's investment decisions using very simple extensions of the model and example applied above.

III 'Exploration' Prior to Investment

Suppose that, in the first period, the firm can obtain additional information about the distribution $f(x)$ before deciding the level of investment. Specifically, with expenditure of amount C, the firm will learn whether x is uniformly distributed in the 0-½ interval (the 'low' state) or in the ½-1 interval (the 'high' state). *Ex ante,* these states are equally likely.

If the 'low' state is revealed, then $F(x) = 2x$ so the solution to (9) gives the optimal value of k:

$$k^{\mathrm{L}} = \tfrac{1}{2}F(k^{\mathrm{L}}) = \tfrac{1}{2}\left(\frac{q-1}{q}\right) = \tfrac{1}{2}k^* \qquad (13)$$

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equation (6a). If r^* is set so that this expression has a zero value for $k = k^*$, as required for neutrality, it must have a positive value for all $k < k^*$ {since x_0 increases with k [see (4)] and $F(x_0)$ increases with x_0} and increasingly so as k falls below k^*. That is, on all *intramarginal* units of investment the neutral RRT provides a greater expected tax credit than the PRT and must therefore raise less expected revenue.

where k^* again represents the optimal value of k in the absence of the additional information.

If the 'high' state is revealed, then $F(x) = 2x\text{-}1$ and (9) yields

$$k^{\mathrm{H}} = \tfrac{1}{2}\left[1+F(k^{\mathrm{H}})\right] = \tfrac{1}{2}\left(1+\frac{q-1}{q}\right) = \tfrac{1}{2}(1+k^*) \tag{14}$$

Clearly, and as shown in Appendix 2, the opportunity to make a more informed decision about the level of investment increases the *ex ante* value of the firm's expected profit. In the absence of taxes, the firm will wish to acquire the additional information so long as the cost, C, is not greater than the resulting gross increase in expected profit.

The questions that now arise are: what will be the effect of the RRT on the incentive to obtain the additional information and, if it is obtained, what then will be the incentive effects on capacity investment in the alternative states? Consideration of the first question is postponed by assuming to start with that the information is freely available.

Given the additional information, the value of r^* that would be neutral with respect to capacity investment depends on which state is realised. There is no threshold rate that will provide neutrality in both states. In this particular example, the threshold rate that would have been neutral in the absence of the information will continue to be neutral in the 'low' state, but any value of $r^* > r$ will provide incentives to over-investment in the 'high' state.

In the 'low' state $F(x) = 2x$ and $k^{\mathrm{L}} = ½k^*$. Referring back to equation (8), a doubling of $F(\bullet)$ while halving the optimal value of k leaves the RHS unchanged for given values of the other parameters, which means that the solution for r^* will be unchanged.

In the 'high' state, on the other hand, the firm will have guaranteed output equal to ½ and, for any value of $q \geq 1$, this will yield guaranteed present value net revenue that is at least as great as the cost of the investment k^{H} associated with that value of q. The consequence is that the firm is guaranteed to be able to utilise its full tax deduction with $r^* = r$, so $F(x_0) = 0$ and the solution to (7) <u>is</u> $r^* = r$.

Proof: The firm's guaranteed net revenue is ½p while the tax deduction available, with $r^* = r$, is $ck^{\mathrm{H}}(1+r)$. Dividing both of these by $c(1+r)$ and substituting in the value of k^{H} from (14) gives the excess of guaranteed revenue over the tax deduction, expressed in present value terms and per unit of c, as

$$\mathrm{X} = \tfrac{1}{2}q - [1-\tfrac{1}{2}(1/q)]$$

With $q = 1$, X = 0. With $q > 1$, $d\mathrm{X}/dq = \tfrac{1}{2} - \tfrac{1}{2}(1/q^2) > 0$.

Thus, if we imagine that the RRT would have been neutral if the additional information had not been available, it will still be neutral in the 'low' state but will encourage over-investment by the firm in the 'high' state. Table 1 shows the levels of investment that would be undertaken in the 'high' state at different values of q and t, with r^* as given by equation (12). For each value of q, the first row of the Table indicates the level of investment in the absence of tax. As expected, the amount of over-investment encouraged increases with the tax rate. With the qualification discussed below, the effect decreases with larger values of q because the excess of the threshold rate over the riskless rate of interest declines as q increases.

When the tax rate rises to the point at which it is just worthwhile to set $k^{\mathrm{H}} = 1$, the firm is obtaining an expected tax credit from marginal investment that is exactly equal to the marginal investment cost. If, as is the case for the three higher values of q shown in Table 1, the firm's guaranteed revenues still exceed its total deductions when this point is reached, then it will be indifferent between stopping at $k^{\mathrm{H}} = 1$ and increasing its expenditure until the deductions exhaust its guaranteed revenue. Thus, with q = 3.33 and t = 0.7, the firm will be indifferent between values of k^{H} in the range 1~1.17 while, with $q = 10$ and $t = 0.9$, the firm will be indifferent between values of k^{H} in the range 1~4.5 (corresponding values for $q = 5$ are 1~2 at $t = 0.8$). Exactly at the tax rate which generates this point of indifference, the firm might as well (but also might as well not) set $k^H = 1$ and pay more tax rather than less. However, once the tax rate rises slightly above this, the firm will want to set k^H above the upper end of the range. The range is larger the greater is q because the amount of guaranteed revenue increases with q. Thus, once the tax rate is sufficiently high that <u>totally</u> wasteful investment becomes worthwhile, the amount of this will be greater the more 'profitable' is the project

Table 1: Levels of k^H with $1+r^* = q(1+r)/(q-1)$

	'Profitability'			
Tax Rate, *t*	**2.5**	**3.33**	**5**	**10**
0	0.800	0.850	0.900	0.950
0.1	0.812	0.857	0.903	0.951
0.3	0.840	0.878	0.911	0.952
0.5	0.877	0.914	0.925	0.956
0.7	0.927	**1.0~1.17**	0.958	0.963
0.9	1.000	1.426	2.222	**1.0~4.5**

The above illustrates the rather obvious point that RRT parameters that would be neutral when investment is undertaken on the basis of one set of information will no longer be neutral if the information set changes. The importance of this is that a major part of the business of mining firms involves improving the information about the size and nature of mineral deposits before investing in the mining of those deposits. We turn, now, to the question of the effects of the RRT on incentives to acquire such information.

Let the cost of obtaining the information, C, be equal to the resulting increase in gross expected pre-tax profit so that, in the absence of tax, the firm would be indifferent about acquiring the information. Suppose, also, that r^* is set so that the RRT would be neutral if the information were not obtained. Initially, we ignore the incentive effects already discussed by assuming that, if the information is obtained, the firm will choose the values of k^{H} and k^{L} that would be optimal in the absence of tax.

Given these conditions, for all $t > 0$ and for all $q > 2$, the increase in the firm's gross expected *after-tax* profit will exceed the exploration cost. That is, in all cases in which the firm has any expected tax liability, the RRT provides it with a positive incentive to undertake an exploration investment that would be marginal in the absence of tax. The formal analysis is given in Appendix 2 but the result, of necessity, is because the firm has lower expected tax payments if it conducts the exploration than if it does not. This, in turn, is because the additional deduction provided by the exploration expenditure, and accumulated at a threshold rate that is overly generous when the firm finds itself in the 'high' state, eliminates more expected tax liability than is created by the addition to its expected income.

The 'subsidy' to exploration will be greater the higher is the tax rate. It will also be greater the lower is q, since this increases the value of r^*. The qualification to this is that, as q becomes close to 2, the 'subsidy' diminishes sharply because the amount of expected tax reduction is constrained by the small expected tax liability of the firm in the absence of the exploration.

For values of $q \geq 3$, equation (A12) of Appendix 2 gives the increase in gross expected after-tax profit, expressed as a proportion of the exploration cost, as

$$1 + t\left(\frac{q}{(q-1)^2} - \frac{\frac{1}{8}}{q-1}\right) > 1$$

where the second, 'subsidy rate', term is linear in the tax rate and positive but tending to zero as q becomes very large.[7] For example, with $t = 0.5$, the 'subsidy rate' is 34.4 per cent when $q = 3$, declining to 14.1 per cent when $q = 5$. More useful than thinking about the rate of

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[7] For values of q in the range $2 < q < 3$, the 'subsidy rate' is lower than this expression would suggest, increasingly so as q decreases towards a value of 2 where the subsidy rate is zero.

subsidy on a *fixed* exploration cost, though, is to consider the maximum expenditure that it would be worth incurring to obtain the information under the RRT, relative to the maximum worthwhile in the absence of tax. Table 2 shows values of this ratio for a range of values of q and t, indicating, for example, that with $t = 0.75$ and $q = 2.5$, the maximum exploration cost that would be worthwhile under the RRT would be twice as great as in the absence of tax.

Table 2: Incentive for Over-Exploration

	'Profitability', *q*			
Tax Rate, *t*	**2.5**	**3.33**	**5**	**10**
0	1.00	1.00	1.00	1.00
0.25	1.25	1.19	1.10	1.04
0.5	1.61	1.60	1.30	1.10
0.75	2.08	2.65	1.94	1.34

The incentive to over-exploration is somewhat larger than indicated above when account is taken of the effects of the RRT on the subsequent capacity investment decision. So far, the firm has been constrained to choose the same k^H or k^L as would be chosen in the absence of tax, but this will not be expected profit maximising. In the 'high' state there will still be the incentives to over-invest discussed previously and, now, the RRT will not have a neutral effect on the investment decision in the 'low' state either. This is because the existence of a tax deduction for the exploration expenditure reduces the probability of obtaining a deduction for any marginal unit of capacity investment expenditure, so the firm will set k^L lower than in the absence of tax. The corresponding effect in the 'high' state will be that the incentive to over-invest is slightly moderated. Since adjustment of the levels of k^H and k^L in response to these effects increases the firm's expected after-tax profit, allowing for them also increases the firm's incentive to undertake the exploration.

It should be emphasised that the preceding discussion should not be interpreted to mean that the RRT will *generally* cause firms to undertake exploration activities that are not worthwhile. The particular kind of exploration here considered is that which improves knowledge of the deposit prior to a mining investment which is *already* expected to earn positive profits. This 'development' exploration is quite distinct from 'indicative' exploration whose purpose is to discover whether a potentially profitable mining opportunity exists at all.[8]

The conclusion of this section is that, once we allow the possibility of the firm engaging in 'development' exploration to improve information about the deposit before committing to mine development, there can be no neutral value of the RRT threshold rate. This is not only because any given threshold rate can, at best, be neutral with respect to only one of the possible states that additional information might reveal, but also because the threshold rate that would be neutral in the absence of additional information provides incentives for over-investment in this form of exploration.

IV. The Timing of Minerals Extraction

Returning to the original model, with the first period capacity investment being based on the information that $f(x)$ is a uniform distribution in the 0-1 interval, we now allow the firm to defer production and earning of net revenue beyond the second period if it wishes. The net price of output is assumed constant over time so, with $r > 0$, it would never be worthwhile to delay production in the absence of tax. The question is whether this continues to be true in the presence of the RRT?

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8 An example of 'indicative' exploration would be where, with probability z, a deposit of some description is revealed to exist and, with probability 1-z, no deposit is revealed. If *this* exploration project is marginal in the absence of tax, it will be deterred by the RRT whenever the threshold rate is set low enough to allow any possibility of tax payments arising.

In those cases in which the firm can ever have any tax liability ($q > 2$) the advantage of delay is that accumulation of the deduction provided by its first period expenditure, at a threshold rate greater than the riskless rate of interest, reduces the present value of expected tax payments. The disadvantage is that the present value of the expected after-tax revenue that it would obtain from immediate production is also reduced. If the former effect is larger than the latter, delay will be worthwhile.

It is natural to imagine that investment in mine capacity might provide the firm with some information about the actual size of x without requiring production to take place. We consider the extreme possibilities, first assuming that the firm learns the size of x exactly and, later, that it gains no additional information about the distribution of x.

Suppose that the firm invests the amount k^* (as previously defined) in the first period and learns the exact value of x. Let z denote the output (minimum of x and k^*) that can then be produced. This production can occur in the second period or it can be deferred for some length of time. With deferral of n periods, after-tax net revenue at the date of production will be

$$R_n = pz(1-t) + tck^*(1+r^*)^{n+1} \qquad \text{s.t. } R_n \leq pz$$

Thus, after-tax net revenue will be increased by deferral up to the point at which the accumulated deduction eliminates all tax liability.

Taking account of the interest cost of postponing the earning of net revenues, the net benefit of deferring production for the nth period, given that it has already been deferred for n-1 periods and that the firm's tax liability has not yet been eliminated, is

$$\begin{aligned} R_n - R_{n-1}(1+r) &= tck^*(1+r^*)^{n+1} - (1+r)tck^*(1+r^*)^n - rpz(1-t) \\ &= (r^*-r)tck^*(1+r^*)^n - rpz(1-t) \qquad (15) \end{aligned}$$

This expression applies to any case in which a firm subject to RRT has incurred deductible expenditure (ck^*) and has certain net revenue (pz) that exceeds the current accumulated value of its deductions. The important point to note is that the positive first term increases with n

while the negative second term is constant. Consequently, if deferral of production is ever worthwhile, it will continue until the whole tax liability has been eliminated. This increasing marginal net benefit of deferral means that it is not straightforward to specify the conditions under which the firm will want to defer production since negative marginal net benefits from initial deferral may be more than offset by subsequent positive marginal net benefits.

In the uniform distribution case, (15) can be written as

$$R_n - R_{n-1}(1+r) = \left(\frac{t}{q}(1+r^*)^n - rqz(1-t)\right)c(1+r) \tag{16}$$

For any n, the incentive to defer production for an additional period is more likely to be positive the greater is t and the smaller are z and q - that is, the greater is the additional tax credit to be obtained and the smaller are the after-tax profits being deferred. A lower value of q also means that r^* will be greater, which further increases the likelihood that the incentive to defer will be positive. Other things being equal, the incentive to defer is more likely to be negative the higher is r, because this increases the present value cost of deferring income while reducing the present value benefit of the additional tax credit.

For illustration, Table 3 shows the maximum values of r at which it would be worth engaging in tax eliminating deferral of production for varying values of t and q in the uniform distribution case. For these calculations, the value of z was set at its maximum possible level (= k^*) so, in that respect, these are 'worst case' scenarios for deferral.

It should be noted that r is the interest rate applying over the interval between the 'first' and 'second' periods (between investment and earliest possible production) so the values shown in Table 3 can only be translated to annual interest rates if some assumption is made about the length of that interval. If it was 1 year and the real interest rate was 3 per cent per annurn, the RRT would cause the firm to defer production until all tax liability had been eliminated in every case shown in Table 3 except for the combination of highest value of q and lowest tax rate. On the other hand, if the interval was 5 years and the real interest rate was 5 per cent per annum (giving a compounded interest rate of 27.7 per cent), tax eliminating deferral would be worthwhile only with the highest value of the tax rate and only for the three lowest values of q.

Table 3: Maximum Values of *r* at which Tax Eliminating Deferral of Production is Worthwhile with Output = *k**

	'Profitability', *q*			
Tax Rate, *t*	**2.5**	**3.33**	**5**	**10**
0.25	0.090	0.080	0.038	0.013
0.5	0.200	0.224	0.118	0.038
0.75	0.333	0.750	0.317	0.105

Now suppose that, after making the investment, the firm still only knows that x is uniformly distributed in the 0-1 interval. The major difference is that deferral now decreases the firm's tax payments with a probability that is always less than 1 and that declines as the already accumulated deduction becomes larger. The reasonable conjecture is that, for given values of q and t, the maximum interest rate at which deferral will be worthwhile will be lower than shown in Table 3 and that, if production is deferred, this is less likely to result in the elimination of all possible tax liability.

For the uniform distribution example, equation (A7) of Appendix 2 gives the present value (in period 1) of expected after-tax profit from production in period two as

$$E(\pi_T^*) = c\{q[(1-t)(k^*-½k^{*2}) + t(x_0-½x_0^2)] - k^*\}$$

where x_0 is the level of output below which the firm pays no tax. Dropping out the cost of the investment, ck^*, and multiplying by $(1+r)$ gives the period 2 value of (undeferred) expected after-tax net revenue as

$$E(R_0) = p[(1-t)(k^*-½k^{*2}) + t(x_0-½x_0^2)] \qquad (17a)$$

After n periods of deferral, the tax-free level of output would have increased to $x_n = x_0(1+r^*)^n$, so long as $x_n \leq k^*$, and the expression for expected net revenue would be

$$E(R_n) = p[(1\text{-}t)(k^*\text{-}½k^{*2}) + t(x_n\text{-}½x_n^{\,2})] \quad (17b)$$

The first term inside the bracket is independent of *n*, so the interest charge on deferring earning this amount for an additional period is constant. The critical matter is what happens, as *n* increases, to the increment in the second term (the expected tax credit) net of the interest charge on the existing tax credit. This is given by

$$pt[(x_n\text{-}½x_n^{\,2}) - (1+r)(x_{n\text{-}1}\text{-}½x_{n\text{-}1}^{\,2})] \quad (18)$$

For given *p* and *t* and with $r = 0$, (18) is positive with a greater value the smaller is *q* and increases with *n* up to a value of $x_n = ½$ and then decreases with *n*. As *r* increases above zero, the maximum value of (18) is both reduced and reached at lower values of x_n. These effects are greater the higher is *q*. With high enough values of *r* and/or *q* (outside the range shown in Table 3) there would be no value of *n* at which (18) was positive.

These properties again suggest that a higher value of *q* will require a lower value of *r* in order to make deferral worthwhile, and that the marginal net benefit of deferral will be increasing over some initial range. However, since the latter effect now reverses at values of $x_n \leq ½$ while the tax eliminating value of x_n is greater than ½ for all values of *q* (> 2) where there is any possible tax liability, the previous result that deferral eliminates all tax liability need no longer hold. In particular, this appears less likely the greater is *q*, since higher values of x_n are required to eliminate the tax liability as *q* increases.

In fact, though, numerical analysis indicates a range of cases where deferral does eliminate all tax liability and, even when it doesn't, that the outcome may not be very different. For a variety of values of *q*, *t*, and *r*, Table 4 shows the present value of expected tax receipts, given optimal deferral by the firm, as a percentage of expected tax receipts in the absence of deferral. Almost all of the values are either zero (indicating complete elimination of tax liability) or 100 (indicating no deferral). In the cases where deferral occurs but does not completely eliminate tax liability, a large proportion of expected government revenue is lost. To illustrate, consider the case of q = 5 and t = 0.75. All tax liability is eliminated with r = 0.03. As r is raised to 0.10 and then to 0.20, 1.34 per cent and then 17.9 per cent of present value expected tax revenue is retained. The interest rate would need to be raised to 0.25 to prevent any deferral but, just before that point, the government would still be losing over 65 per cent of its present value expected revenue. The explanation lies in the initially increasing marginal benefit of deferral. This means that the firm jumps to multi-period deferral as soon as deferral becomes marginally worthwhile, significantly reducing both the amount of tax expected to be paid and the present value of those payments.

Table 4: Present Value of Deferred Expected Tax Revenue as Percentage of Expected Tax Revenue Without Deferrment

Tax Rate, *t*	**Interest Rate, *r***	**'Profitability', *q***			
		2.5	**3.33**	**5**	**10**
0.25	**0.20**	100	100	100	100
	0.10	0.008	100	100	100
	0.03	0	0	10.5	100
0.5	**0.20**	0	100	100	100
	0.10	0	0	100	100
	0.03	0	0	0	6.16
0.75	**0.20**	0	0	17.9	100
	0.10	0	0	1.34	100
	0.03	0	0	0	1.54

Comparison of Table 4 with Table 3 confirms the general proposition that a lower interest rate is required to induce deferral when the firm gains no additional information about the size of the deposit. However, with r = 0.03, both Tables show that deferral will take place, eliminating all revenue in Table 3 and almost all revenue in Table 4, in all cases except for the combination of the highest value of q with the lowest tax rate.

The results of this section suggest that, as soon as we allow the actual timing of production to be determined by the firm in light of the RRT parameters imposed, there is potentially significant scope for increasing expected after-tax profits at the expense of the government's revenue by delaying production. While this result is stronger when, after its initial investment, the firm gains knowledge about the size of the deposit and, therefore, about the value of the additional tax credits it will obtain, it still applies quite generally even when the firm gains no such additional information.

It should also be noted that, once it is worthwhile for the firm to defer production, the RRT threshold rate that would have had a neutral effect on the level of investment in the absence of deferral will no longer do so. The opportunity to extend the period of accumulation effectively increases the value of r^* above that required for investment neutrality, so the firm also has incentives to expand the level of' investment above the otherwise optimal level, k^*.

V. Timing of the Mining Investment

The example so far has been that of a mining investment which is to take place ‘now’, ignoring the issue of how the date that ‘now’ represents would be chosen by the firm and, consequently, what the effect of the RRT on that choice might be. The final modification of the baseline model introduces these considerations explicitly.

If all of the variables in the basic model remained constant over time, the optimal date at which to conduct the investment in the absence of tax, and with $r > 0$, would be ‘as early as possible’, but this is not very helpful. Given its expectations about the future values of key variables, the firm will seek to maximise the present value of expected profit through its choice of investment date. For illustrative purposes, we assume constant information about the deposit, that c is constant and equal to 1, but that the value of q increases over time at a perfectly anticipated, constant proportional rate α, starting from q = 1 at some arbitrarily defined date, $n = 0$.

In the absence of tax, if $E(\pi)$ is the firm's expected profit from investment at time n, it will be optimal to postpone the investment so long as

$$\frac{dE(p)}{dn} \Big/ E(p) > r \qquad \text{where} \qquad \frac{dE(p)}{dn} = \frac{dE(p)}{dq} \cdot \frac{dq}{dn} = \frac{dE(p)}{dq} aq$$

so the first order condition for determining the optimal investment date is

$$\frac{dE(p)}{E(p)} \cdot \frac{q}{dq} = \frac{r}{a} \tag{19}$$

Because expected profit increases more rapidly than q, but decreasingly so as q becomes larger,[9] the value of the 'elasticity' expression on the LHS of (19) decreases with q but is always greater than 1, so a solution requires $\alpha < r$.[10] The value of this 'elasticity' in the uniform distribution case is $(q+1)/(q-1)$. Thus, for example, with $r/\alpha = 2$ the optimal date at which to conduct the investment, n^*, would be that at which $q = 3$.

To examine the effect of the RRT on the choice of investment date requires some specification of the threshold rate. The simplest experiment is to assume that r^* is continuously adjusted so that, at any point in time, the RRT has a neutral effect on the level of investment, given that the investment is undertaken at that time. That is, r^* is continuously adjusted to satisfy equation (12). Recalling that after-tax expected profit with the 'neutral' RRT is larger than with a PRT at the same tax rate, but by a decreasing amount the higher is q, and noting that after-tax profit with the PRT will increase over time at the same rate as pre-tax profit, it follows that after-tax

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9 With a *fixed* level of investment, and hence fixed investment cost and expected level of output, expected net revenue would increase at the same rate as q and expected profit would increase at a faster rate, but decreasingly so as the gap between profit and net revenue became smaller. Allowing the level of investment to increase with q does not alter this basic result.

10 $\alpha < \mathrm{r}$ also ensures that, in the absence of tax, the firm will wish to produce at the earliest possible date after the investment is undertaken. As previously seen, the RRT may provide incentives for the firm to defer production but, for the purposes of this section, it is constrained not to do so.

profit with the 'neutral' RRT must always be rising at a slower rate than pre-tax profit as q increases. This means that the value of q at which (19) is satisfied will be lower with the RRT in place than in the absence of tax and, therefore, that the RRT will cause the investment to be conducted at an earlier date than is optimal.

A more reasonable experiment, though, is to suppose that r^* is fixed at the level that would generate investment neutrality <u>if the investment occurred at the optimal date n^*</u>. This would mean that, at earlier dates and with lower values of q, the RRT would be less generous than in the previous experiment, and increasingly so the greater the distance from n^*. Consequently, the rate of increase in after-tax expected profit would be higher at all dates prior to n^*, with the effect that the incentive to early investment would be reduced. It would not be eliminated, however, since the rate of increase in after-tax expected profit <u>at n^*</u> would remain the same as in the previous experiment (*ie*. less than r).

The ultimate limitation on the incentive to early investment is that, before some date $\underline{n}$ the value of q will be sufficiently low that the firm cannot have any tax liability. One possibility is that the firm's optimal strategy is to invest exactly at that date, with the result that the RRT generates no revenue.

Table 5 illustrates these effects for the uniform distribution example and for the second experiment described above. It should be noted that, with $r/\alpha \geq 3$, the value of q at the optimal date n^* will be $q \leq 2$ so that the RRT which would be neutral with respect to investment at that date will raise no revenue and, thus, cannot distort the timing of investment. For values of r/α equal to 2.5, 2, and 1.5, Table 5 shows the value of q at which investment would be undertaken with different tax rates. E(T)% gives the present value of expected tax payments, evaluated at $n = 0$, as a proportion of those that would be received if the timing of investment were not affected by the RRT.

Table 5: Effect of RRT on Timing of Investment and Expected Tax Revenue				
		Ratio of Interest Rate to Rate of Increase in ***q***, ***r***/α (***r*** = 0.05 with variable α)		
Tax Rate, *t*		**2.5**	**2**	**1.5**
0	***q***	2.3	3	5
	E(T)%	100.0	100.0	100.0
0.25	***q***	2.1	2.9	4.8
	E(T)%	85.1	97.9	99.0
0.5	***q***	1.7	2.6	4.3
	E(T)%	0.0	92.3	96.2
0.75	***q***	1.7	1.5	3.4
	E(T)%	0.0	0.5	82.7

As expected, the distorting effects of the RRT are greater the higher is the tax rate. They are also greater the smaller is α relative to the interest rate and, at least for the tax rates shown, are relatively insignificant when investment would optimally take place at a high value of q. This accords with earlier results in which the distorting effects of the RRT are smaller the more 'profitable' is the investment to be undertaken. Notably, though, with $r/\alpha = 2.5$ and $t \geq 0.5$, the incentive is for the firm to invest exactly at the date when it is about to incur some expected tax liability, and the same effect occurs for $r/\alpha = 2$ and $t \geq 0.75$. Thus, there are again instances in which the RRT provides incentives for the firm to act so as to eliminate the possibility of paying any tax.

VI. Conclusions

The preceding sections have shown that the RRT that would have a neutral effect on the level of mining investment in the absence of opportunities to obtain additional information about the deposit to be exploited, to vary the timing of investment, or to vary the rate of extraction of minerals will not be neutral once these opportunities are available. The 'neutral' RRT will encourage excessive investment in information about the deposit, premature investment in mining, and a slowing of the rate of extraction

Although the analysis has been confined to the case of a risk-neutral investor and has used the simplest possible example for illustrative purposes, it should be clear that the qualitative nature of the results is perfectly general. The sorts of incentive effects of the RRT that have been described will apply with any distribution of x and regardless of the investor's attitude to risk.

None of this is necessarily to suggest that the RRT is not a superior means of collecting government revenues from mining activity than more traditional forms of royalty. Rather, the comparison between alternatives should not be based on any 'near neutrality' presumption about the RRT. The alternative taxing mechanisms need to be evaluated over all of the dimensions of decision making in which they may have distorting effects.

As all of the examples in this paper indicate, the distorting effects of the RRT are greater the higher is the threshold rate and the higher the tax rate. For highly profitable mining operations, the 'one-dimensionally neutral' RRT imposed at a moderate rate of tax has relatively small impacts on other aspects of decision making since the excess of the required threshold rate over the riskless rate of interest is not large. However, except when substantial exogenous changes in world market conditions or in information about prospectivity occur, the mining project that is highly profitable *ex ante* is a rare beast. Rather, highly profitable ventures are the occasional products of exploration activity whose *ex ante* expected return is relatively modest. To make the RRT approximately neutral with respect to such indicative exploration would require a high threshold rate and, to allow the government to capture a significant share of the rents from successful exploration outcomes, a relatively high tax rate. But, as the preceding analysis shows, this is exactly the combination likely to stimulate subsequent excessive investments.[11]

On the other hand, general application of the RRT with a modest threshold rate and tax rate may, despite its deterrent effects on initial exploration activity, be less distorting and/or raise more expected tax revenue than the kinds of royalty arrangements more commonly employed. Fraser and Kingwell (1997) use the model outlined in Section II to compare expected revenue from the RRT with that from an *ad valorem* royalty when the RRT parameters are set to give the same deterrent effect on investment as the royalty. There is a continuum of t,r^* pairs that will generate a given marginal investment distortion, since the greater deterrent effects of a higher tax rate can be offset by a higher threshold rate. Fraser and Kingwell find, in general, that the 'investment preserving' RRT will raise greater expected revenue than the royalty and will do so by a greater amount the higher the values of t and r^*. The obvious qualification suggested by this paper is that the distorting effects of the two mechanisms, and their implications for expected tax revenue, need to be evaluated over a wider range of dimensions of the firm's decision making than the simple baseline model provides. In particular, the suggestion that the RRT will raise greater expected revenue the higher the values of t and r^* needs to be treated with a great deal of caution once we allow for the kinds of effects analysed in Sections III-V.

Two final issues deserve brief mention. If it *were* desired to impose an entirely neutral form of tax on mining activity, the Pure Rent Tax offers that possibility. Because the PRT simply has the effect of reducing the firm's equity participation in proportion to the tax rate, it will be neutral with respect to all dimensions of the firm's decision making. Moreover, even in the

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[11] Recognition of this problem lay behind the 1990 amendments to the RRT that applies to offshore petroleum in Australia, where the threshold rate applying to mine development expenditure was reduced below that applying to exploration expenditure.

highly restricted cases in which the RRT could be neutral, the PRT will raise the same expected revenue while being levied at a lower tax rate so that, if there are incentives for the firm to avoid tax in ways not canvassed in this paper, they will be smaller under the PRT. The reasons for government preference for the RRT thus require explanation. Some discussion of that question is included in Smith (1997).

The second issue is that attention to achieving (near) neutrality of the mechanism by which governments raise revenue from mining activity is appropriate if, in the absence of tax, firms would be expected to make decisions that maximised the value of the resources they exploit. That would be a reasonable presumption if mineral rights were privately owned and firms were free to choose the timing and extent of activity. In fact, however, mineral rights are state-owned in Australia (and in many other countries) and mining companies gain access to resources through leasing arrangements that provide incentives for distorted patterns of exploration and mining activity independently of the associated mineral taxes.[12] Neutrality of the latter is, under these conditions, unlikely to be optimal. Rather, the appropriate focus should be on improving the incentive effects of the leasing arrangements taken in their totality.

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12 For discussion of the distortions associated with leasing arrangements and possible ways of improving those arrangements see Fane and Smith (1986), Industries Assistance Commission (1991), and Smith (1997).

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Appendix 1: Cash Bidding and the RRT with Risk-Averse Investors

Consider a single investment decision where the investor is risk-averse. As indicated in the Introduction, if there is any set of RRT parameters that would be neutral and expected revenue raising, involving a value of $t < 1$, then for every higher value of t (up to $t = 1$) there will be an associated neutral value of r^* which, for the reason given in footnote 5, increases as t increases. However, the increase in r^* will be less than would be needed to compensate for the effect of the higher tax rate on expected revenue, so expected tax revenue increases as we move to successively higher t,r^* pairs. This is because a greater concentration of revenue collection on the most favourable possible pre-tax outcomes reduces the variance in the firm's after-tax expected profits. The risk-averse firm will be prepared to exchange some reduction in expected profit for that reduction in variance, while keeping the level of investment constant.

If the RRT were combined with allocation of the lease by competitive cash bidding, the firm would be prepared to bid the certainty equivalent of its expected after-tax profits in order to obtain the lease. Since after-tax expected profit decreases with higher t,r^* pairs, the bid amount will also decrease. Fraser's (1998) numerical results suggest there may be circumstances where the sum of the bid plus expected tax revenue (the government's total expected receipts) may decrease. The following shows that this cannot be correct.

Suppose we start from a neutral t,r^* pair at which expected tax revenue is E(T) and where the firm would be prepared to bid the amount B to acquire the lease. If we now move to a higher neutral t,r^* pair such that expected tax revenue rises to E(T)+$1, Fraser's result suggests that the firm may now be prepared bid less than B-$1. In fact, risk aversion requires the firm always to bid more than B-$1.

As indicated above (and confirmed by Fraser's analysis) the increase in the t,r^* pair reduces the variance of the firm's after-tax expected profit. Let this effect be denoted $\Delta\sigma^2$. Now think of the movement from the lower to the higher t,r^* pair as involving two steps. In step 1, with the firm constrained to keep investment constant, the values of *t* and *r** are adjusted so as to leave expected tax revenue unchanged at E(T), but such that the variance of after-tax profit is reduced by $\Delta\sigma^2$. In step 2 we move to the higher of the two neutral *t*,*r** pairs, so this second move involves keeping the variance unchanged but increasing expected tax revenue to E(T)+$1. After step 1, the firm faces the same expected after-tax profit but with lower variance. Since it is risk-averse, the certainty equivalent of the expected profit must have increased so it will now bid more than B to obtain the lease. Step 2 then takes away $1 in expected after-tax profit, leaving the variance unchanged. The certainty equivalent of this is $1, so the firm will reduce its bid by $1. The combination of these two effects must be that the bid has decreased by less than $1 and, consequently, that the government's total expected receipts have increased.

Fraser’s (1998) discussion of combining up-front cash bidding with the RRT largely assumes the government (or, more accurately, the community it represents) to be risk-neutral. Otherwise, the government would want to strike a balance between the increased revenue uncertainty of choosing a higher neutral *t*,*r** pair and the increase in expected revenue provided (as discussed, for example, in Garnaut and Emerson, 1984). Fraser’s concluding remarks give partial recognition to that possibility, but there is no recognition of the fact that, if the government is indeed risk-neutral, setting RRT parameters so as to be neutral with respect to the decisions of risk-averse investors will not be an appropriate objective.

Appendix 2: RRT and the Incentive for 'Over-Exploration'

Expected Profit in the Absence of Tax

In the absence of tax, the present value of expected profit in the uniform distribution example is

$$\begin{aligned} \mathrm{E}(\pi_0) &= \frac{p}{1+r}\left[F(k)\tfrac{1}{2}(k + x_{\min}) + (1 - F(k))k\right] - ck \\ &= c\left(q\left[F(k)\tfrac{1}{2}(k + x_{\min}) + (1 - F(k))k\right] - k\right) \end{aligned} \tag{A1}$$

where $x_{\min}$ is the minimum possible value of x.

With the resource stock uniformly distributed in the 0-1 interval, $x_{\min} = 0$, $F(k) = k$, and $k = k^* = (q\text{-}1)/q$. In that case, (A1) gives:

$$\mathrm{E}(\pi_0{}^*) = c\left\{q\left(k^* - \tfrac{1}{2}k^{*2}\right) - k^*\right\} \tag{A2}$$

With the stock distributed in the 0-½ interval, $x_{\min} = 0$, $F(k) = 2k$, and $k = k^{\mathrm{L}} = ½k^*$. Expected profit would then be

$$\mathrm{E}(\pi_0^{\mathrm{L}}) = c\left\{q\left(\tfrac{1}{2}k^* - \tfrac{1}{4}k^{*2}\right) - \tfrac{1}{2}k^*\right\} \tag{A3}$$

With the stock distributed in the ½-1 interval, $x_{\min} = ½$, $F(k) = 2k\text{-}1$, and $k = k^{\mathrm{H}} = ½+½k^*$. Expected profit would then be

$$\mathrm{E}(\pi_0^{\mathrm{H}}) = c\left\{q\left(\tfrac{1}{2} + \tfrac{1}{2}k^* - \tfrac{1}{4}k^{*2}\right) - \left(\tfrac{1}{2} + \tfrac{1}{2}k^*\right)\right\} \tag{A4}$$

Starting with x uniformly distributed in 0-1, the availability of additional information which would reveal, with equal probability, that x was uniformly distributed in 0-½ or in ½-1 would increase expected profit, gross of the cost of acquiring the information, by the excess of the mean of (A3) and (A4) over (A2). This is

$$\Delta\mathrm{E}(\pi_0) = c\left\{q\left(\tfrac{1}{4} - \tfrac{1}{2}k^* + \tfrac{1}{4}k^{*2}\right) - \left(\tfrac{1}{4} - \tfrac{1}{2}k^*\right)\right\} \tag{A5}$$

which, since $k^* = (q\text{-}1)/q$, reduces to

$$\Delta\mathrm{E}(\pi_0) = c\tfrac{1}{4}\left((q-1)/q\right) = c\left(\tfrac{1}{4}k^*\right) \tag{A6}$$

In order for the firm to be indifferent about acquiring the additional information in the absence of tax, then, the cost of obtaining that information would need to be $C = \Delta E(\pi_0) = c(¼k^*)$.

Expected Profit with the RRT

With the resource uniformly distributed in 0-1, the neutrality requirement for the RRT, with q > 2, was given by (12) as

$$1+r^* \ = \ \frac{q}{q-1}(1+r) \ = \ \frac{(1+r)}{k^*}$$

while the level of output at which the firm's revenue would exactly equal its available tax deduction was given by (4) as

$$x_0 \ = \ \frac{c}{p}k^*(1+r^*) \qquad \text{which, by substitution, gives} \qquad x_0 \ = \ \frac{c}{p}(1+r) \ = \ \frac{1}{q}$$

With probability $F(x_0)$ (= x_0), output will be less than or equal to x_0 and the firm will pay no tax. Expected output in that case will be ½x_0. With probability $F(k^*)$-$F(x_0)$, output will lie between x_0 and k^*, with expected output being ½(x_0+k^*), and with probability 1-$F(k^*)$ output will be equal to k^*. In these 1-$F(x_0)$ of cases the firm will pay tax but will have a present value tax credit equal to $ptx_0/(1+r) = cqtx_0$. Thus, the present value of expected profit is

$$\begin{aligned} E(\pi_T^*) \ &= \ cq\left\{F(x_0)\tfrac{1}{2}x_0 + (1-t)\left\langle\left(F(k^*) - F(x_0)\right)\tfrac{1}{2}(k^*+x_0) + \left(1-F(k^*)\right)k^*\right\rangle + t\left(1-F(x_0)\right)x_0\right\} - ck^* \\ &= \ c\left\{q(1-t)\left(k^* - \tfrac{1}{2}k^{*2}\right) + qt\left(x_0 - \tfrac{1}{2}x_0^{\,2}\right) - k^*\right\} \end{aligned} \qquad \text{(A7)}$$

This can be compared with the expected after-tax profit that the firm would have with a Pure Rent Tax levied at the same rate, t. With $x_0 = 1/q$ and $k^* = (q\text{-}1)/q$, (A7) can be rewritten as

$$\begin{aligned} E(\pi_T^*) \ &= \ c\left\{ q(1-t)\left(k^* - \tfrac{1}{2}k^{*2}\right) + t\left(q - \frac{\tfrac{1}{2}}{q}\right) - k^*\right\} \\ &= \ c\left\{ q(1-t)\left(k^* - \tfrac{1}{2}k^{*2}\right) - k^*(1-t) + \frac{\tfrac{1}{2}t}{q}\right\} \\ &= \ E(\pi_0^*)(1-t) \ + \ \frac{\tfrac{1}{2}ct}{q} \end{aligned} \qquad \text{(A8)}$$

The first term in the final line of (A8) is expected after-tax profit under the PRT, so the second term represents the extent to which expected RRT revenue falls below that generated by the PRT. As noted in the text, the difference between the expected tax revenues decreases with q because the excess of r^* over r decreases with q.

Now suppose that, with the same RRT parameters as above, the firm has the opportunity to discover whether x is uniformly distributed in the 0-½ interval or in the ½-1 interval, incurring the cost $C = c(¼k^*)$ to obtain this additional information. The level of capacity investment undertaken in the 'high' and 'low' states is assumed to be the same as in the absence of tax, so incentive effects of the RRT on that investment decision are ignored.

If the firm finds itself in the 'low' state, where $F(x) = 2x$, it will invest the amount $k^L = ½k^*$ and will have a total tax deduction, including the exploration cost, of $c¾k^*(1+r^*)$. The level of output at which revenue would exactly match this deduction is $x_0^L = ¾x_0$. Making the appropriate substitutions in the first line of (A7) and simplifying yields expected after-tax profit, gross of the exploration cost, as

$$\begin{aligned} E(\pi_T^L) &= c\left\{q(1-t)\left(\tfrac{1}{2}k^* - \tfrac{1}{4}k^{*2}\right) + qt\tfrac{3}{4}\left(x_0 - \tfrac{3}{4}x_0^{\,2}\right) - \tfrac{1}{2}k^*\right\} \\ &\text{s.t:}\quad \tfrac{3}{4}\left(x_0 - \tfrac{3}{4}x_0^{\,2}\right) \le \left(\tfrac{1}{2}k^* - \tfrac{1}{4}k^{*2}\right) \end{aligned} \qquad \text{(A9)}$$

where the condition that the expected tax credit base cannot exceed maximum possible output is satisfied for values of $q \geq 2.5$.

If the firm finds itself in the 'high' state, where $F(x) = 2x\text{-}1$, it will invest the amount $k^H = ½+½k^*$ and will have a total tax deduction of $c(½+¾k^*)(1+r^*)$. The level of output at which its revenue would exactly match this deduction is

$$x_0^H = ¾x_0 + ½/(q\text{-}1) = ¾/q + ½/(q\text{-}1)$$

For values of $q \geq 3$, $x_0^H \leq ½$ so the firm is guaranteed to earn revenue which exceeds its tax deduction: that is $F(x_0^H) = 0$. Then, with probability $F(k^H)$ output will lie between ½ and k^H, with expected output equal to ½(½+k^H) and, with probability 1-$F(k^H)$, output will be equal to k^H. The firm will pay tax in all cases, but will have a guaranteed tax credit of present value amount equal to qtx_0^H. Expected after-tax profit, gross of exploration cost, will be given by

$$\mathrm{E}(\pi_T^H) \;=\; c\left\{ q(1-t)\left(\tfrac{1}{2}+\tfrac{1}{2}k^*-\tfrac{1}{4}k^{*2}\right) + qt\left(\tfrac{3}{4}x_0 + \frac{\tfrac{1}{2}}{q-1}\right) - \left(\tfrac{1}{2}+\tfrac{1}{2}k^*\right)\right\} \tag{A10}$$

For $q \geq 3$, the increase in *gross* expected after-tax profit resulting from the opportunity to undertake the exploration is given by the excess of the mean of (A9) and (A10) over (A7) as

$$\Delta\mathrm{E}(\pi_T) \;=\; c\left\{ q(1-t)\tfrac{1}{4}\left(1-2k^*+k^{*2}\right) + qt\tfrac{1}{4}\left(\frac{1}{q-1} - \tfrac{7}{8}x_0{}^2 - x_0\right) + \tfrac{1}{4}(2k^*-1)\right\}$$

With $k^* = (q\text{-}1)/q$ and $x_0 = 1/q$, this gives

$$\begin{aligned}\Delta\mathrm{E}(\pi_T) \;&=\; c\left\{ (1-t)\tfrac{1}{4}\frac{1}{q} + t\tfrac{1}{4}\left(\frac{q}{q-1} + \tfrac{7}{8}\frac{1}{q} - 1\right) - \tfrac{1}{4}\left(1-\frac{2}{q}\right)\right\} \\ &=\; \tfrac{1}{4}c\left\{ t\left(\frac{1}{q-1} - \frac{\tfrac{1}{8}}{q}\right) + \frac{q-1}{q}\right\}\end{aligned} \tag{A11}$$

Dividing through by the exploration cost, $C = c(¼k^*) = ¼c(q\text{-}1)/q$, gives the increase in gross expected profit as a proportion of C as

$$1 \;+\; t\left(\frac{q}{(q-1)^2} - \frac{\tfrac{1}{8}}{q-1}\right) \tag{A12}$$

where the second term is positive but decreasing with q. For example, with $q = 3$ and $t = 0.5$, the exploration that would be marginal in the absence of tax provides an increase in gross expected after-tax profit that is 1.34 times the exploration cost under the RRT.

Matters are more complicated when $2 < q < 3$. Since $x_0^H > ½$ in these cases, the firm is not guaranteed to be able to use its full tax deduction in the 'high' state, so (A10) overstates expected profit for values of q in this range. Taking account of this qualification generates unwieldy expressions, but the overall effect is that (A12) overstates the proportionate 'subsidy' to exploration by an increasingly large amount as the value of q decreases towards 2. At that point, the ‘subsidy’ is zero since the firm would have no tax liability in the absence of the exploration and continues to have no tax liability in either state if the exploration is undertaken. The result that the ‘subsidy’ becomes positive as soon as q exceeds 2 can be seen by noting that $x_0^L \geq k^L$ with $q \leq 2.5$ and $x_0^H \geq k^H$ with $q \leq 2.175$, so that the firm will pay no tax in either the ‘high’ or ‘low’ state with values of q very close to 2 and, therefore, the (small) expected tax liability that the firm would have in the absence of the exploration is eliminated.